Facets of synchronization: discontinuous coupling and the minimax flow problem

by

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Outline

- Will discuss work in [1, 2, 3, 4].

1. **Tool to achieve synchronization**: Discontinuous coupling to synchronize piecewise-smooth systems.
2. **Application of synchronization**: Sync. used to solve minimax flow problem in flow networks.

[1] M. Coraggio, P. De Lellis, S. J. Hogan, M. di Bernardo, “Synchronization of networks of piecewise-smooth systems”, IEEE Control System Letters, 2018

[2] M. Coraggio, P. De Lellis, M. di Bernardo, “Convergence and synchronization in networks of piecewise-smooth systems via distributed discontinuous coupling”, Automatica, 2022

[3] M. Coraggio, P. De Lellis, M. di Bernardo, "Distributed discontinuous coupling for convergence in heterogeneous networks", IEEE Control System Letters, 2020

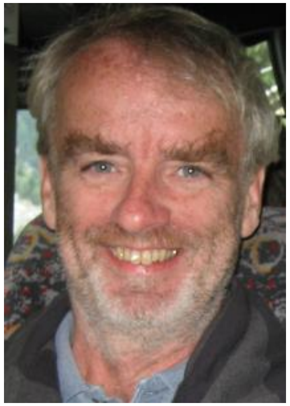
[4] M. Coraggio, S. Jafarpour, F. Bullo, M. di Bernardo, "Minimax flow over acyclic networks: distributed algorithms and microgrid application", IEEE Transactions on Control of Network Systems, 2022

Acknowledgments

Mario di Bernardo

Pietro De Lellis

Francesco Bullo



John Hogan

Saber Jafarpour

1 | Synchronization of piecewise-smooth systems

Introduction
to the problem

Synchronization

- Sync. appears in many natural and engineered systems.

- In some cases, it is desired, in others not.
  Hence, it's crucial to understand when and how it occurs.
- Often studied with *master stability fun.* [1], *Lyapunov fun.* [5].

[1] L. M. Pecora, T. L. Carroll, “Master stability functions for synchronized coupled systems,” Phys. Rev. Lett., 1998

[2] A. Pikovskij et al., "Synchronization: a universal concept in nonlinear sciences", Cambridge Univ. Press, 2003

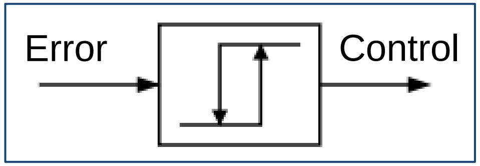
[3] A. Arenas et al., “Synchronization in complex networks,” Phys. Rep., 2008

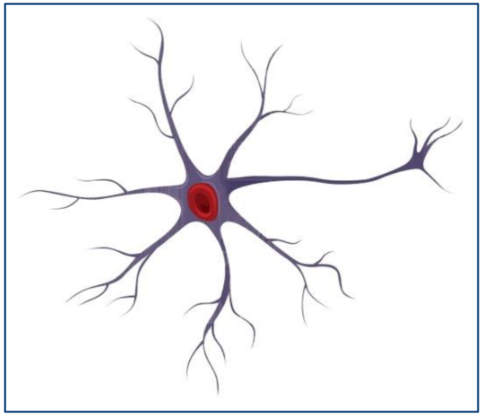
[4] L. Scardovi, R. Sepulchre, “Synchronization in networks of identical linear systems,” Automatica, 2009

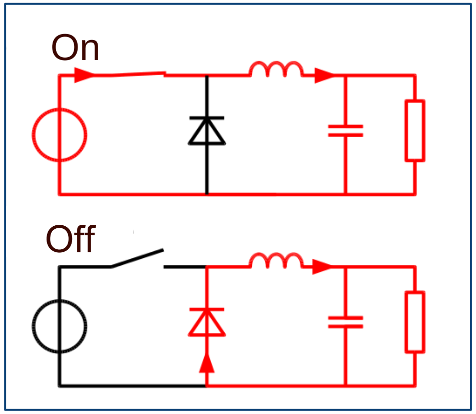
[5] P. DeLellis, M. di Bernardo, G. Russo, “On QUAD, Lipschitz, and contracting vector fields for consensus and synchronization of networks,” IEEE Transactions on Circuits and Systems I: Regular Papers, 2011

Piecewise-smooth (PWS) systems

- Typical assumption: nodes in the network have *smooth* dynamics (ODE).
- **Not true** in many cases:
  - switching control systems,
  - mechanical gears,
  - neuron and cardiac cells,
  - power converters…

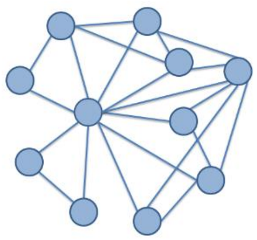


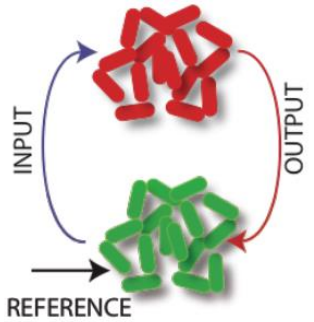


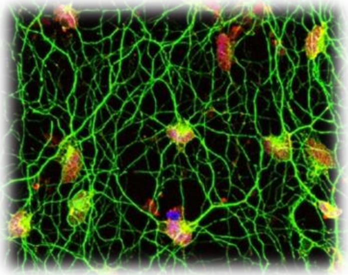
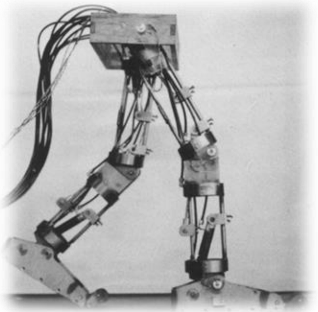


Networks of Piecewise-smooth systems

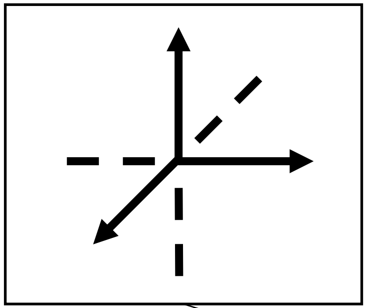
- In several real-world networks, nodes might be modeled as **piecewise-smooth systems** (discontinuous dynamics):
  - drivelines of vehicles,
  - power grids,
  - cell populations,
  - cooperative robot tasks,
  - biological neuron networks.

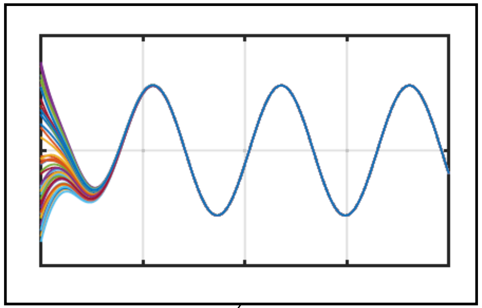


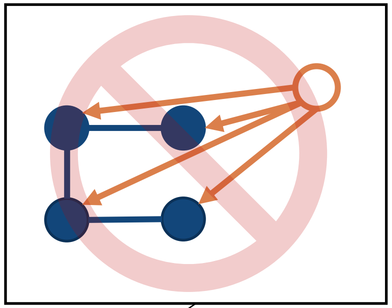



Relevant previous results







Paper	Global sync.	Asymptotic sync.	Control	Lack of centralised control
[1]	✓	✓	Yes	✗
[2]	✓	✗	No	✓
[3]	✗	~ some cases	No	✓
[4]	✓	✓	Yes	✓

[1] X. Yang, Z. Wu, J. Cao, “Finite-time synchronization of complex networks with nonidentical discontinuous nodes,” Nonlin. Dyn., 2013.

[2] P. De Lellis, M. di Bernardo, D. Liuzza, “Convergence and sync. in heterogeneous networks of smooth and piecewise smooth systems,” Automatica, 2015.

[3] S. Coombes, R. Thul, “Synchrony in networks of coupled non-smooth dynamical systems: Extending the master stability function”, Eur. J. Appl. Math., 2016.

[4] M. Coraggio, P. De Lellis, M. di Bernardo, “Convergence and synchronization in Networks of PWS Systems via distributed discontinuous coupling”, Automatica, 2022.

Problem formulation

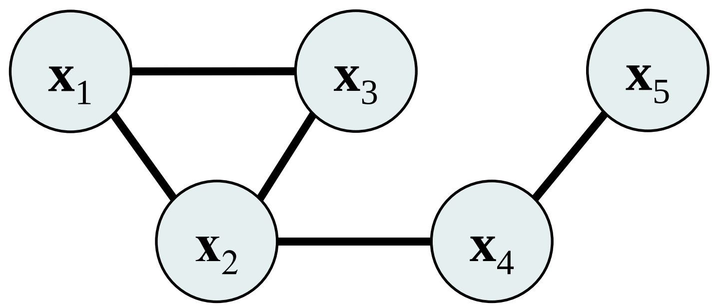
- Network model:

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - \sum_{j=1}^{N} L_{ij}\mathbf{g}(\mathbf{x}_i, \mathbf{x}_j), \quad i = 1, \ldots, N.$$

coupling

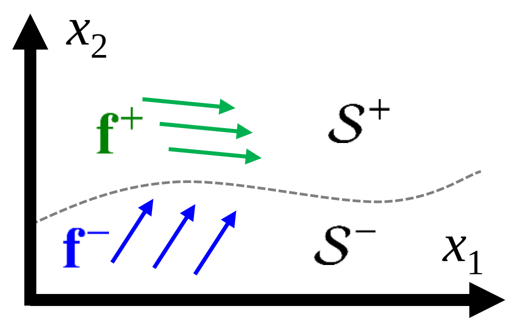
L : Laplacian matrix of graph

internal node dynamics



- **f** is in general **discontinuous**, e.g.,

$$\mathbf{f}(\mathbf{x}) = \begin{cases} \mathbf{f}^+(\mathbf{x}), & \mathbf{x} \in \mathcal{S}^+ \\ \mathbf{f}^-(\mathbf{x}), & \mathbf{x} \in \mathcal{S}^- \end{cases}$$



Problem formulation

- We seek sufficient conditions for **synchronization** of the nodes:

$$\lim_{t \to \infty} \left\| \mathbf{x}_i(t) - \mathbf{x}_j(t) \right\| = 0, \quad \forall i, j.$$

- We want a synchronization that is:

1. Global; (achievable from all initial conditions)
2. Complete; (zero asymptotic synchronization error)
3. Without a centralised control.

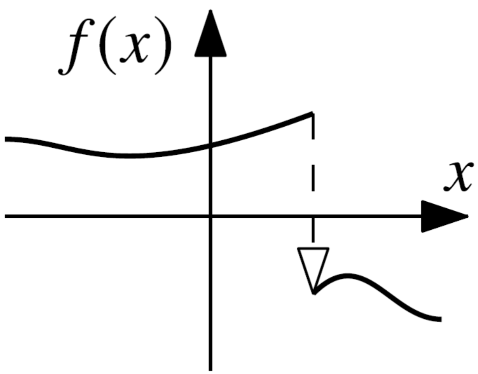
Main results

Node dynamics: QUAD

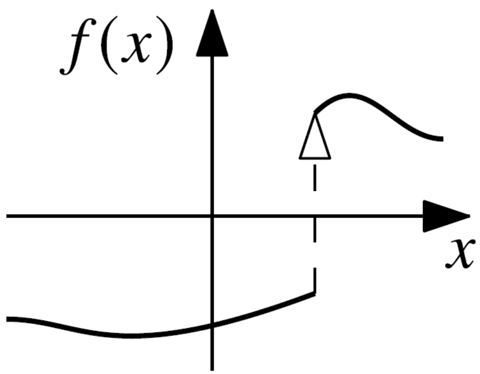
- First, we need some regularity condition on the dynamics $\mathbf{f}$.
- **QUADness** is a typical one, used in smooth networks:
  (slightly more flexible than *one-sided Lipschitz continuity*)

$$(\mathbf{v}_1 - \mathbf{v}_2)^{\mathrm{T}} \mathbf{P} [\mathbf{f}(\mathbf{v}_1) - \mathbf{f}(\mathbf{v}_2)] \leq (\mathbf{v}_1 - \mathbf{v}_2)^{\mathrm{T}} \underline{\mathbf{Q}} (\mathbf{v}_1 - \mathbf{v}_2) \quad \forall \mathbf{v}_1, \mathbf{v}_2.$$

- Some discontinuous functions are QUAD.



QUAD function.



Non-QUAD function.

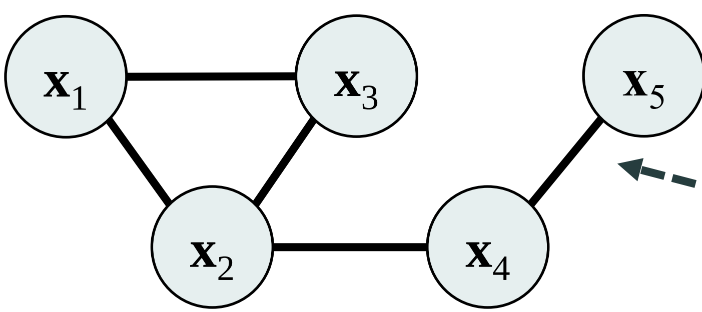
Coupling: linear diffusive

- Consider a **linear diffusive coupling**:

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - c \sum_{j=1}^{N} L_{ij} \boldsymbol{\Gamma}(\mathbf{x}_j - \mathbf{x}_i), \quad i = 1, \ldots, N,$$

coupling strength

- We seek a threshold value on c to attain synchronization.



e.g., coupling for $\mathbf{x}_5$

$$c\,\Gamma(\mathbf{x}_4 - \mathbf{x}_5)$$

$$(\mathbf{v}_1 - \mathbf{v}_2)^{\mathrm{T}} \mathbf{P} [\mathbf{f}(\mathbf{v}_1) - \mathbf{f}(\mathbf{v}_2)] \leq (\mathbf{v}_1 - \mathbf{v}_2)^{\mathrm{T}} \mathbf{Q} (\mathbf{v}_1 - \mathbf{v}_2) \quad \forall \mathbf{v}_1, \mathbf{v}_2.$$

Theorem. Assume:

- $\mathbf{f}$ is QUAD($\mathbf{P}$, $\mathbf{Q}$), with $\mathbf{P} > 0$,
- $\boldsymbol{\Gamma} > 0$.

The network synchronizes if $c > c^*$, where

$$c^* \triangleq \frac{\mu_2(\mathbf{Q})}{\lambda_2(\mathbf{L}) \quad \mu_2^-(\mathbf{P}\boldsymbol{\Gamma}))}.$$

agent dynamics

graph topology

coupling protocol

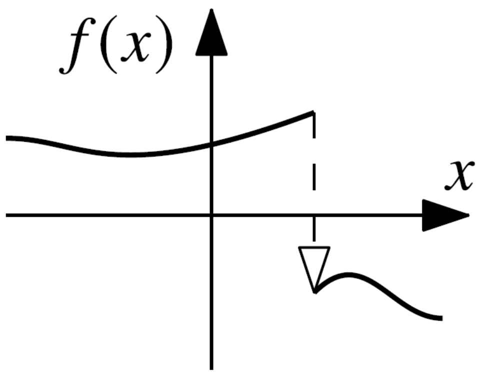
- Proof through a *common Lyapunov function*.

μ: matrix measure (logarithmic norm)
$\mu_2(\mathbf{A}) = \lambda_{\max}(\mathrm{sym}(\mathbf{A}))$,
$\mu_2^-(\mathbf{A}) = -\mu_2(-\mathbf{A}) = \lambda_{\min}(\mathrm{sym}(\mathbf{A}))$,

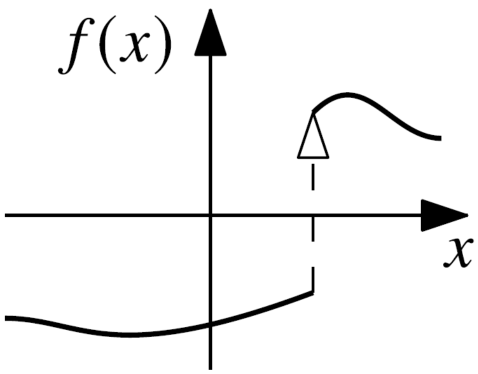
[1] M. Coraggio, P. De Lellis, S. J. Hogan, M. di Bernardo, “Synchronization of Networks of Piecewise-Smooth Systems,” IEEE Control Syst. Lett., 2(4), 653–658, 2018.

New assumption for node dynamics

- Problem: many PWS functions are not QUAD.



QUAD, σ-QUAD.



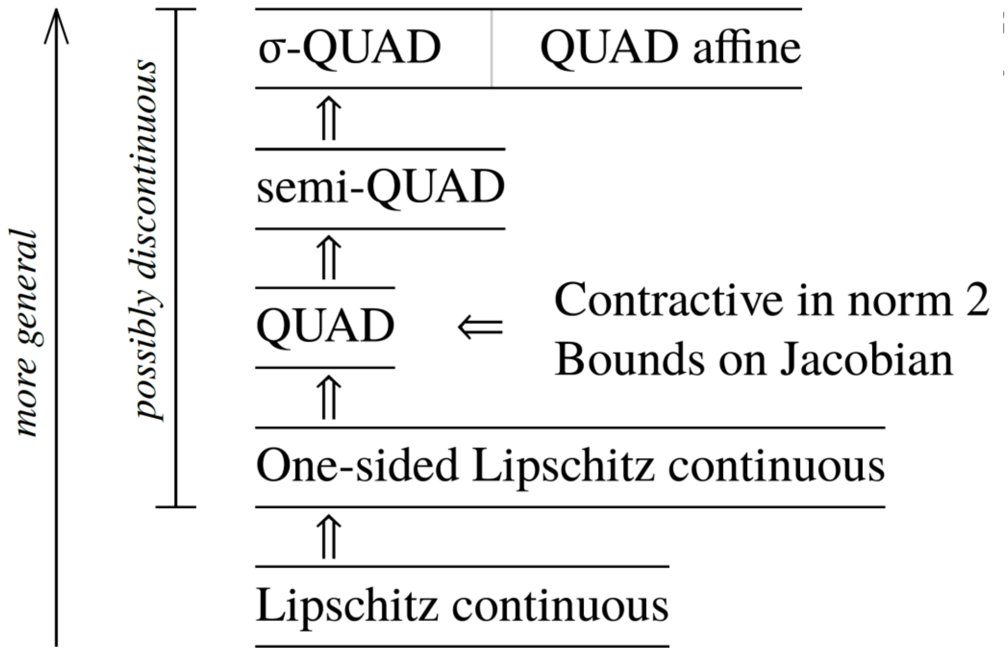
Not QUAD, σ-QUAD.

- We define **σ-QUADness**, a less restrictive assumption on $\mathbf{f}$:

$$(\mathbf{v}_1 - \mathbf{v}_2)^{\mathrm{T}} \mathbf{P} [\mathbf{f}(\mathbf{v}_1) - \mathbf{f}(\mathbf{v}_2)] \leq (\mathbf{v}_1 - \mathbf{v}_2)^{\mathrm{T}} \mathbf{Q} (\mathbf{v}_1 - \mathbf{v}_2) \qquad \forall \mathbf{v}_1, \mathbf{v}_2$$

- Now, $\mathbf{f}$ can have any number of finite jumps.

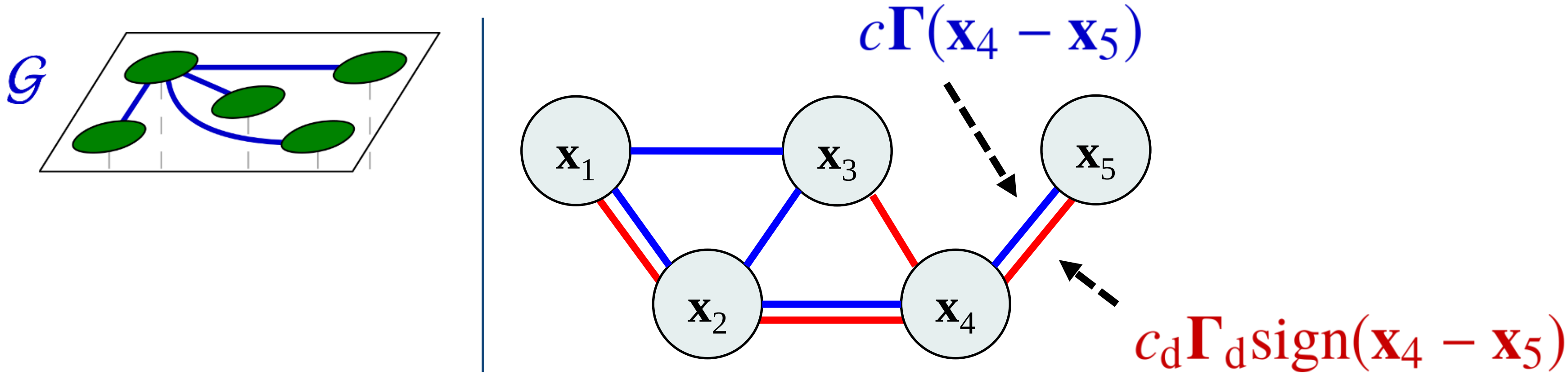
Lipschitz-like properties



Lipschitz continuity:	$\lVert \mathbf{f}(\mathbf{v}_1) - \mathbf{f}(\mathbf{v}_2) \rVert \le Q \lVert \mathbf{v}_1 - \mathbf{v}_2 \rVert$
One-sided Lipschitz:	$(\mathbf{v}_1 - \mathbf{v}_2)^\top [\mathbf{f}(\mathbf{v}_1) - \mathbf{f}(\mathbf{v}_2)] \le Q \lVert \mathbf{v}_1 - \mathbf{v}_2 \rVert^2$
QUADness:	$(\mathbf{v}_1 - \mathbf{v}_2)^\top \mathbf{P} [\mathbf{f}(\mathbf{v}_1) - \mathbf{f}(\mathbf{v}_2)] \le (\mathbf{v}_1 - \mathbf{v}_2)^\top \mathbf{Q} (\mathbf{v}_1 - \mathbf{v}_2)$

Discontinuous coupling layer

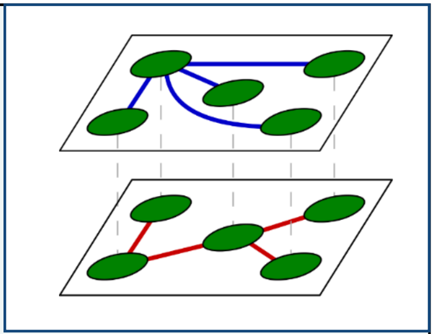
- Start with linear diffusive coupling layer.
- Add a **discontinuous coupling** layer (even different edges).



- We have a *multiplex* network:

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - c\sum_{j=1}^{N} L_{ij}\boldsymbol{\Gamma}(\mathbf{x}_j - \mathbf{x}_i) - c_\mathrm{d}\sum_{j=1}^{N} L_{ij}^\mathrm{d}\boldsymbol{\Gamma}_\mathrm{d}\mathrm{sign}(\mathbf{x}_j - \mathbf{x}_i).$$

Synchronization of PWS QUAD systems



Theorem. Assume

- $\mathbf{f}$ is σ-QUAD($\mathbf{P}$, $\mathbf{Q}$, $\mathbf{M}$), with $\mathbf{P} > 0$,
- $\mathbf{\Gamma}, \mathbf{\Gamma}_\mathrm{d} > 0$.

The network synchronizes if $c > c^*$ and $c_\mathrm{d} \geq c_\mathrm{d}^*$, where

$$c^* = \frac{\mu_2(\mathbf{Q})}{\lambda_2(\mathbf{L}) \quad \mu_2^-(\mathbf{P}\mathbf{\Gamma})}, \qquad c_\mathrm{d}^* = \frac{\mu_\infty(\mathbf{M})}{\delta_{\mathcal{G}_\mathrm{d}} \quad \mu_\infty^-(\mathbf{P}\mathbf{\Gamma}_\mathrm{d})}.$$

agent dynamics

graph topology

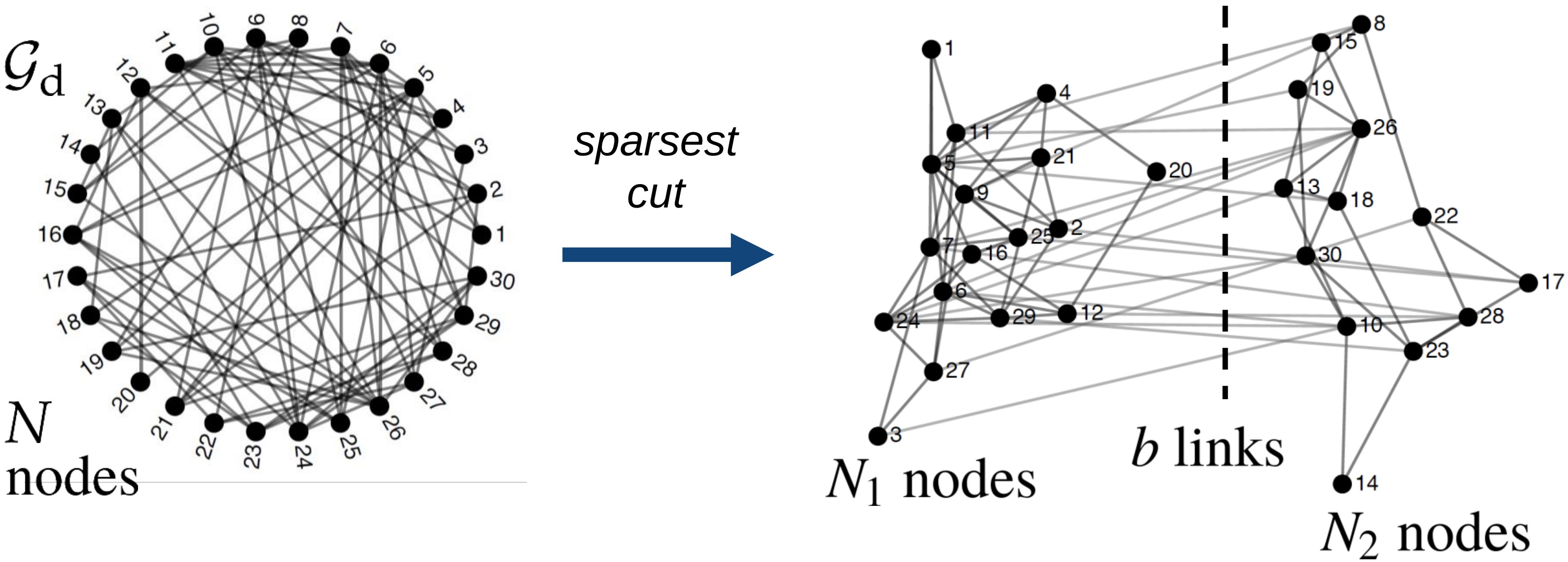
coupling protocol

$$\mu_\infty(\mathbf{A}) = \max_i\left(A_{ii} + \textstyle\sum_{j=1, j\neq i}^n |A_{ij}|\right),$$
$$\mu_\infty^-(\mathbf{A}) = -\mu_\infty(-\mathbf{A}) = \min_i\left(A_{ii} - \textstyle\sum_{j=1, j\neq i}^n |A_{ij}|\right).$$

[1] M. Coraggio, P. De Lellis, M. di Bernardo, "Convergence and synchronization in networks of piecewise-smooth systems via distributed discontinuous coupling", Automatica, 2022

Minimum density

- $\delta_{\mathcal{G}_\mathrm{d}}$: ***minimum density*** of $\mathcal{G}_\mathrm{d}$; computed from *sparsest cut*.



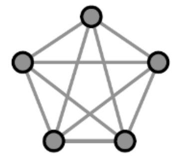
$$\delta_{\mathcal{G}_\mathrm{d}} \triangleq \frac{N}{2} \min \frac{b}{N_1 N_2}$$

NP-hard, but…

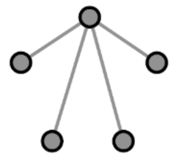
- We devised an algorithm and found simple formulae for selected topologies (complete, star, ring, path, k-nearest-neighbours).

Minimum density for selected topologies

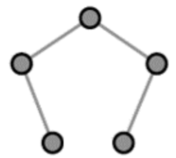
- Complete: $$\delta_{\mathcal{G}} = \frac{N}{2}$$



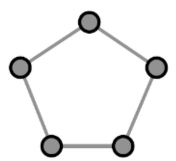
- Star: $$\delta_{\mathcal{G}} = \frac{N}{2(N-1)}$$



- Path: $$\delta_{\mathcal{G}} = \begin{cases} 2/N, & N \text{ even} \\ 2N/(N^2-1), & N \text{ odd} \end{cases}$$



- Ring: $$\delta_{\mathcal{G}} = \begin{cases} 4/N, & N \text{ even} \\ 4N/(N^2-1), & N \text{ odd} \end{cases}$$

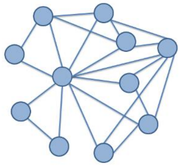


- *l*-nearest neighbours: $$\delta_{\mathcal{G}} = \begin{cases} \frac{4\sum_{k=0}^{l-1}(l-k)}{N}, & N \text{ even} \\ \frac{4N\sum_{k=0}^{l-1}(l-k)}{N^2-1}, & N \text{ odd} \end{cases}$$

Examples and numerical considerations

Example 1: Theorem application

- Consider $N = 30$ chaotic relay systems $\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i)$, with



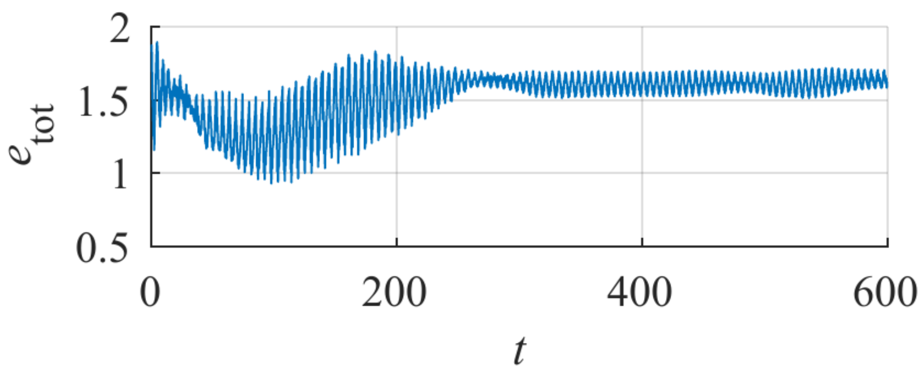
$$\mathbf{f}(\mathbf{x}_i) = \begin{bmatrix} 1.51 & 1 & 0 \\ -99.922 & 0 & 1 \\ -5 & 0 & 0 \end{bmatrix} \mathbf{x}_i - \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \operatorname{sign}(x_{i,1}).$$

- These systems are σ-QUAD with

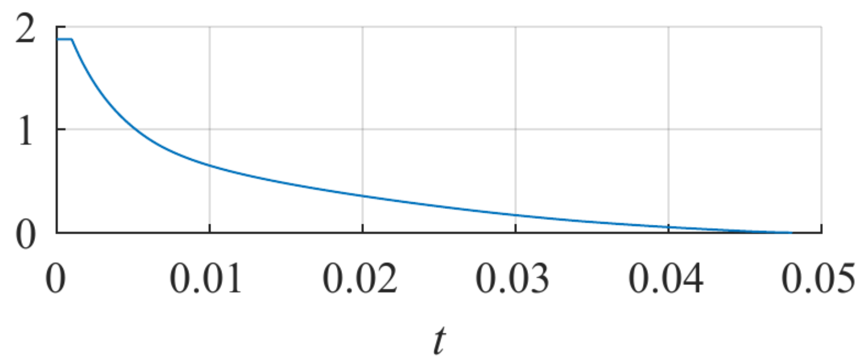
$$\mathbf{P} = \mathbf{I}, \quad \mathbf{Q} = \begin{bmatrix} 1.51 & 1 & 0 \\ -99.922 & 0 & 1 \\ -5 & 0 & 0 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Example 1: Theorem application

- Coupling protocol: $\mathbf{\Gamma} = \mathbf{\Gamma}_\mathrm{d} = \mathbf{I}$;
  Diffusive layer: $\lambda_2(\mathbf{L}) = 1$;
  Discontinuous layer: $\delta_{\mathcal{G}\mathrm{d}} = 1.29$.
- $c^* = \mu_2(\mathbf{Q}) \,/\, \lambda_2(\mathbf{L}) = 50.31$, $\quad c_\mathrm{d}^* = \mu_\infty(\mathbf{M}) \,/\, \delta_{\mathcal{G}\mathrm{d}} = 3.10$.



No synchronization
($c = 0.1 < c^*$, $\quad c_\mathrm{d} = 0.001 < c_\mathrm{d}^*$).



Synchronization
($c = 51 > c^*$, $\quad c_\mathrm{d} = 3.2 > c_\mathrm{d}^*$).

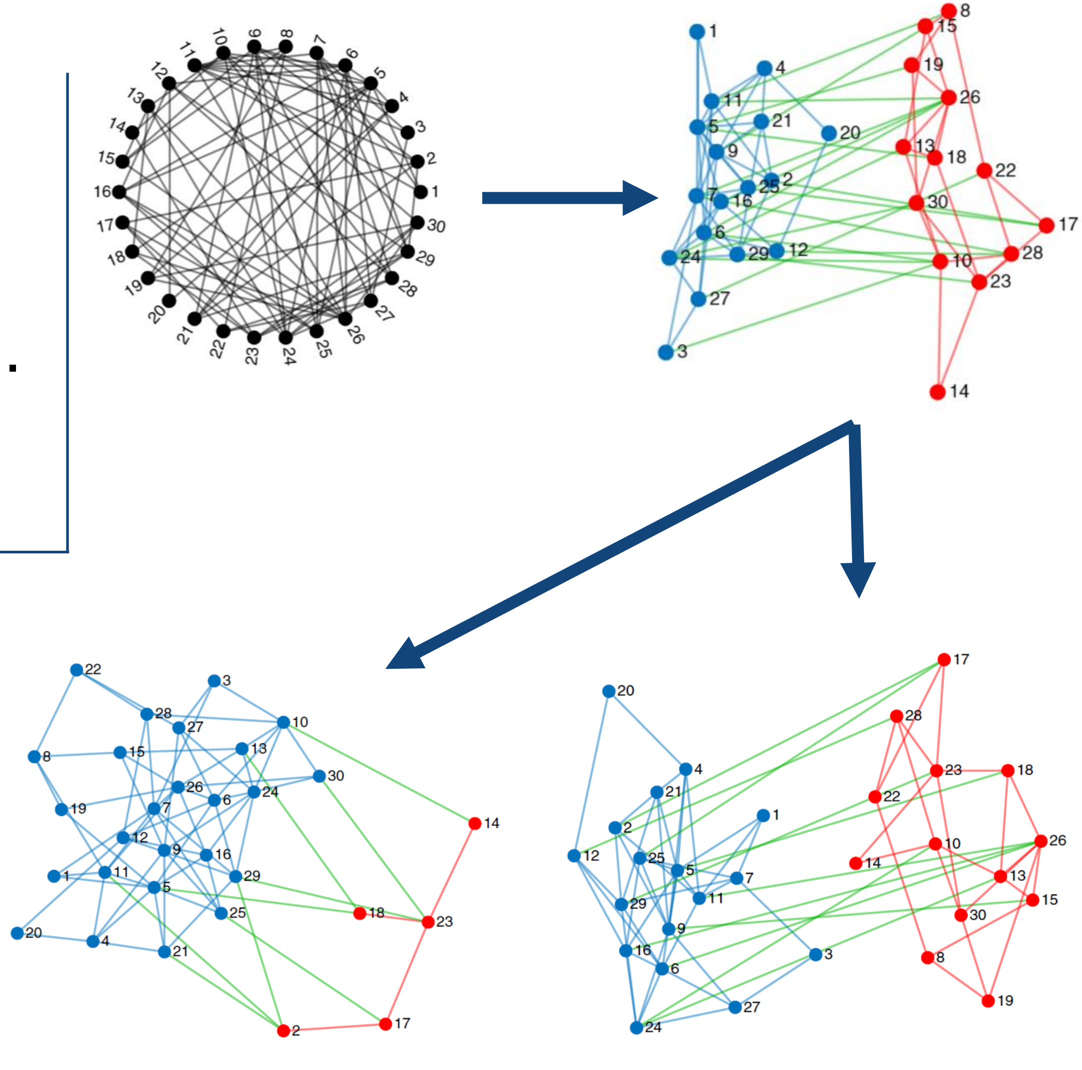
Example 2: Resilience to faults

- Resilience: how c_d^* changes when edges are removed?

a) Remove 4 blue & 4 red.

b) Remove 8 green.

- $\delta_{\mathcal{G}_{d,a}} > \delta_{\mathcal{G}_{d,b}}$ (a better than b). So *here* inter-cluster edges are more important to have a low threshold c_d^*.

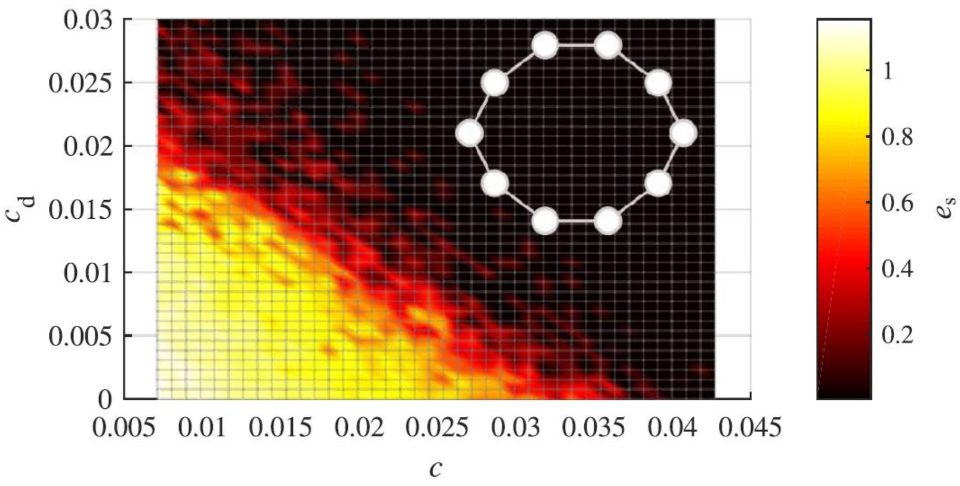


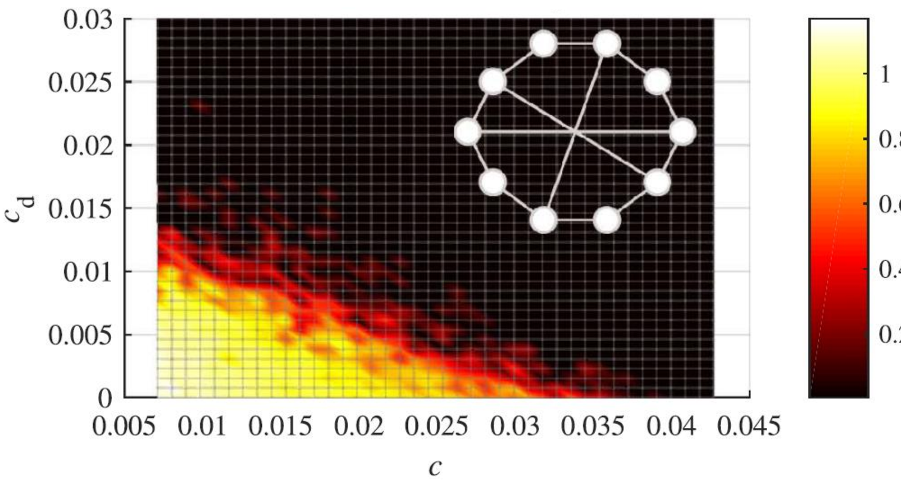
(a) (b)

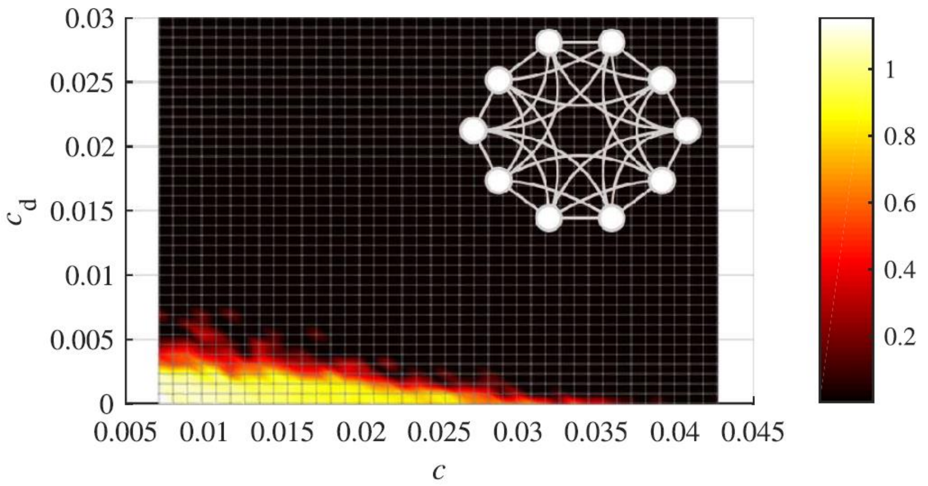
Example 3: Relation between layers

- $N = 10$ Sprott circuits;
  $\mathbf{L}$: 3-nearest-neighbours;
  $\mathbf{L}_\mathrm{d}$: variable.

- Different synchronizability with different topologies.

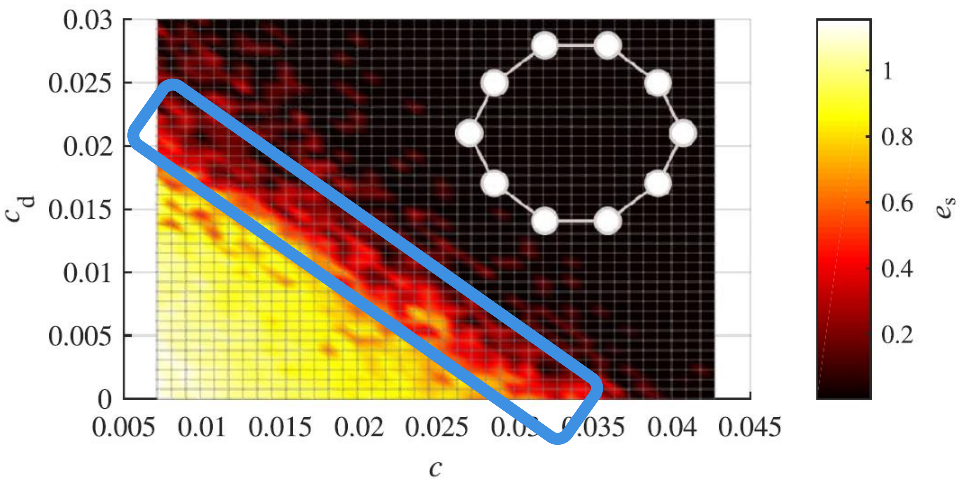






Remarks

- What is the relation *between* the gains? Can we find smaller thresholds c^*, c_d^* that depend on each other?



- We found that discontinuous coupling can also be used to:
  - **Synchronize heterogeneous agents** [1].
  - Compensate disturbances.

[1] M. Coraggio, P. De Lellis, M. di Bernardo, "Distributed discontinuous coupling for convergence in heterogeneous networks", IEEE Control System Letters, 2020

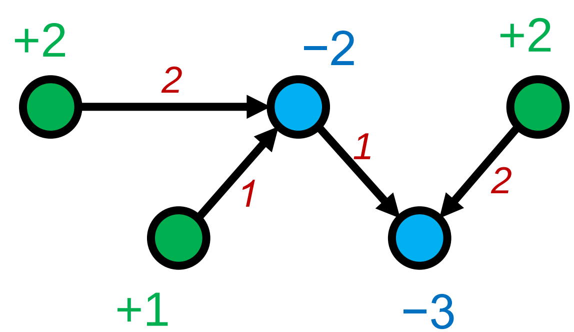
2 | The minimax flow problem in flow networks

Introduction
to the problem

Flow networks

- Flow networks are ubiquitous in nature and engineering.

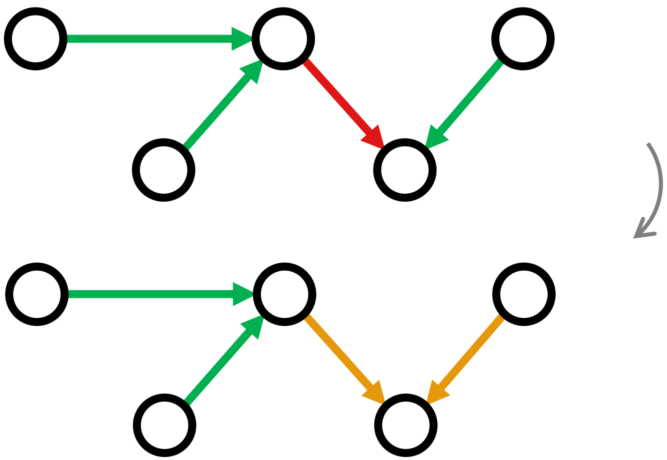
- Some commodity flows between nodes, which are suppliers or consumers;



Line faults

- Each edge has a *maximum capacity* for the flow.
- If the capacity is met, a fault occurs.

- We wish to maximize robustness to such faults.
- To do so, we aim to **minimize the maximum flow**.



: close to failure : fine : distressed

State of the art

- *Minimax problems on graphs* have a long history and many variations exist (e.g. [1]).
- Distributed solutions [2] are important in many applications, such as new-generation power grids.

- We could not find in the literature our exact problem.
- Yet, in [3], a very similar one is considered, providing only an approximate solution.

[1] P. L. Hammer, "Time-minimizing transportation problems: Time-minimizing transportation," Naval Research Logistics Quarterly, 1969.

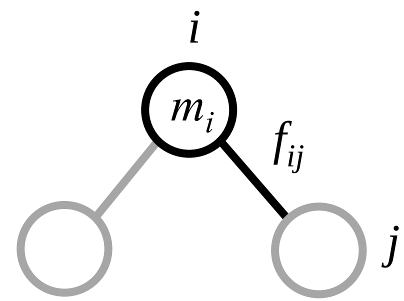
[2] A. Nedić and J. Liu, "Distributed Optimization for Control," Annu. Rev. Control Robot. Auton. Syst, 2018

[3] S. Z. Anaraki and M. Kalantari, "Acceleration of distributed minimax flow optimization in networks," 2011 45th Annual Conference on Information Sciences and Systems, 2011.

Problem statement

- We consider acyclic graphs.
- Flow network model (static):

$$\sum_{\text{edges } \{i,j\}} f_{ij} = m_i, \ \forall \text{ nodes } i; \qquad \left(\sum_{\text{all nodes } i} m_i = 0 \right).$$



- Cost function: $J \triangleq \max_{\text{all edges } \{i,j\}} \frac{|f_{ij}|}{\bar{f}_{ij}}$

$m_i \in \mathbb{R}$: commodity
$f_{ij} \in \mathbb{R}$: flow
$\bar{f}_{ij} \in \mathbb{R}_{>0}$: capacity

- Optimization problem:

$$\min_{m_i \text{ for supplier nodes } i} J,$$

$$\text{s.t. } m_{i,\min} \le m_i \le m_{i,\max}, \ \forall \text{ supplier nodes } i.$$

Main results

Maximum downstream flows

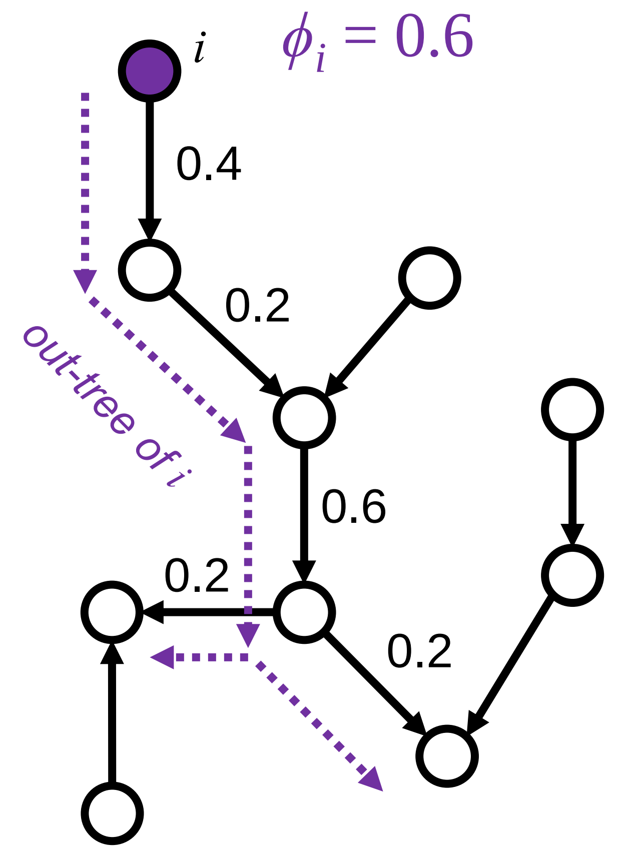
Definition. For each node i, ***maximum downstream flow*** ϕ_i is the maximum flow in the *out-tree* of node i.

$$\phi_i := \max_{(j,k)\in\mathcal{D}_i} \frac{f_{jk}}{\bar{f}_{jk}} \geq 0$$

- ϕ_is give global information but can be computed with local information: (exploiting acyclicity of the graph)

$$\phi_i = \max_{j\in\text{out-neig}(i)} \left\{ \phi_j, \beta_{ij}\frac{f_{ij}}{\bar{f}_{ij}} \right\}$$

$$\dot{\hat{\phi}}_i(t) = -k_\phi \left(\hat{\phi}_i(t) - \max_{j\in\text{out-neig}(i)} \left\{ \beta_{ij}\frac{f_{ij}}{\bar{f}_{ij}}, \hat{\phi}_j(t) \right\} \right), \quad \hat{\phi}_i(0) = 0$$



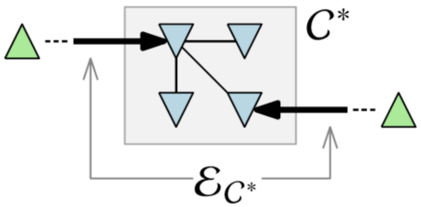
Example.

$\mathcal{D}_i$: downstream of i (excluding branches with consumers only)
β_{ij} : Boolean to exclude uncontrollable flows

Main results

Theorem. If the maximum downstream flows ϕ_i of all suppliers are equal, then the max flow (J) is minimized.

- We turned the optimization problem into a synchronization problem.
- Proof based on graph-theoretic arguments.



▲ Suppliers
▼ Consumers

- To achieve synchronization, we use a **distributed control law** akin to:

$$\dot{m}_i = -k_\phi(\phi_i - \phi_{\text{avg}}).$$

ϕ_{avg} : average of ϕ_i for suppliers

Application to (electrical) microgrids

- The voltage phase angles $\delta_i(t)$ have the dynamic

$$\begin{cases} D_i \dot{\delta}_i(t) = P_i - \sum_{j=1}^{N} A_{ij} \sin\left(\delta_i(t) - \delta_j(t)\right), & i \in \text{suppl.} \\ 0 = P_i - \sum_{j=1}^{N} A_{ij} \sin\left(\delta_i(t) - \delta_j(t)\right), & i \in \text{cons.} \end{cases}$$

D_i : droop coefficient
P_i : power
A_{ij} : weighted incidence matrix

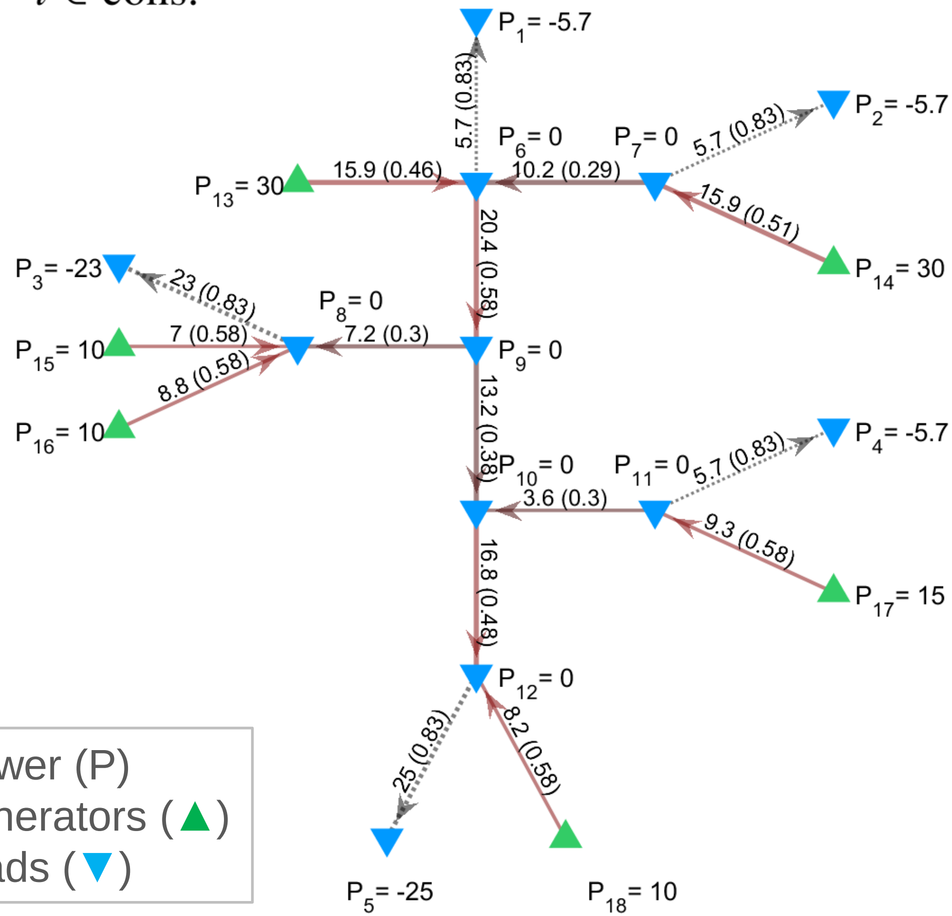
- The steady state solution is a flow network

$$\mathbf{Bf} = \mathbf{P} - \omega \mathbf{D}$$

$\mathbf{B}$: incidence matrix
$\mathbf{f}$: flows
$\mathbf{P}$: power
$\omega = \mathrm{d}\delta_i/\mathrm{d}t$
$\mathbf{D}$: droop coeff.s

- We minimax $\mathbf{f}$, by varying $\mathbf{P}$.

Commodity = power (P)
Suppliers = generators (▲)
Consumers = loads (▼)



Control law on powers P_i

$$\gamma_i(t) \triangleq \begin{cases} 1, & \text{if } P_{\min,i} < P_i(t) < P_{\max,i} \\ 0, & \text{otherwise} \end{cases} \qquad i \in \mathcal{V}_\text{s}$$

$$\hat{\phi}^{\text{n-sat}}_{\text{avg},i}(t) \triangleq \text{mean}\left\{\{\hat{\phi}_i(t)\} \cup \left\{\hat{\phi}_j(t)\right\}_{j\in\mathcal{V}_\text{s}|j\neq i,\gamma_j=1}\right\}, \qquad i\in\mathcal{V}_\text{s}$$

$$\hat{\phi}^{\text{sat}}_{\max,i}(t) \triangleq \max\left\{\hat{\phi}_j(t)\right\}_{j\in\mathcal{V}_\text{s}|j\neq i,\gamma_j=0}, \qquad i\in\mathcal{V}_\text{s}$$

Recall:
$\dot{m}_i = -k_\phi(\phi_i - \phi_{\text{avg}})$.

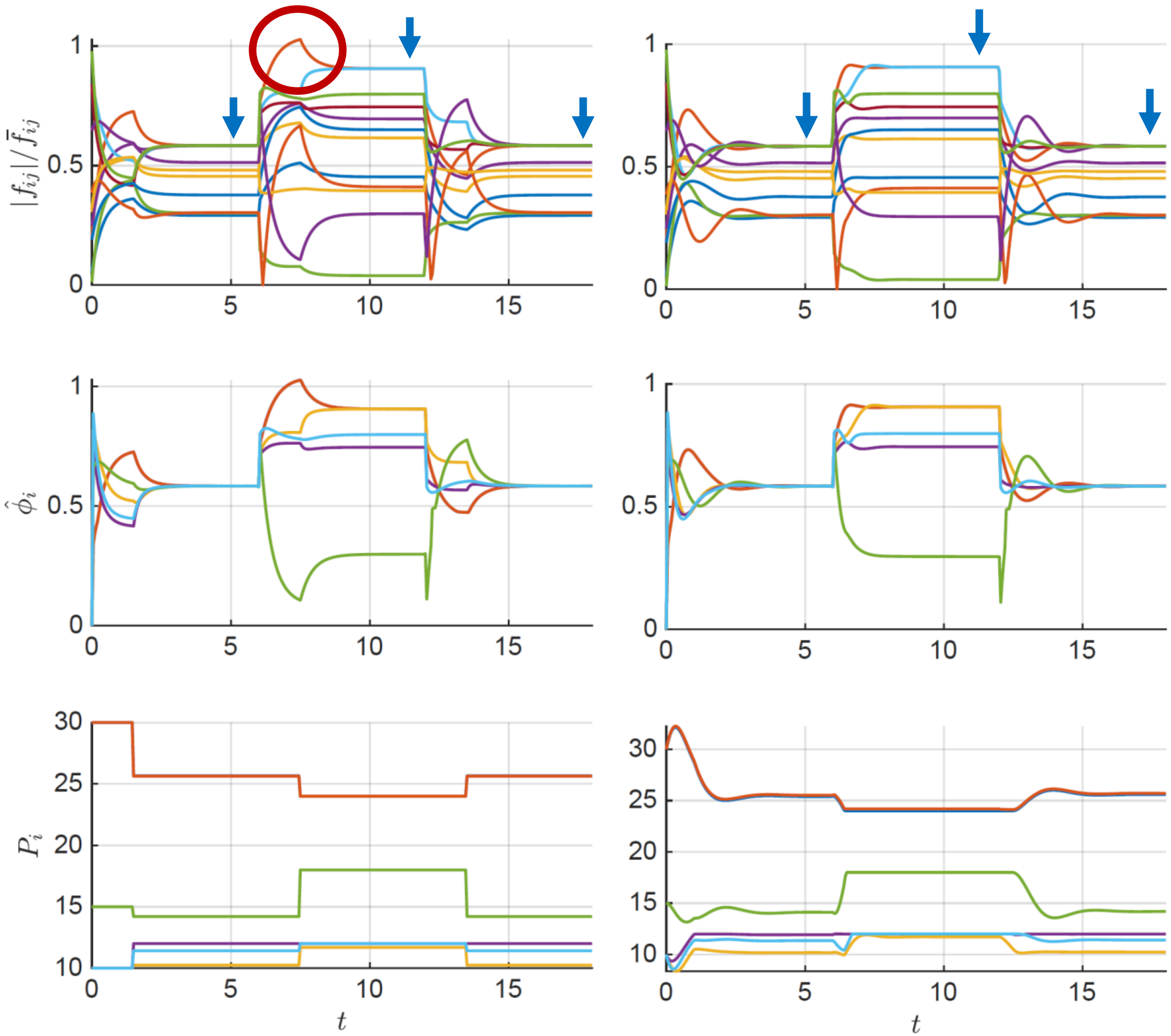
$$\dot{P}_i = \begin{cases} & \text{if } \gamma_k = 1 \quad \forall k \in \mathcal{V}_\text{s} \\ P_i, & \text{if } (\exists k \in \mathcal{V}_\text{s} : \gamma_k = 0) \wedge \\ & \quad (\gamma_i = 1 \vee \zeta_i = 1) \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{P}_i \triangleq -k_P\left(\hat{\phi}_i - \hat{\phi}^{\text{n-sat}}_{\text{avg},i}\right) - k^\gamma_P\left(\hat{\phi}^{\text{n-sat}}_{\text{avg},i} - \hat{\phi}^{\text{sat}}_{\max,i}\right)$$

$$\zeta_i \triangleq \begin{cases} 1, & \text{if } \quad (P_i \leq P^{\min}_i \ \wedge\ \tilde{P}_i > 0) \vee \\ & \qquad (P_i \geq P^{\max}_i \ \wedge\ \tilde{P}_i < 0) \\ 0, & \text{otherwise} \end{cases}$$

Simulation results

- Our strategy achieves **same steady state cost J** as a centralised solution to the optimization.
- … and **better transient**.



Centralised solution.

Our distributed strategy.

Conclusion

RECAP:

- **Discontinuous coupling** can synchronize piecewise-smooth (and heterogeneous) systems.
- **σ-QUADness** is an extension of one-sided Lipschitz for discontinuous functions
- The **minimum density** is an additional metric to capture the synchronizability of a graph.
- In flow networks, we defined the **maximum downstream flows**, which measure the stress caused by a supplier onto the network.
- The minimax flow problem can be solved through synchronization of the maximum downstream flows.

OPEN CHALLENGES:

- Find less conservative (possibly related) coupling thresholds c^*, c_d^*.
- Solve the minimax flow problem in cyclic networks.

Thank you for your attention.

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