

Twisting Signals for Joint Radar-Communications

Kumar Vijay Mishra



ELLIIT Focus Period Symposium

Linköping University, Sweden

23 April 2026





Could Nobel-Winning Laser Tech Make Sci-Fi 'Tractor Beams' a Reality?

2018

BBC

Star Trek style 'tractor beam' created by scientists

2013

LIVESCIENCE

Tractor beams inspired by sci-fi are real, and could solve the looming space junk problem

2025

The tight-beam backscatter we picked up

Courtesy: The Expanse (Season 5)

was probably a communication with Marco.

- NJTech: Wanhua Lv
- ARL/UT Austin: Brian M. Sadler
- UIS Colombia: Henry Arguello, Edwin Vargas, Roman Jacome, Jonathan Arley
- Rice: Ashutosh Sabharwal, West Chen, Nate Raymondi, Kris Li
- IIT Delhi: Arpan Chattopadhyay, Himali Singh

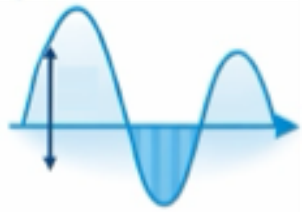


The National Academies of

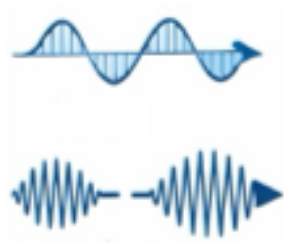
SCIENCES
ENGINEERING
MEDICINE

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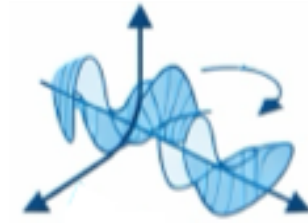
Characteristics of Electromagnetics Waves



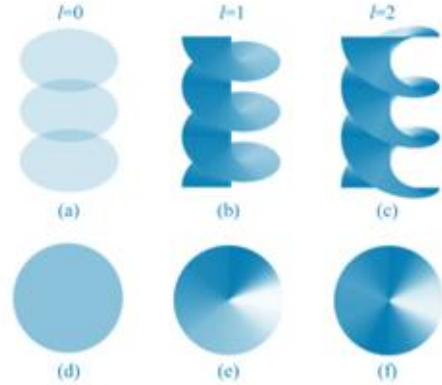
Amplitude



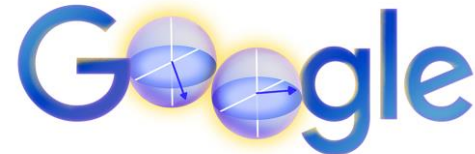
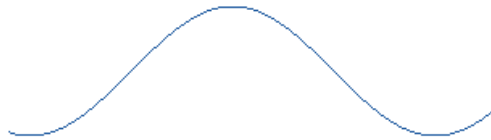
Frequency



Polarization
(Spin Angular
Momentum)



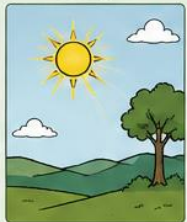
Orbital Angular Momentum



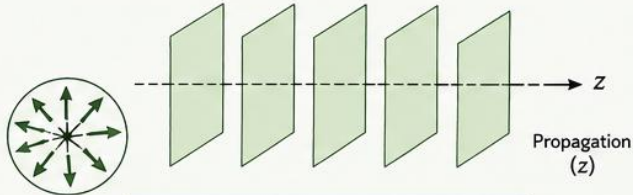
SAM is independent of OAM

1 UNPOLARIZED-UNTWISTED (No SAM, No OAM)

EXAMPLE:
Sunlight



Sunlight is randomly polarized and has planar wavefronts.



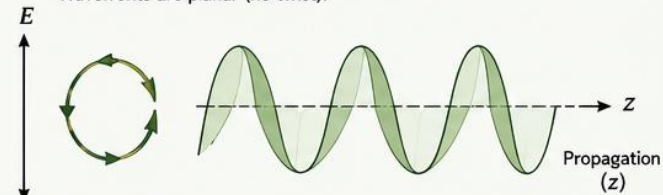
Random polarization (no preferred direction).
Wavefronts are planar (no twist).

2 POLARIZED-UNTWISTED (SAM $\neq 0$, OAM = 0)

EXAMPLE:
GPS signal
(Right-hand circularly polarized)



Electric field rotates in time (circular polarization).
Wavefronts are planar (no twist).



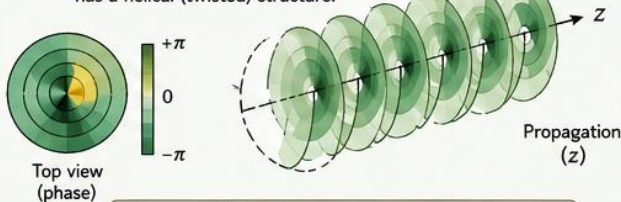
Circular polarization (rotating electric field).
Carries spin angular momentum (SAM $\neq 0$), but no OAM.

3 UNPOLARIZED-TWISTED (OAM) (No SAM, OAM $\neq 0$)

EXAMPLE:
Microwave beam
with OAM
(unpolarized)



No preferred polarization, but the phase has a helical (twisted) structure.



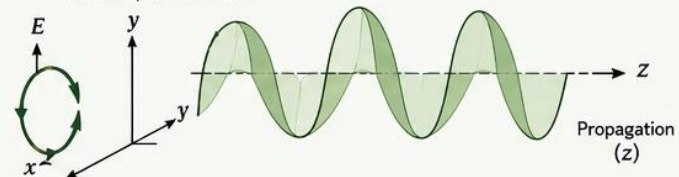
Phase $\propto e^{jl\phi}$ (helical). No fixed polarization.
Carries orbital angular momentum (mode index $l \neq 0$).

4 POLARIZED-TWISTED (OAM + SAM) (SAM $\neq 0$, OAM $\neq 0$)

EXAMPLE:
Circularly polarized
vortex beam
(optical OAM mode)



Electric field is circularly polarized and the phase is helical.



Circular polarization (SAM $\neq 0$) and helical phase (OAM $\neq 0$).
Carries both spin (SAM) and orbital (OAM) angular momentum.

OAM History



Foundations in physics

Angular momentum in quantum systems



Discovery of optical vortices

Phase singularities and structured light



Applications in communications and sensing

From free-space links to radar systems



Quantum structured light

OAM in quantum states and experiments



Emerging frontiers

From visualization to biomedical impact

1930s

Conceptual roots in atomic physics and the quantization of angular momentum.

1970s–80s

Optical vortices and phase singularities (Nye and Berry).

1992 **Physical Review A**

Les Allen et al. show that OAM is independent of spin angular momentum (SAM).

2014

Twisted light sends images across Vienna.

nature

2016

Physicists twist light, send 'Hello World' message between islands.

SCIENTIFIC AMERICAN

2018

NTT demonstrates 100 Gbps wireless transmission using OAM multiplexing for the first time.

MicroWave Journal

2018

Nobel Prize in Physics — optical tweezers and optical manipulation.

2022

Nobel Prize in Physics — OAM states of photons used in quantum experiments.

2023

Nobel Prize in Physics — attosecond pulses for ultrafast probing of OAM physics.

2024

OAM-based optical techniques poised to transform medical diagnostics and imaging.

PHYS ORG

1930

1970

1990

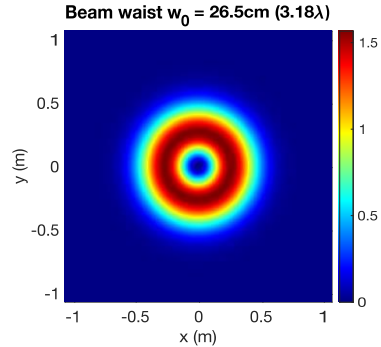
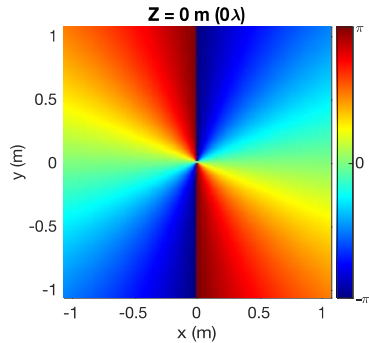
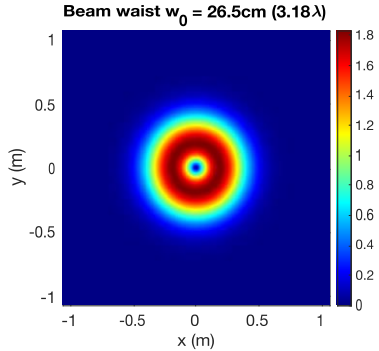
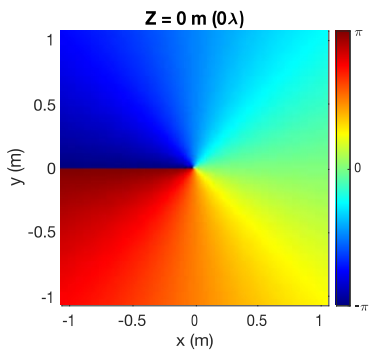
2010

2020

2024+

Historical Notes

Different OAM modes are mutually orthogonal \longrightarrow New way to achieve orthogonality



$$\int_0^{\infty} \int_0^{2\pi} LG(r, \phi, z; l_1, p_1) LG^*(r, \phi, z; l_2, p_2) d\phi dr = \begin{cases} 0 & \text{if } l_1 \neq l_2 \text{ or } p_1 \neq p_2 \\ 1 & \text{if } l_1 = l_2 \text{ and } p_1 = p_2 \end{cases}$$

Outline

Motivation & Background

OAM-Aided Automotive ISAC

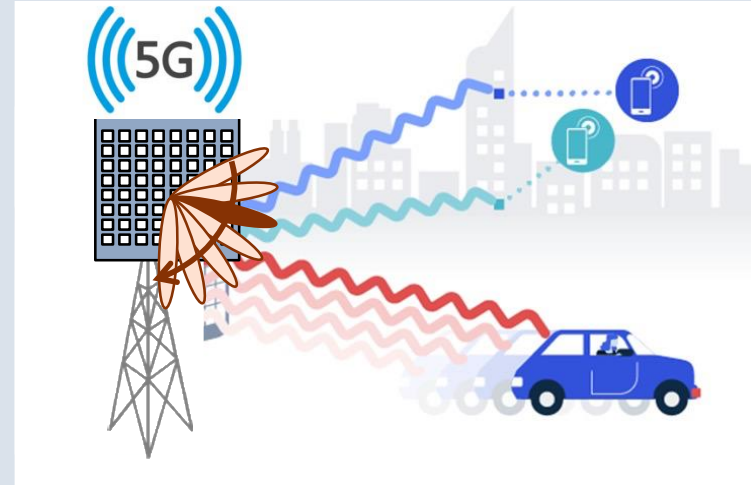
Co-Pulsing FDA Radar

Spectrum and EW

Courtesy: Stranger Things (S05E01)



Laguerre-Gaussian Beams



Which Beam to Use?

◆ Laguerre–Gaussian (LG) Vortex Beam: Easy to generate via metasurfaces

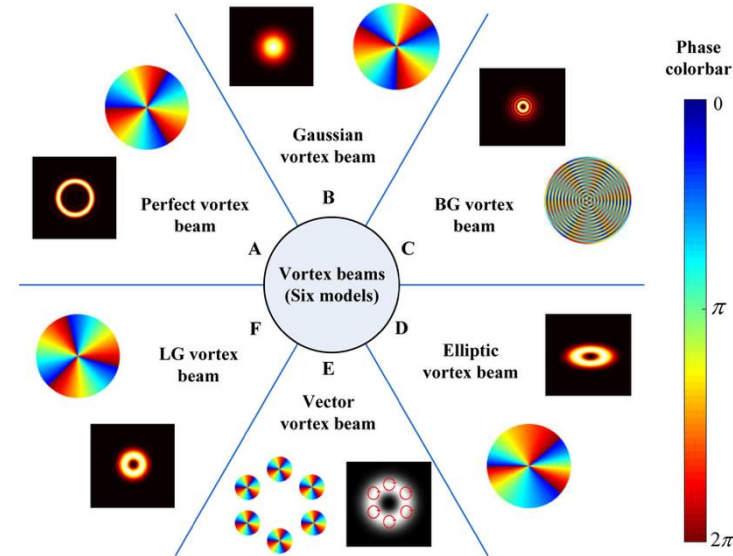
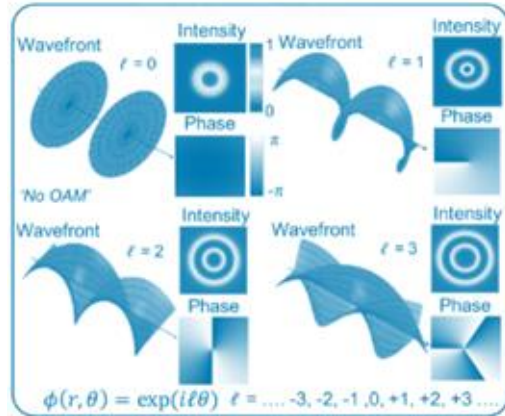
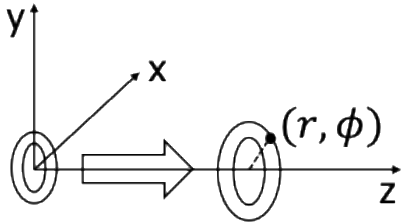
★ Beam waist w_0 (determines $w(z)$): no clear guideline

Describes amplitude

$$E_{(l,p)}(r, z, \phi) = \sqrt{\frac{2p!}{\pi(p+|l|)!}} \frac{1}{w(z)} \left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|} \exp\left(\frac{-r^2}{w^2(z)}\right) L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)$$

$$\exp\left(\frac{-ik_0 r^2 z}{2(z^2 + z_R^2)}\right) \exp\left(-il\phi + i(2p + |l| + 1) \arctan\left(\frac{z}{z_R}\right)\right)$$

Describes phase



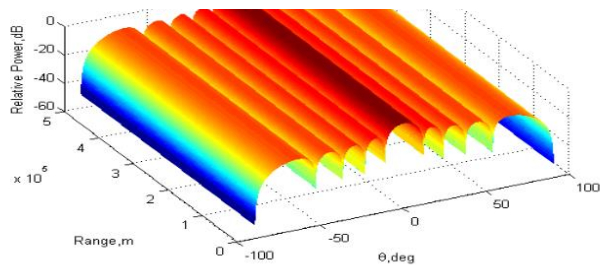
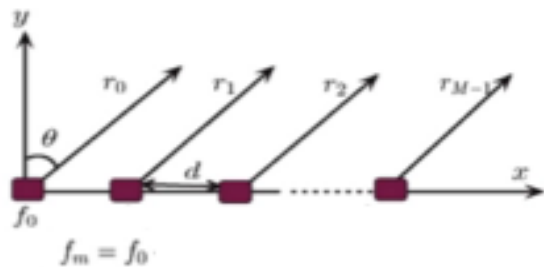
OAM Modes for Different p and l

Azimuthal index l	Radial index $p \rightarrow$	$p = 0$	$p = 1$	$p = 2$	$p = 3$
	$l = 0$	Intensity			
Phase		constant ($l = 0$)	constant ($l = 0$)	constant ($l = 0$)	constant ($l = 0$)
$l = 1$	Intensity				
	Phase				
$l = -1$	Intensity				
	helical (-1)				
$l = 2$	Intensity				
	helical (+2)				

L-G Beams may not be Ideal!

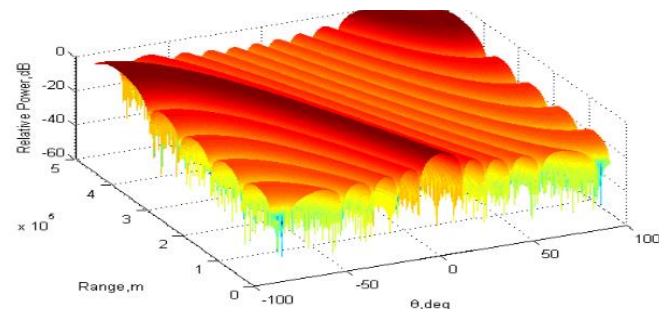
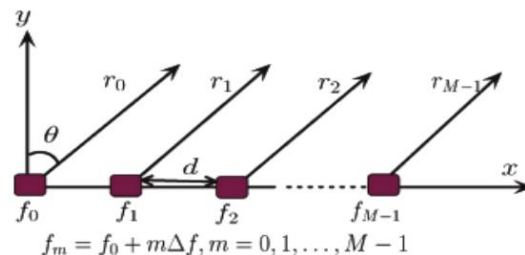
- ◆ L-G Beams are susceptible to turbulence and interference
- ◆ OAM modes encode azimuth but no range information
- ◆ Solution: Use Frequency-Diverse Arrays (FDAs)

Phased Array (PA)



Angle-Dependent Beampattern

Frequency Diverse Array (FDA)



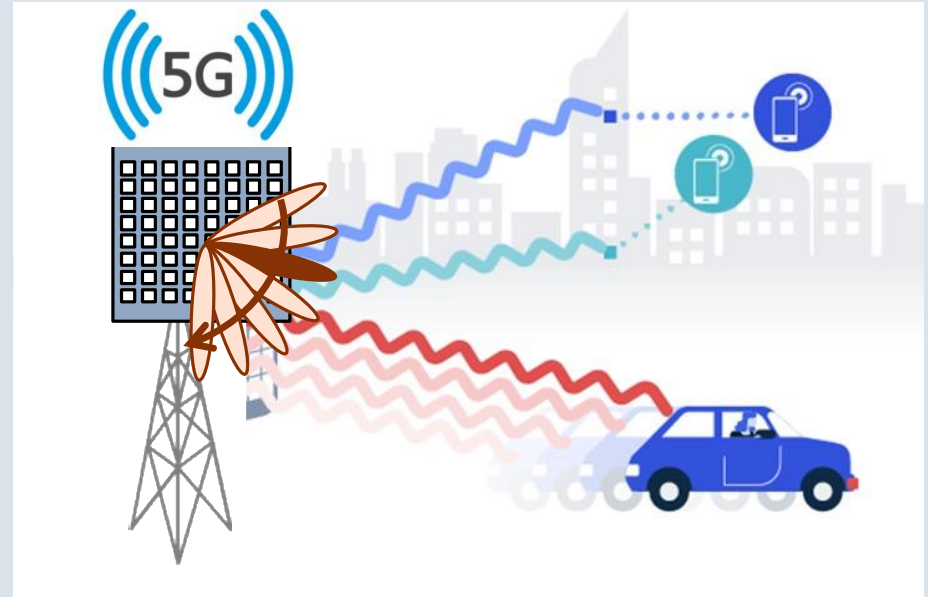
Range-Angle-Dependent Beampattern

... but in **classical** far-field!

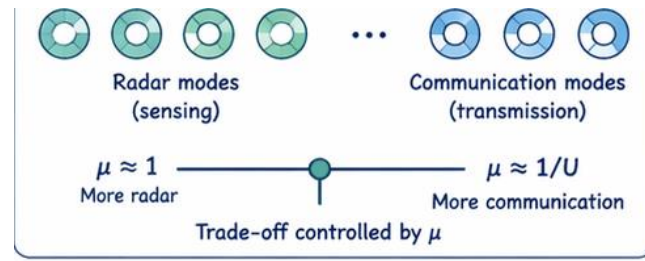
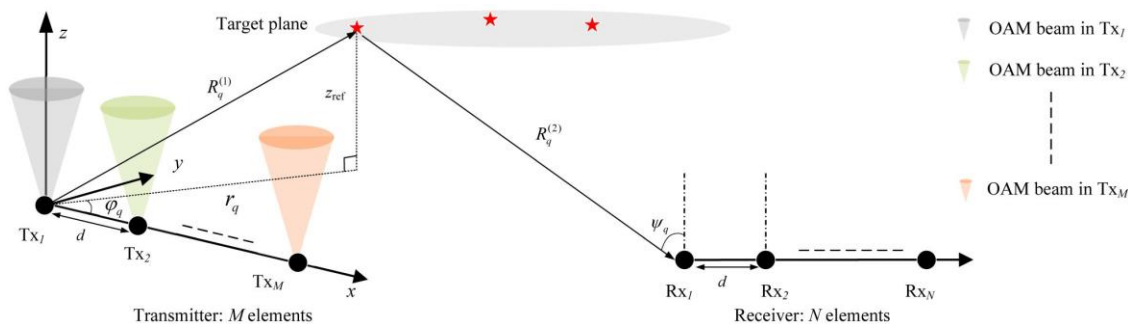
Courtesy: Species II (1998)



Signal Processing



Bistatic OAM-FDA ISAC



1. TRANSMIT SIGNAL

FDA-Enhanced Transmitting Vehicle

Uniform Linear Array with M elements (FDA)

$f_m = f_0 - m\Delta f, m = 0, 1, \dots, M-1$

OAM Modes (ℓ) (Laguerre-Gaussian Beams)

$\ell_{-L}, \dots, \ell_0, \dots, \ell_{+L}$

Data Symbols (Mapped to OAM Modes)

$a_{-L}, \dots, a_0, \dots, a_{+L}$

$U = 2L + 1$ modes

Transmit signal from m -th antenna for mode ℓ_u :

$$s_{\ell_u, m}(t) = a_u e^{j2\pi f_m t}, \quad m = 0, 1, \dots, M-1, \quad u = -L, \dots, L$$

2. CHANNEL MODEL

Transmitting Vehicle

Target q ($R_q, \phi_q, \psi_q, r_q, v_q$)

Receiving Vehicle (V2V)

Uniform Linear Array with N elements

Physical Meaning

- R_q : Range
- ϕ_q : Azimuth (DoD)
- ψ_q : Azimuth (DoA)
- r_q : Radial distance
- v_q : Radial velocity

--- Direct path

--- Received path

Baseband signal at receiver for mode ℓ_u :

$$x_{\ell_u}(t) = \sum_{q=1}^Q \underbrace{\tilde{\rho}_q(t)}_{\text{Propagation gain}} \underbrace{a_u}_{\text{OAM-dependent phase}} e^{j\Phi_{\ell_u q}} \underbrace{\mathbf{b}(R_q)}_{\text{Range steering (FDA)}} \otimes \underbrace{\mathbf{a}_R(\psi_q)}_{\text{Angle steering (ULA)}} + \underbrace{\mathbf{n}(t)}_{\text{Noise}}$$

OAM-dependent phase

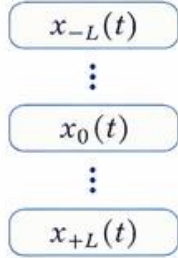
$$\Phi_{\ell_u q} = -\pi \frac{r_q^2 - r_{\max}^2}{\lambda_0 R_{\ell}} - \ell_u \phi_q$$

Range and Angle Estimation

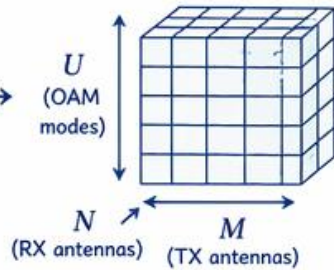
3. RECEIVED SIGNAL (Across OAM Modes)

Stack signals of all modes

$$u = -L, \dots, L$$



Data Tensor Construction
(M transmit \times N receive \times U modes)



Tensor Form

$$\mathcal{X} = \llbracket \mathbf{B}, \mathbf{A}_R, \mathbf{A}_U \rrbracket + \mathcal{N}$$

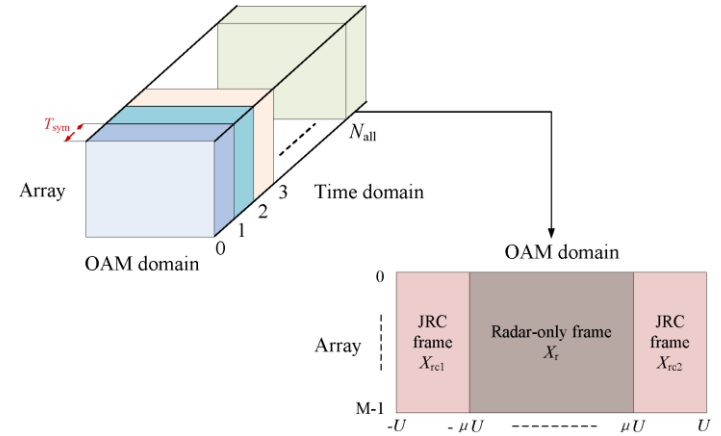
where

$\mathbf{B} \in \mathbb{C}^{M \times Q}$: Range steering (FDA)

$\mathbf{A}_R \in \mathbb{C}^{N \times Q}$: Angle steering (ULA)

$\mathbf{A}_U \in \mathbb{C}^{U \times Q}$: OAM steering

\mathcal{N} : Noise tensor



Range-Azimuth-DoA estimation:

- FDA structure allows auto-pairing of bistatic range and angles

Doppler and Comms Estimation

■ Doppler estimation: Take the derivative of phase and convert the angular frequency to Doppler velocity

■ Use MRC to

1 Build slow-time data cube

Collect M pulses: $y_m(t)$, $m = 0, \dots, M-1$

After steps in III.A, obtain OAM–range–angle bin for each target (R_k, ψ_k, ϕ_k) .

Form slow-time sequence:

$$z_k[m] = Y_{l_k}(R_k, \psi_k, \phi_k; m), \quad m = 0, \dots, M-1$$

2 Doppler FFT

$$Z_k[\nu] = \sum_{m=0}^{M-1} z_k[m] e^{-j2\pi\nu m/M}$$

$$\text{Peak at } \nu_k \Rightarrow v_k = \frac{\lambda \nu_k}{2}$$

3 Rotational velocity (from OAM phase evolution)

The phase of $z_k[m]$ contains a rotational Doppler term due to OAM:

$$z_k[m] \approx A_k e^{j(2\pi f_{D,k} T_r m + l_k \omega_k T_r m + \varphi_{0,k})}$$

Unwrap phase $\angle z_k[m] = \beta_k m + \varphi_{0,k}$

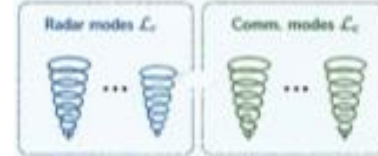
$$\Rightarrow \omega_k = \frac{\beta_k - 2\pi f_{D,k} T_r}{l_k T_r}$$

- T_r : pulse repetition interval
- $f_{D,k} = \frac{2v_k}{\lambda}$
- ω_k : rotational velocity (rad/s)

1 OAM-based mode-division multiplexing (MDM)

Allocate radar modes \mathcal{L}_r and communication modes \mathcal{L}_c ($\mathcal{L}_r \cap \mathcal{L}_c = \emptyset$).

At each range–angle bin, use communication modes $l \in \mathcal{L}_c$.



2 Extract communication signals

For each $l \in \mathcal{L}_c$:

$$Y_l^{(c)}(m) = Y_l(R_k, \psi_k; m)$$

(after removing radar contribution on \mathcal{L}_r)

3 Symbol demodulation

Assume symbols $c_i[p]$ are embedded on mode l .

$$Y_l^{(c)}(m) = \gamma_l \sum_p c_i[p] g(mT_r - pT_s) + n_l(m)$$

Matched filter:

$$r_l[p] = \sum_m Y_l^{(c)}(m) g^*(mT_r - pT_s)$$

Decision:

$$\hat{c}_i[p] = \arg \min_{a \in \mathcal{A}} |r_l[p] - \gamma_l a|$$

\mathcal{A} : modulation alphabet
(e.g., QPSK, 16-QAM)

Output: recovered symbols $\{\hat{c}_i[p], l \in \mathcal{L}_c\}$

Courtesy: The Midnight Sky (2020)

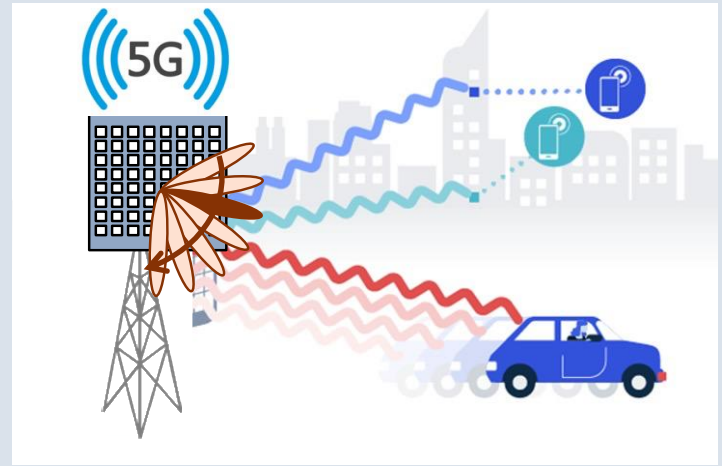


Radar is still up, and...



*-[Ade] What is it?
-[Maya] I can't pull it up on radar.*

Performance Guarantees



Provable Guarantees using Kruskal Rank

Theorem [Lv, Mishra, Hu 2026]

Consider a bistatic FDA system with an $M(N)$ -element transmit (receive) ULA and $d, \Delta f$ be the interelement spatial spacing and frequency increment, respectively. Each transmit element sends out a total of $2U+1$ OAM states with the inter-state spacing δ . Assume that there are Q scatterers with R_{\max} being the maximum bi-static range and the JRC sharing factor is denoted as μ . If

$$\mathbf{C1} \quad d \leq \frac{c}{2f_0},$$

$$\mathbf{C2} \quad \Delta f \leq \frac{c}{R_{\max}},$$

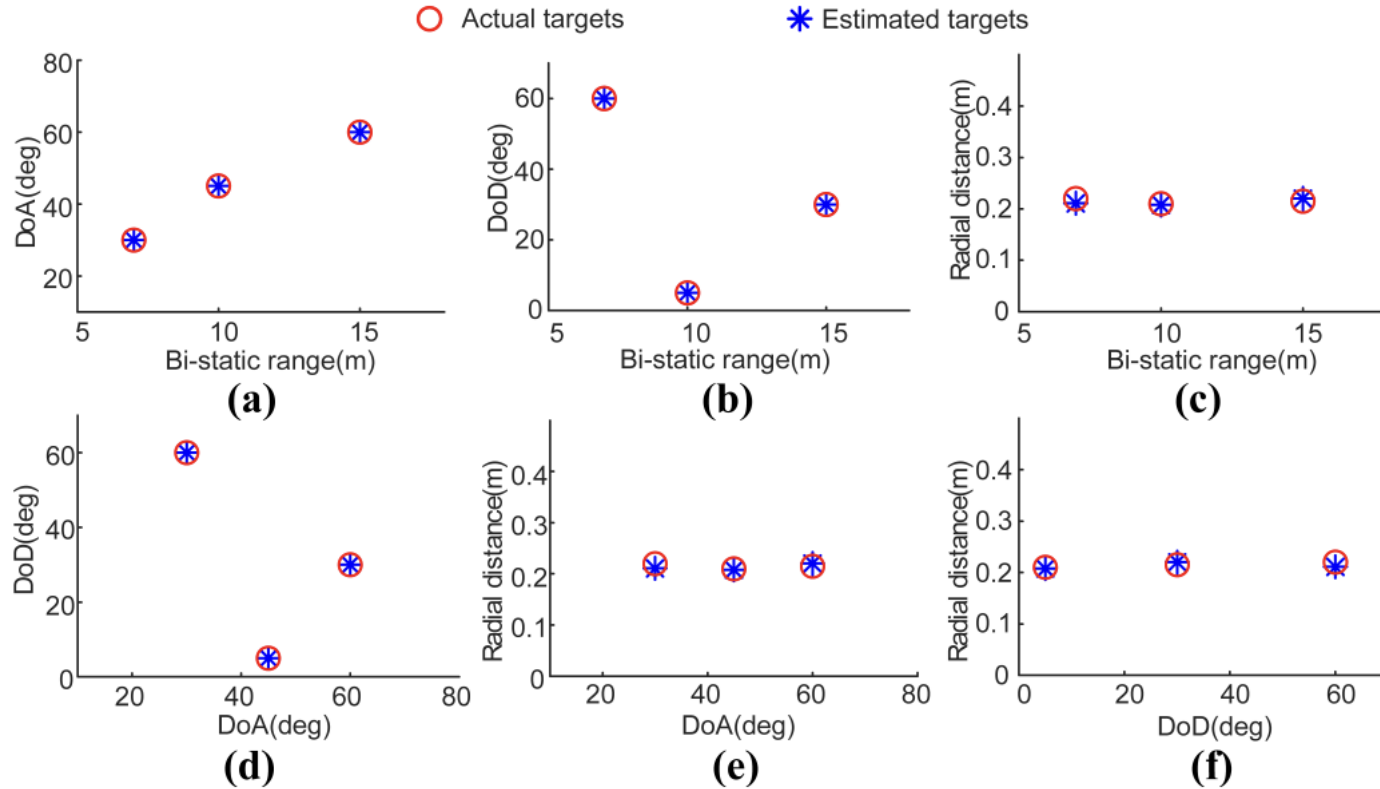
$$\mathbf{C3} \quad N > Q,$$

$$\mathbf{C4} \quad M > Q,$$

$$\mathbf{C5} \quad \frac{1}{U} \leq \mu \leq 1,$$

then the unknown parameters of Q scatterers and communication symbols a_u can be perfectly recovered from the multiplexing covariance matrix \mathbf{R} .

Detection Performance



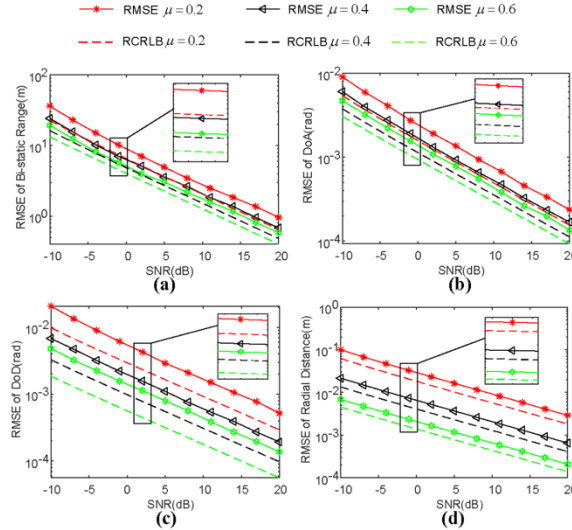
Estimation Performance

CRLBs

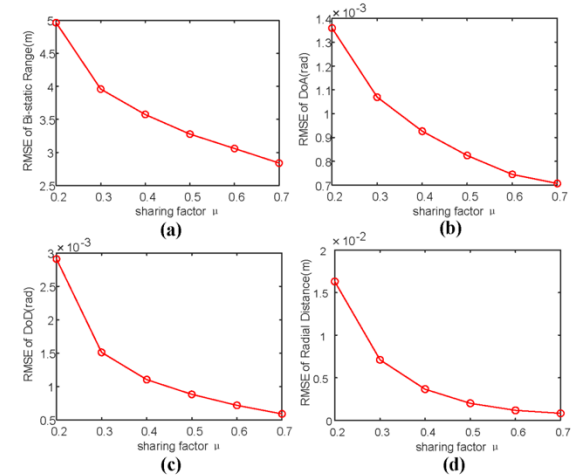
Parameter	CRLB (approx.)
Range R_k	$\text{var}(\hat{R}_k) \geq \frac{c^2}{8\pi^2 \text{SNR} \beta_2}$
DoA ψ_k	$\text{var}(\hat{\psi}_k) \geq \frac{6 \xi}{\text{SNR} N(N^2 - 1)(2\pi \Delta f \tau_k)^2}$
Azimuth ϕ_k (OAM)	$\text{var}(\hat{\phi}_k) \geq \frac{1}{\text{SNR} \sum_{l \in \mathcal{L}_r} l^2}$
Velocity v_k	$\text{var}(\hat{v}_k) \geq \frac{\lambda^2}{8\pi^2 \text{SNR} T_r^2 \sum_{m=1}^M (m - \bar{m})^2}$
Rotational ω_k	$\text{var}(\hat{\omega}_k) \geq \frac{1}{\text{SNR} l_k^2 T_r^2 \sum_{m=1}^M (m - \bar{m})^2}$
Complex amp. α_k	$\text{var}(\hat{\alpha}_k) \geq \frac{\sigma^2}{2MN}$

β_2 : effective bandwidth factor of $g(t)$, τ_k : delay of target k .

RMSE Compared with CRLB



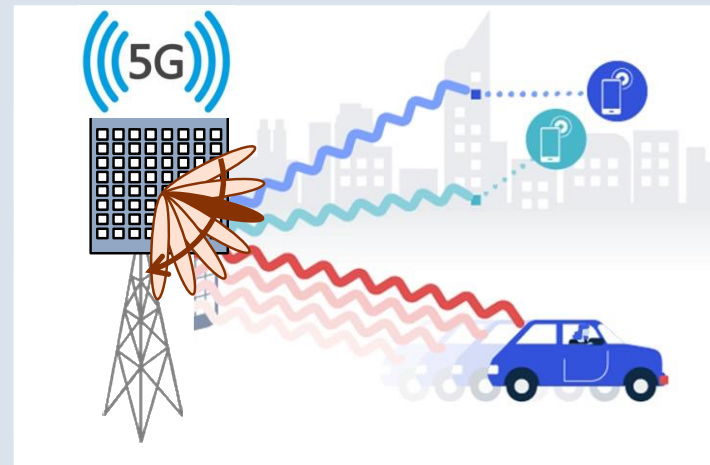
Target RMSE for Various μ



Courtesy: Foundation (S01E03)



Co-Pulsing FDA Radar



FDA Consumes More Spectrum

Introduction of FOs → Additional Bandwidth

Limited work on Doppler estimation

Solutions:

- Sparse FDA – Fewer elements and FOs
- FOs can be optimized based on available bandwidth
 - Logarithmic [Khan et al., 2014]
 - Non-uniform [Basit et al., 2017]
 - Random [Liu et al., 2016]
 - Co-prime FDA [Qin et al., 2017]
 - Integers that share no common factors except 1: (2,3), (4,9), (6, 10, 15)
 - FDA-MIMO [Sedighi et al., 2019]

Our Approach:

- Fewer FOs → Less bandwidth and more EW tolerance
- Employ FDA to jointly extract range-angle parameters

Co-Pulsing FDA Radar

Array ^a	FO	Spectrum ^b	Beampattern ^c	PRI	Antennas ^d	Pulses ^e
L-shaped ULA (U-U) [35]	None	B	Angle	Uniform	$Q + 1$	$Q + 1$
L-shaped Co-prime array (C-U) [36]	None	B	Angle	Uniform	$2\sqrt{Q+1} - 1$	$Q + 1$
L-shaped ULA (U-C)	None	B	Angle	Co-prime	$Q + 1$	$2\sqrt{Q+1} - 1$
L-shaped Co-prime array (C-C)	None	B	Angle	Co-prime	$2\sqrt{Q+1} - 1$	$2\sqrt{Q+1} - 1$
L-shaped FDA (U-Cube)	Linear	$B + (P_s - 1)\Delta f$	Angle-range	Uniform	$Q + 1$	$Q + 1$
L-shaped FDA (UUC)	Linear	$B + (P_s - 1)\Delta f$	Angle-range	Co-prime	$Q + 1$	$2\sqrt{Q+1} - 1$
L-shaped FDA (UCU)	Co-prime	$B + \xi_{P_s-1}\Delta f$	Angle-range	Uniform	$Q + 1$	$Q + 1$
L-shaped FDA (UCC)	Co-prime	$B + \xi_{P_s-1}\Delta f$	Angle-range	Co-prime	$Q + 1$	$2\sqrt{Q+1} - 1$
L-shaped Co-prime FDA (CUU)	Linear	$B + (P_s - 1)\Delta f$	Angle-range	Uniform	$Q + 1$	$Q + 1$
L-shaped Co-prime FDA (CUC)	Linear	$B + (P_s - 1)\Delta f$	Angle-range	Co-prime	$Q + 1$	$2\sqrt{Q+1} - 1$
L-shaped Co-prime FDA (CCU)	Co-prime	$B + \xi_{P_s-1}\Delta f$	Angle-range	Uniform	$2\sqrt{Q+1} - 1$	$Q + 1$
L-shaped Co-prime FDA (C-Cube)	Co-prime	$B + \xi_{P_s-1}\Delta f$	Angle-range	Co-prime	$2\sqrt{Q+1} - 1$	$2\sqrt{Q+1} - 1$

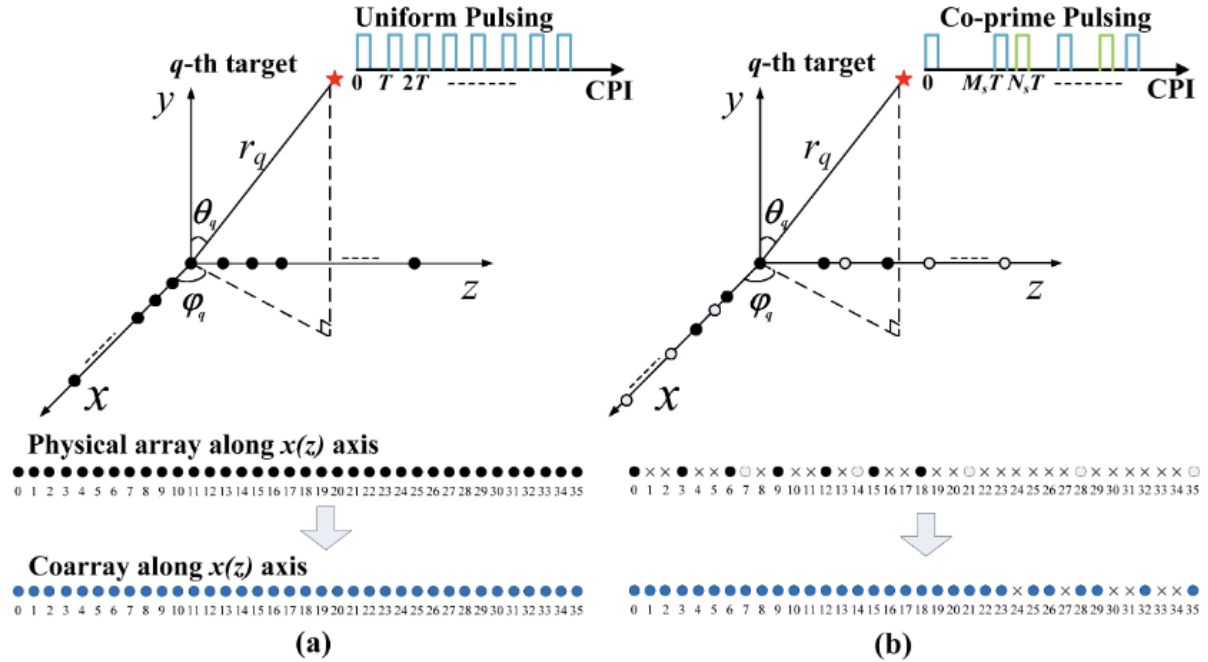
L-shaped array:

- Simpler 2-D FDA structure to perform co-prime sampling in spatial, spectral, and Doppler domains.
- L-shaped previously investigated for ULA [Kishigami et al, 2016] and co-prime array [Elbir, 2020]

Co-Pulsing FDA Radar

CCC Configuration

- Co-Prime Array in L-Shaped Configuration
- Co-Prime Offsets
- Co-Prime Pulsing



- Co-Pulsing imparts additional DoFs to sparse FDA design
- Extraction of Doppler is coupled with other parameters (second-order statistics)

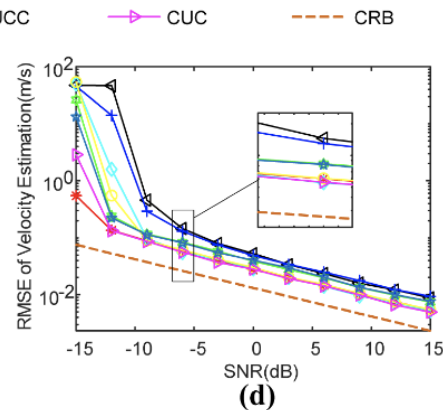
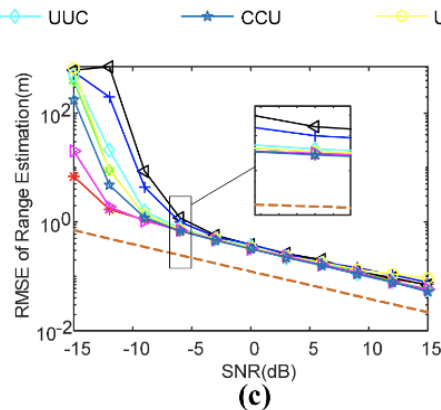
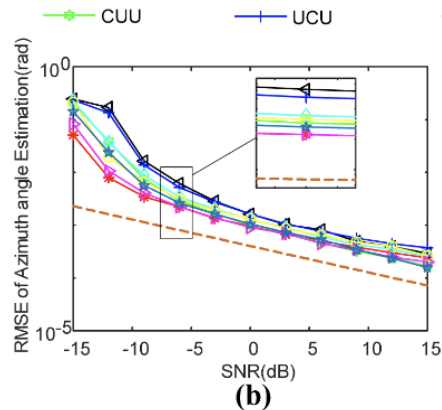
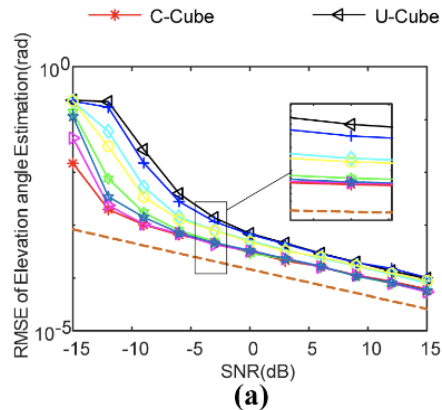
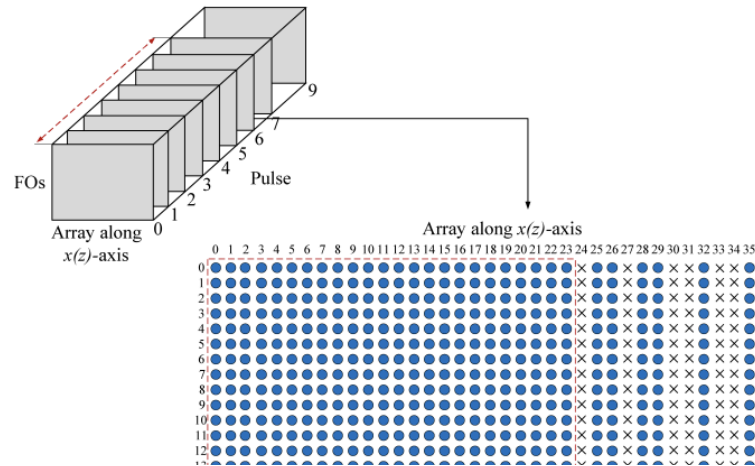
C-Cube Auto-Pairing (CCing)

Ipsso-Facto Joint Estimation

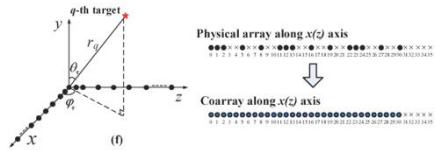
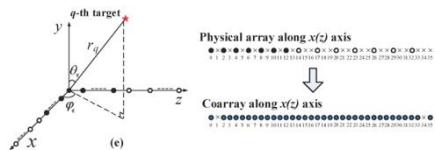
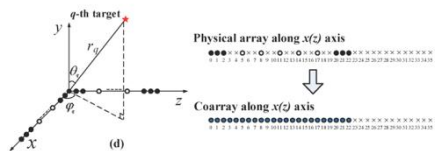
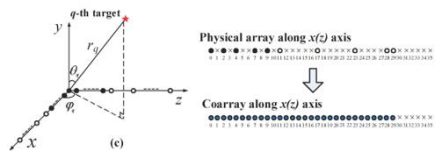
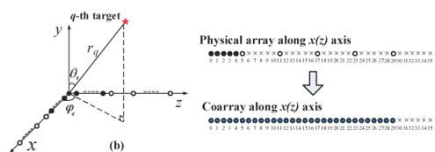
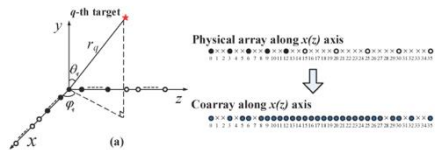
- Parameter retrieval algorithm employs SVD of the concatenated covariance matrix:
 - Elevation and azimuth embodied in left EVs
 - Range and Doppler embodied in right EVs
 - Auto-pairing property between left and right eigenvectors allows for automatic pairing of 2-D DoA with range and Doppler velocity
 - Physical reason: L-shaped + FDA structure (range-angle beampattern)
 - Kruskal rank properties are used to guarantee the recovery conditions
- Difficult to generalize tensor methods for L-shaped arrays; no auto-pairing
 - CCing also works when some target parameters are the same (at the cost of some DoFs)

Co-Pulsing Radar

Range-dependent beampattern: reduce number of pulses, spectrum usage, and antenna elements simultaneously



Relationship with Other Arrays



Array

\mathcal{L}_H^a

L-shaped uniform FDA (U-Cube)

0.76

L-shaped CADiS FDA

0.4302

L-shaped nested FDA

0.6303

L-shaped super-nested FDA

0.5342

L-shaped CNA FDA

0.5859

L-shaped GNA FDA

0.5969

L-shaped multi-coset FDA

0.5985

L-shaped Co-prime FDA (C-Cube)

0.5340

^a Computed for common physical aperture of $35d$ along each axis.

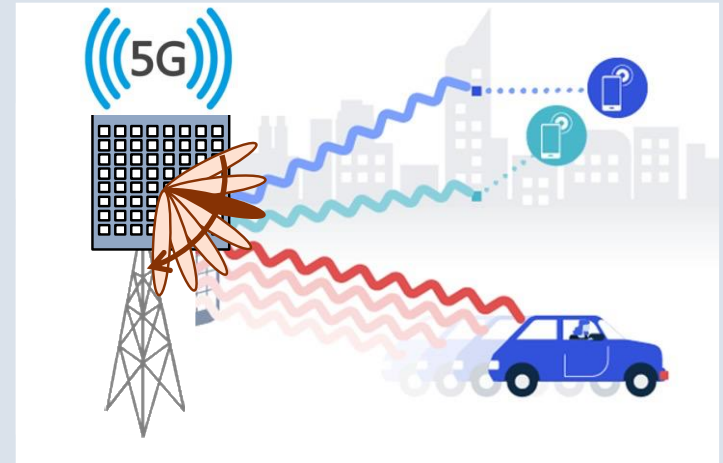
C-Cube offers a trade-off between high DoFs and mild target coupling

A fundamental array structure from which all others could be derived

Courtesy: Independence Day (1996)

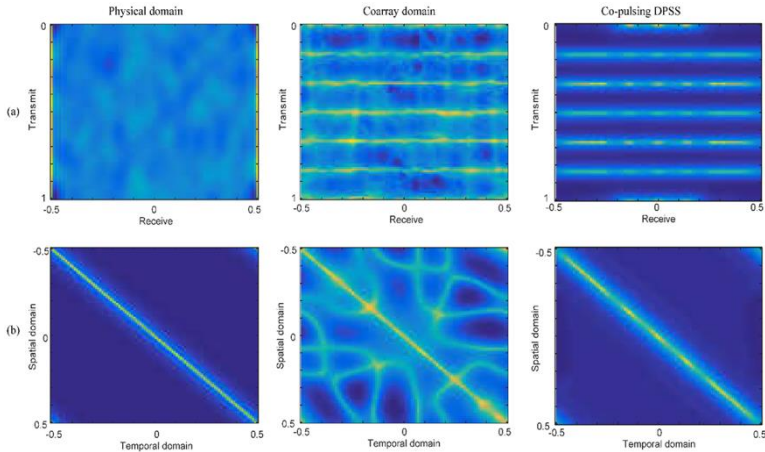
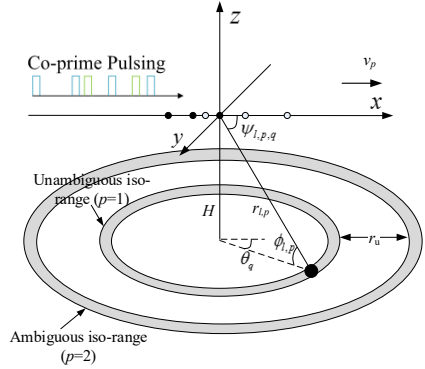


Generalizations

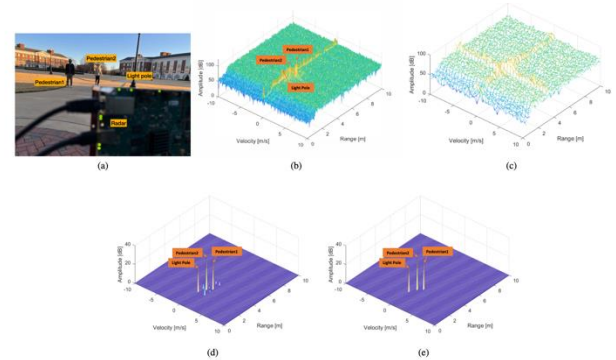
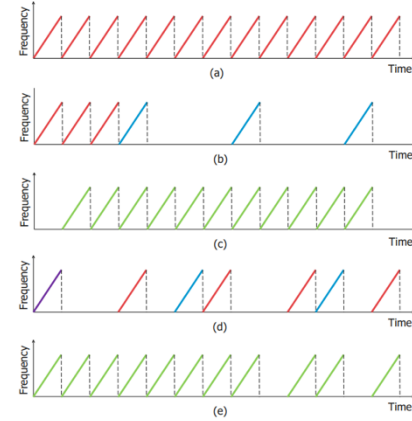


Co-Pulsing Applications

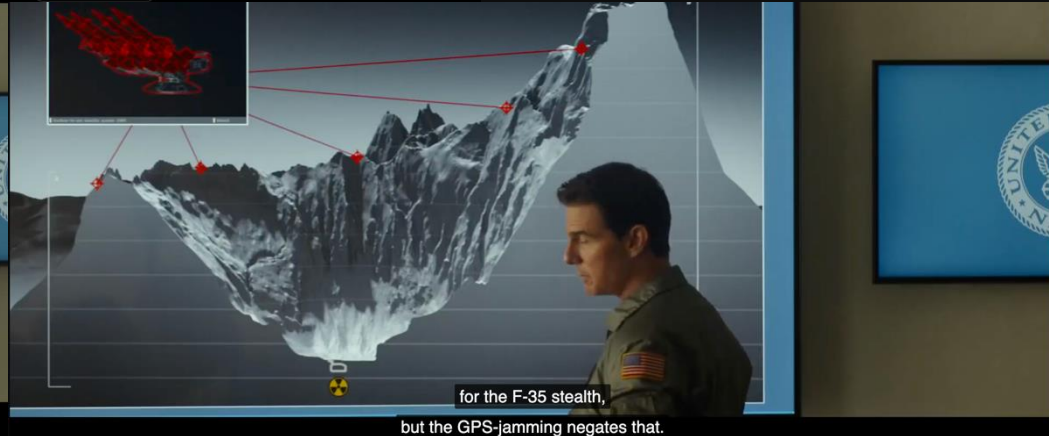
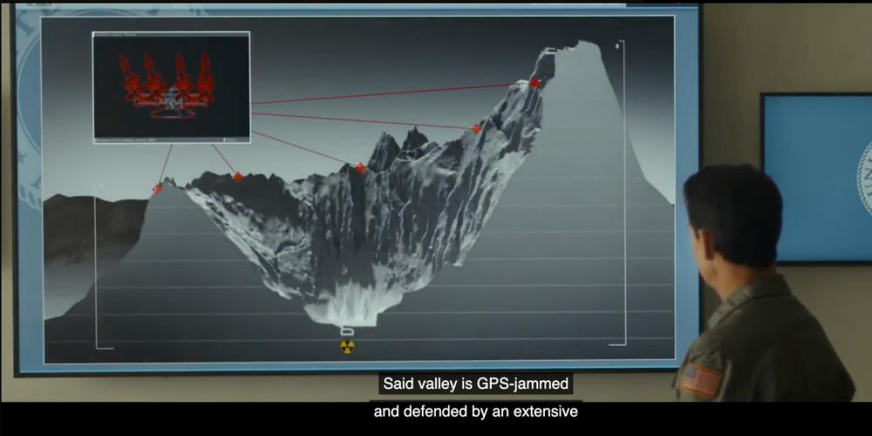
Co-STAP



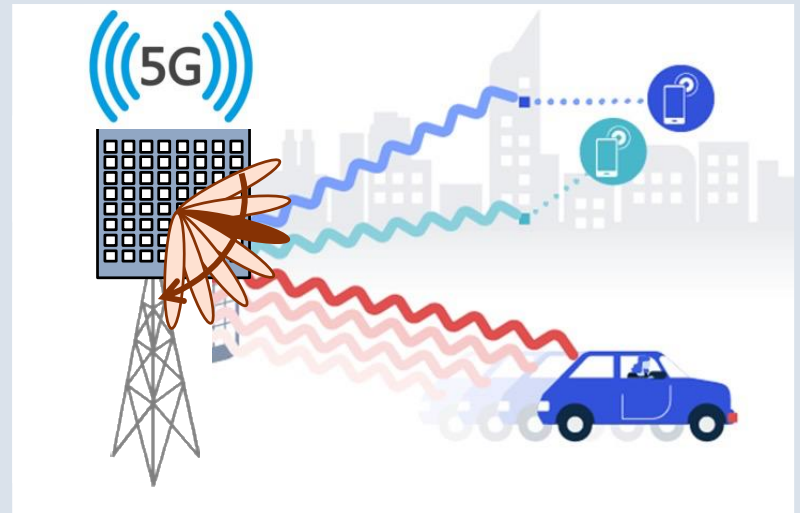
Co-Chirping with FMCW Radar



Courtesy: Top Gun: Maverick (2022)



Spectrum and EW



Understanding Spectrum Congestion is Essential in Modern Warfare



Ukraine Is Using ~~Open Market~~ Starlink for Drone Strikes on Russian Tanks ALEXANDER FREUND

2022

INTERESTING ENGINEERING

By Chris Young

2025

World's first 6G electronic warfare weapon by China can jam F-35 radar in seconds

Researchers claim the new 6G system is capable of jamming US F-35 stealth fighter jet radars.

Lloyd's List  Ece Göksedef

2026

War zone GNSS interference surges across the Middle East Gulf



MATT BURGESS

Attacks on GPS Spike Amid US and Israeli War on Iran

2026



Powered by THE TIMES OF INDIA

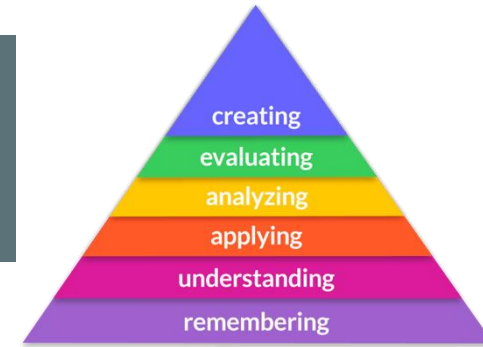
US brings EA-37B 'Compass Call' to Iran war: The electronic attack jet that can jam communications, blind radar

2026

Next-Generation Cognitive Radar

Benjamin Bloom's Taxonomy [1956] applied to cognitive radar

- Level 1: Prior measurements or databases in radar
- Level 2: Learning-based algorithms for adaptive radar
- Level 3: Knowledge-aided processing for a dynamic scene
- Level 4-6: Higher-order cognition to interconnect, decide, synthesize



Next-Generation Cognition

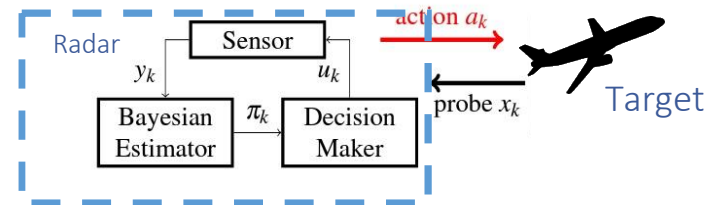
- **Cognitive Radar**: adaptive sensing + optimized processing
- **Super-cognition**: for legacy radars
- **Ultra-cognition**: to build cognitive strategy databases
- **Hyper-cognition**: All submodules operate cognitively

- **Metacognition**
- **Inverse cognition**
- **Inverse-inverse cognition**
- **Cognition masking**

Inverse Cognition

- **Inverse cognition**: Counter-adversarial techniques at the target-end
- Design of inverse stochastic filters
- Stability of inverse filters

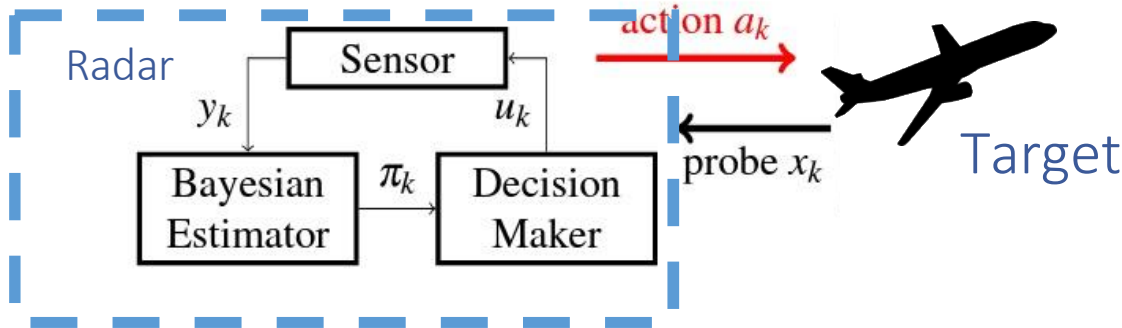
Adversarial Inference



Inverse Cognition

◆ Inverse filtering

- ✦ Radar sends EM pulse, estimates target position from the received signal
- ✦ Radar's action: signal waveform, angular beamforming
- ✦ Target observes radar's action and infers radar's estimate



Radar/them/adversary

Estimates \hat{x}_k ,

Takes action a_k



Target/us/defender
State x_k , control u_k ,
Estimates $\hat{\hat{x}}_k$

H. Singh, A. Chattopadhyay, and K. V. Mishra, "Inverse extended Kalman filter - Part I: Fundamentals," IEEE T-SP, 2023.

H. Singh, K. V. Mishra and A. Chattopadhyay, "Inverse extended Kalman filter - Part II: Highly non-linear and uncertain systems," IEEE T-SP, 2023.

H. Singh, K. V. Mishra and A. Chattopadhyay, "Inverse unscented Kalman filter," IEEE T-SP, 2024.

H. Singh, K. V. Mishra and A. Chattopadhyay, "Inverse cubature and quadrature Kalman filter," IEEE T-AES 2024.

H. Singh, A. Chattopadhyay and K. V. Mishra, "Inverse particle filter," IEEE T-SP 2025.

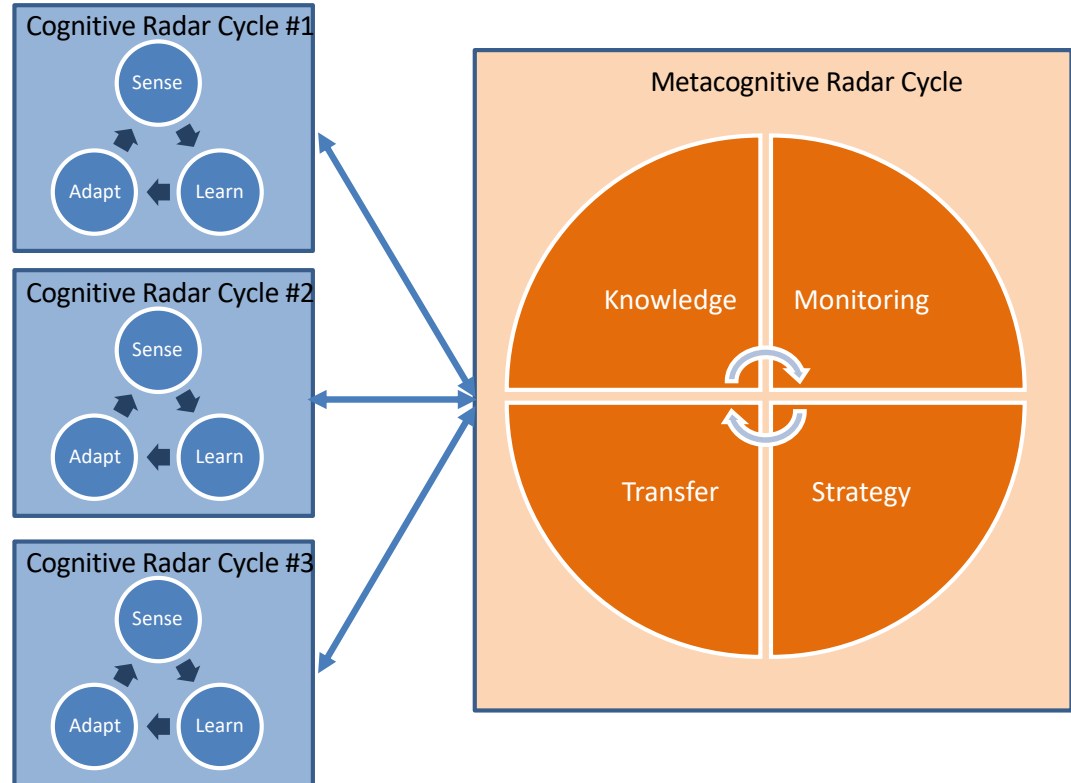
Inverse Stochastic Filters

◆ Inverse filtering algorithms

- ✦ Inverse EKF with unknown input
- ✦ Inverse EKF without unknown input
- ✦ Inverse KF with unknown input
- ✦ Gaussian sum EKF
- ✦ Second-order EKF (SOEKF)
- ✦ RKHS-EKF
- ✦ I-UKF, I-CKF, I-QKF

◆ Stability analysis

- ✦ I-EKF without unknown input
- ✦ I-SOEKF
- ✦ I-KF with unknown input
- ✦ I-UKF, I-CKF, I-QKF



Stability of Inverse Filters

- ◆ **Bounded non-linearity approach:** very similar to standard Lyapunov drift-based techniques
 - ✦ Hard to establish in our problem
 - ✦ Existing work on EKF stability could not establish it completely
 - ✦ Reference: Reif et al., IEEE TAC 1999
- ◆ **Unknown matrix approach:** Compensates for the effect of nonlinearity induced additive offset in the error dynamics by an unknown matrix multiplication

Theorem (Singh, Chattopadhyay, Mishra, 2023)

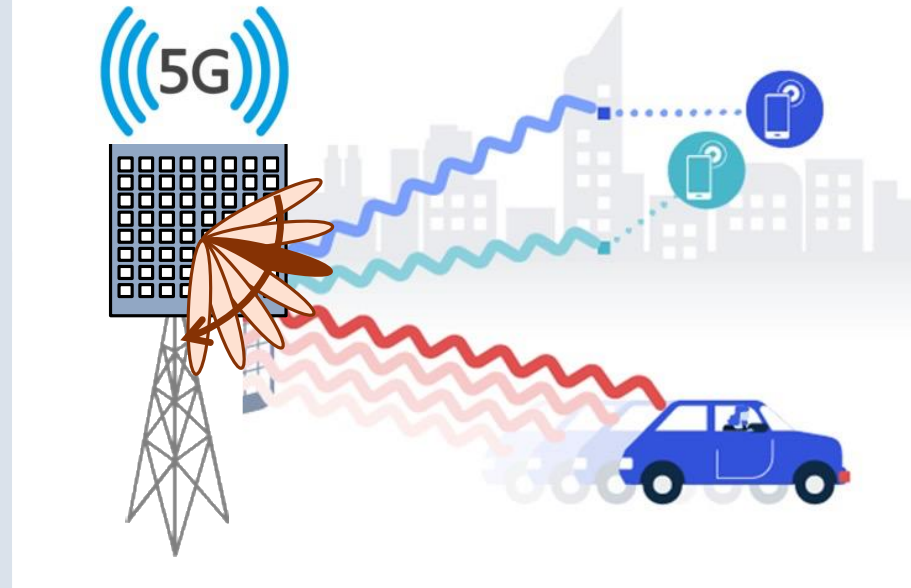
- Suppose that the one step prediction forward EKF is stable as per Reif et al. 1999. Let the following assumptions additionally hold true:
 1. There exist positive real number bounds such that for all $k \geq 0$, the following bounds hold:
$$\epsilon_l \mathbf{I} \preceq \bar{\mathbf{R}}_k \preceq \epsilon_u \mathbf{I}, m_l \mathbf{I} \preceq \bar{\Sigma}_k \preceq m_u \mathbf{I}, \|\mathbf{G}_k\| \leq \bar{g}$$
 2. \mathbf{H}_k is full column rank for every k
 3. There exist suitable positive constants such that, $\|\bar{\chi}(\hat{\mathbf{x}}, \hat{\mathbf{x}})\|_2 \leq \kappa_{\bar{\chi}} \|\hat{\mathbf{x}} - \hat{\mathbf{x}}\|_2^2$ for $\|\hat{\mathbf{x}} - \hat{\mathbf{x}}\| \leq \epsilon_{\bar{\chi}}$ (bounded non-linearity)

Then the error process of the inverse EKF is exponentially bounded in the mean square sense and with probability one, provided that the estimation error is always bounded within a suitable ϵ ball

Courtesy: Independence Day (1998)

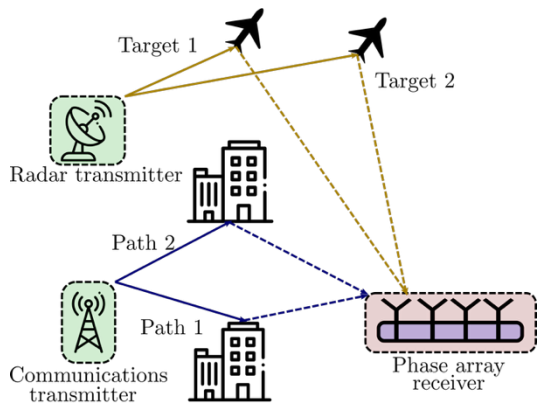


Other EW Applications of ISAC



Dual-Blind Deconvolution

Multi-antenna receiver

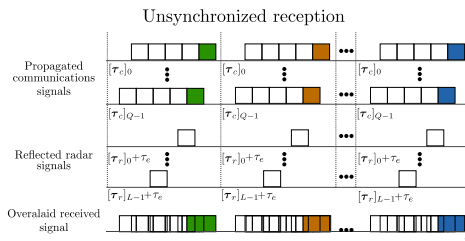
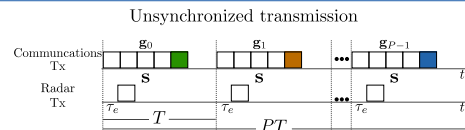


n-tuple BD

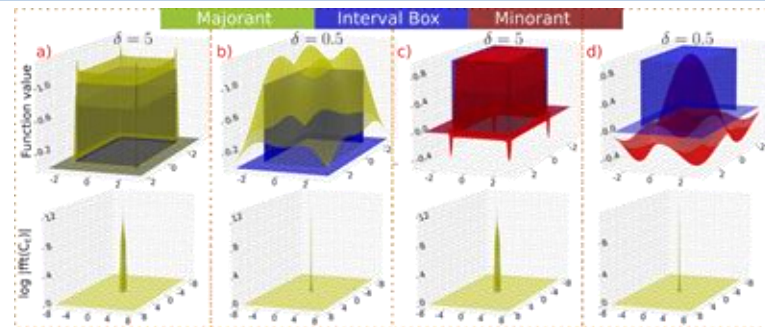
➤ SDP formulation

$$\begin{aligned} & \underset{q, Q}{\text{maximize}} \quad \langle \mathbf{q}, \mathbf{y} \rangle_{\mathbb{R}} \quad \text{subject to} \\ & \begin{bmatrix} Q & \tilde{Q}_{r_1}^H \\ \tilde{Q}_{r_1} & I_J \end{bmatrix} \succeq 0, \quad \begin{bmatrix} Q & \tilde{Q}_{c_1}^H \\ \tilde{Q}_{c_1} & \mu_c I_{PJ} \end{bmatrix} \succeq 0, \\ & \quad \vdots \\ & \begin{bmatrix} Q & \tilde{Q}_{r_{K_r}}^H \\ \tilde{Q}_{r_{K_r}} & I_J \end{bmatrix} \succeq 0, \quad \begin{bmatrix} Q & \tilde{Q}_{c_{K_c}}^H \\ \tilde{Q}_{c_{K_c}} & \mu_c I_{PJ} \end{bmatrix} \succeq 0 \\ & \text{Tr}(\Theta_n \mathbf{Q}) = 0 \end{aligned}$$

Unsynchronized transmission



Low-rank Hankel-type Matrix recovery



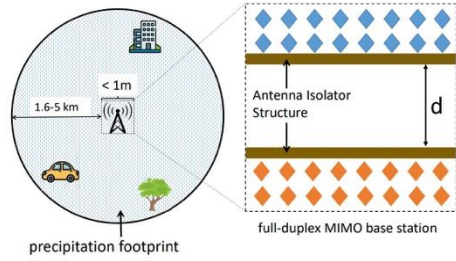
R. Jacome, E. Vargas, K. V. Mishra, B. M. Sadler and H. Arguello, "Multi-antenna dual-blind deconvolution for joint radar-communications via SOMAN minimization." Signal Processing, 2024.

E. Vargas, K. V. Mishra, R. Jacome, B. M. Sadler and H. Arguello, "Joint radar-communications processing from a dual-blind deconvolution perspective," IEEE ICASSP, 2022.

J. Monsalve, E. Vargas, K. V. Mishra, B. M. Sadler and H. Arguello, "Dual-blind deconvolution in ISAC receiver using multi-dimensional Beurling-Selberg functions," RadarConf 2024

Opportunistic ISAC using 5G Base-Stations

Full-Duplex BS for Weather Sensing

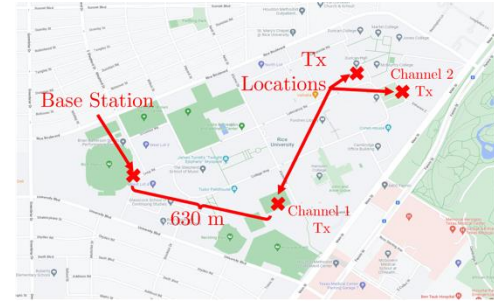


(a) view from the base station

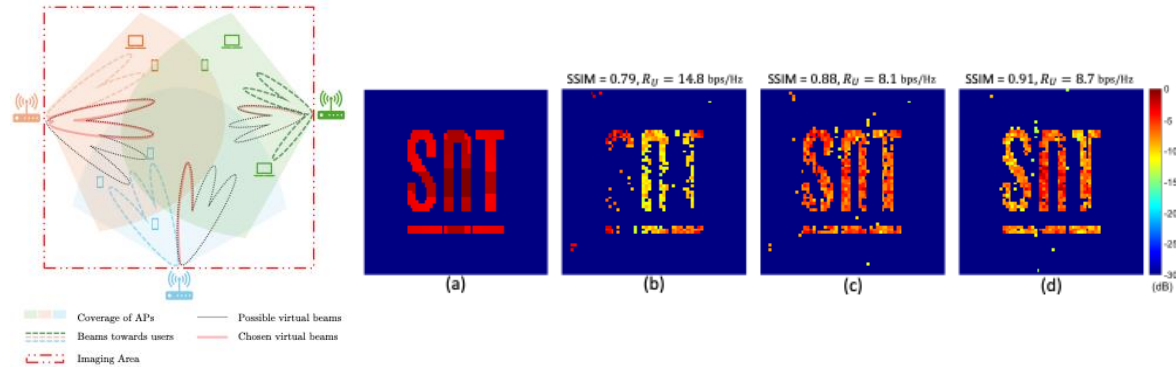


(b) base station configuration

5G BS-Based Aerial Radar Detection



SINC: Synergistic Imaging and Comms



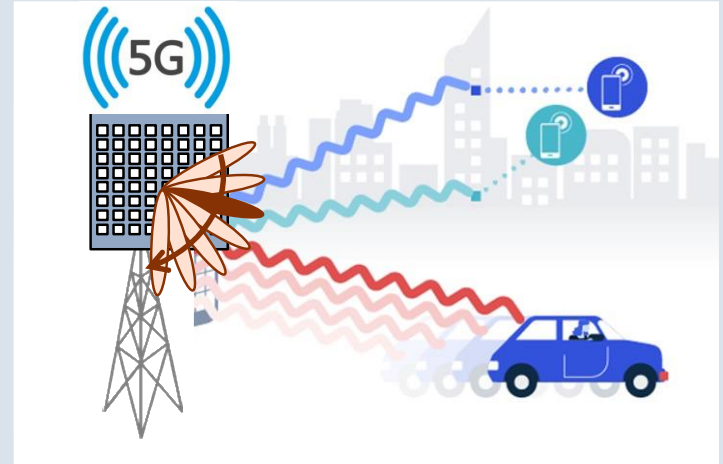
Z Chen, KV Mishra, D Pandey, A Sabharwal, "Near-ground Precipitation Sensing Using Full-duplex MIMO Base Stations," IEEE J-STEAP, 2025.

N. Raymondi, K. V. Mishra, A. Sabharwal, "Detecting aerial radars from massive MIMO base-stations: Design and experimental evaluation," IEEE T-AES, 2026.

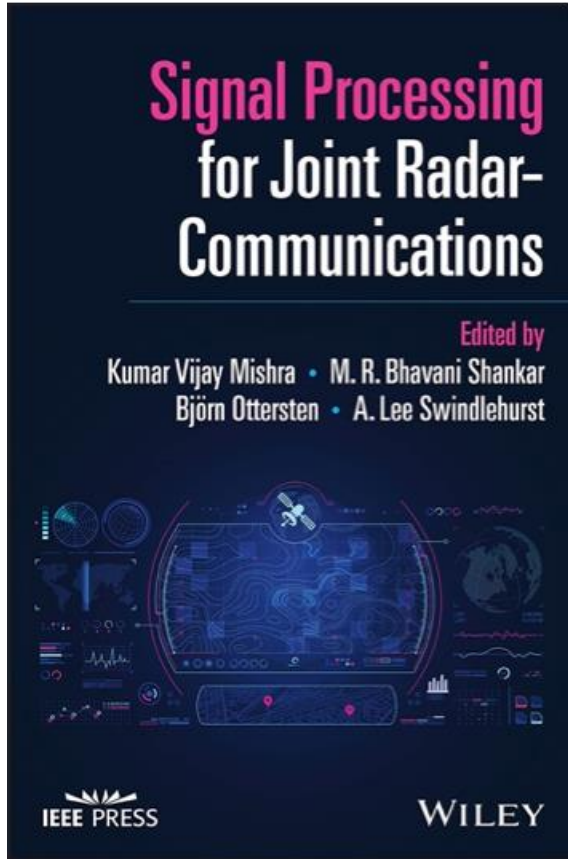
A. Murtada, K. V. Mishra, M. R. B. Shankar, "SINC: Synergistic imaging and communications via distributed MU-MIMO," IEEE SSP, 2025.



More Information



Thank you!



<https://www.linkedin.com/in/vizziee/>

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“for major contributions in radar signal processing and the integration of sensing and communications”

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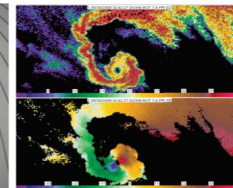
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Volume 1: Precipitation sensing platforms

Edited by
V. N. Bringi, Kumar Vijay Mishra and
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Coming Soon

handbook of statistics 55

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