Control and complex systems

perspectives, challenges and applications

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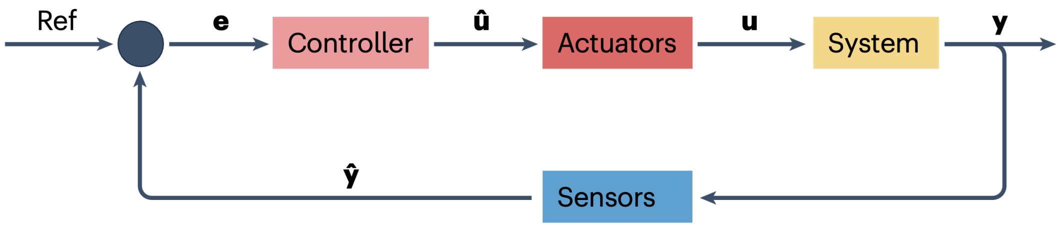
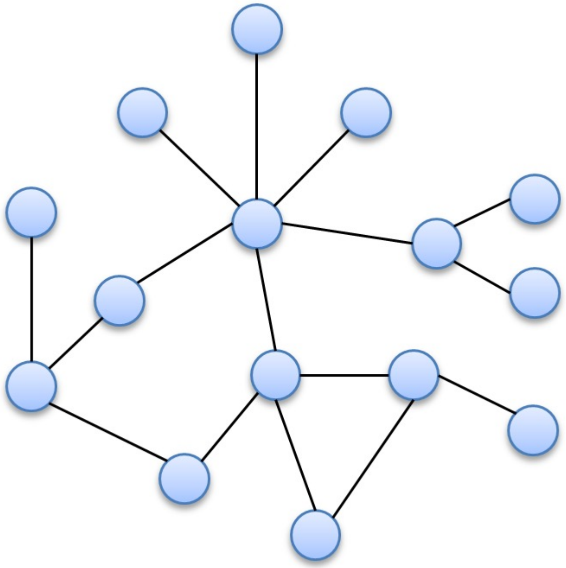
ELLIIT Symposium on Networks Dynamics and Control – Linköping, 22nd September 2023

What's in my talk

- Complex Systems and Control Theory
- *Controlling a complex system*
- Continuification-based control
- *Complex Systems for Control*
- The shepherding problem
- Conclusions and open problems





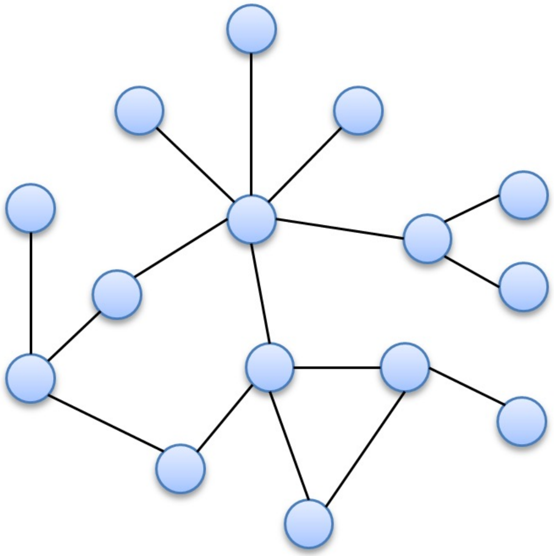
What do I mean by "Complex" System

- A complex system is:
  a) A collection of objects or *agents* with high cardinality..
  b) ..which interact with one another in a nontrivial way..
  c) ..such that their collective behaviour is unexpected or different from, or not immediately predictable from, the aggregation of the behaviour of its individual parts

$$\dot{x}_i = f_i(t, x_i) + u_i(t, X_i)$$

$$x_i \in \mathbb{R}^n \quad f_i \in \mathbb{R}^n \times \mathbb{R} \mapsto \mathbb{R}^n$$

$$u_i = \sigma \sum_{j=1}^{N} a_{ij} \left[h(x_j) - h(x_i) \right]$$

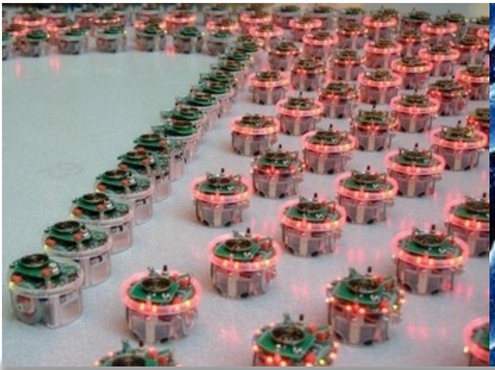
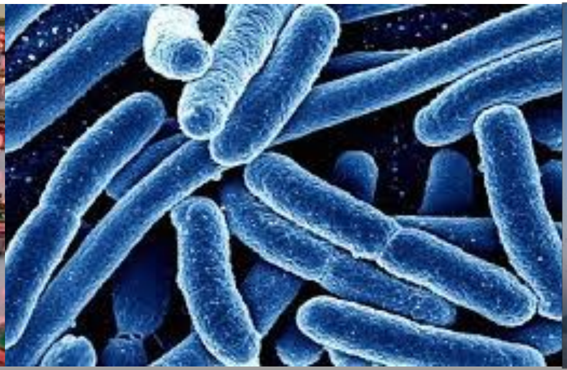


L. Torres, A. Blevins, D. Bassett, T. Eliassi-Rad, *The Why, How and When of Representations for Complex Systems*, SIAM Review 2021

Numerous applications

- From power grids and swarm robotics to biology and epidemiology
- Often, we wish to control the emerging collective behaviour of these systems
- E.g. avoid or induce synchronization, pattern formation, prevent undesired cascading phenomena, achieve crowd control etc

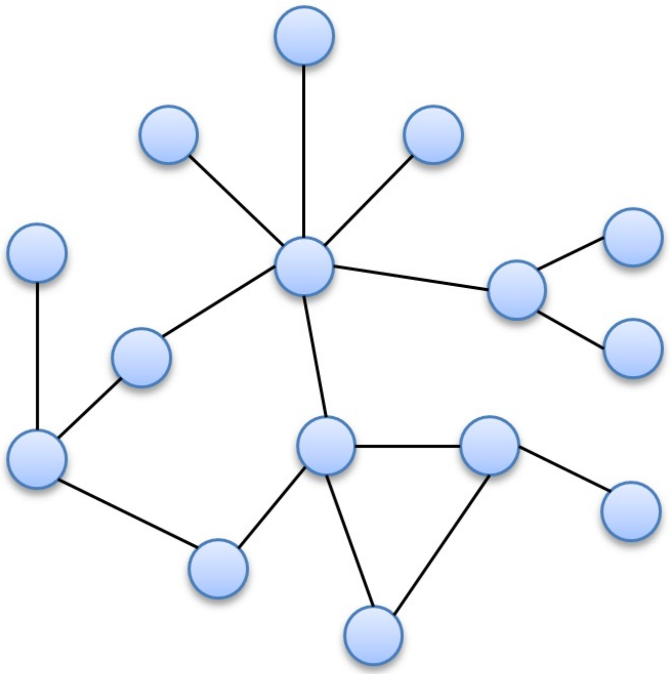
Can we orchestrate in real-time the collective behaviour of a complex system?


Controlling complex systems

Feedback Control = Sense + Compute + Actuate

1. Whom do we sense? *observability*
2. Whom do we control? *controllability*
3. What do we compute? *control design*

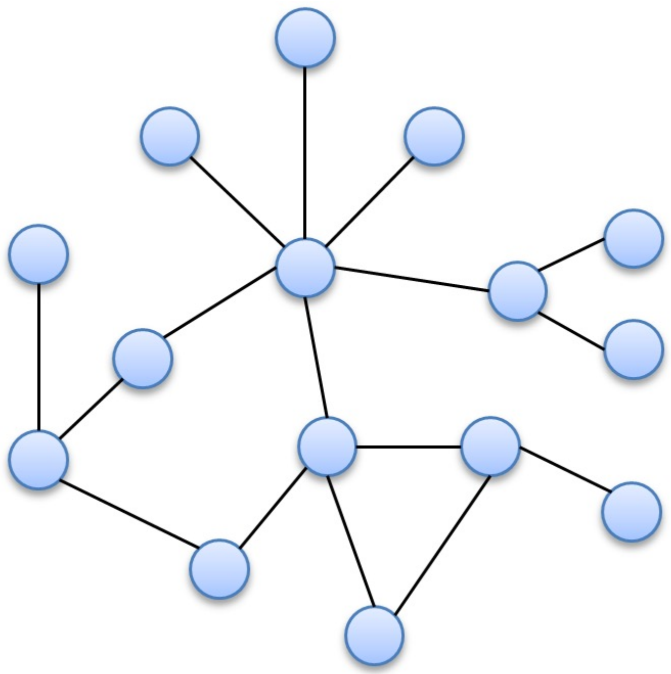


- We want the control strategy to be distributed and to be computed in real-time as a function of the sensed variables
- In Control Theory, we also want to certify stability (proofs of convergence)

Control Design

- To achieve this goal we can act upon
  1. the agents
  2. the links interconnecting them
  3. the topology of the network structure itself
  4. a combination of the above

$$\dot{x}_i = f_i(x_i, t) + \sigma(t) \sum_j \mathcal{L}_{ij}(t) h(x_j, t)$$

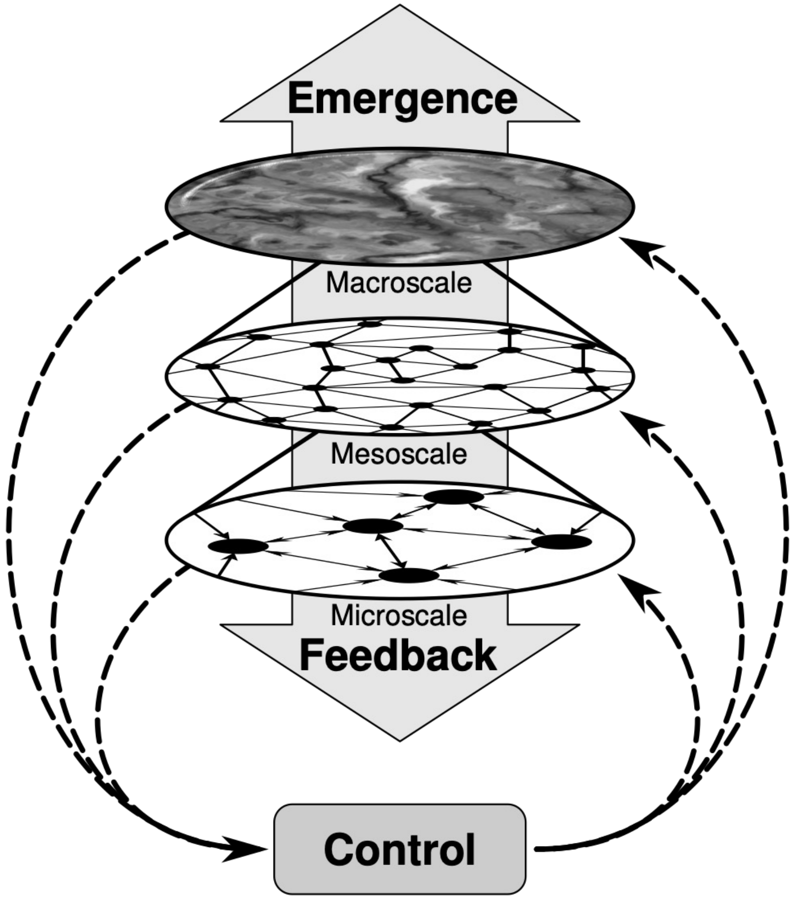


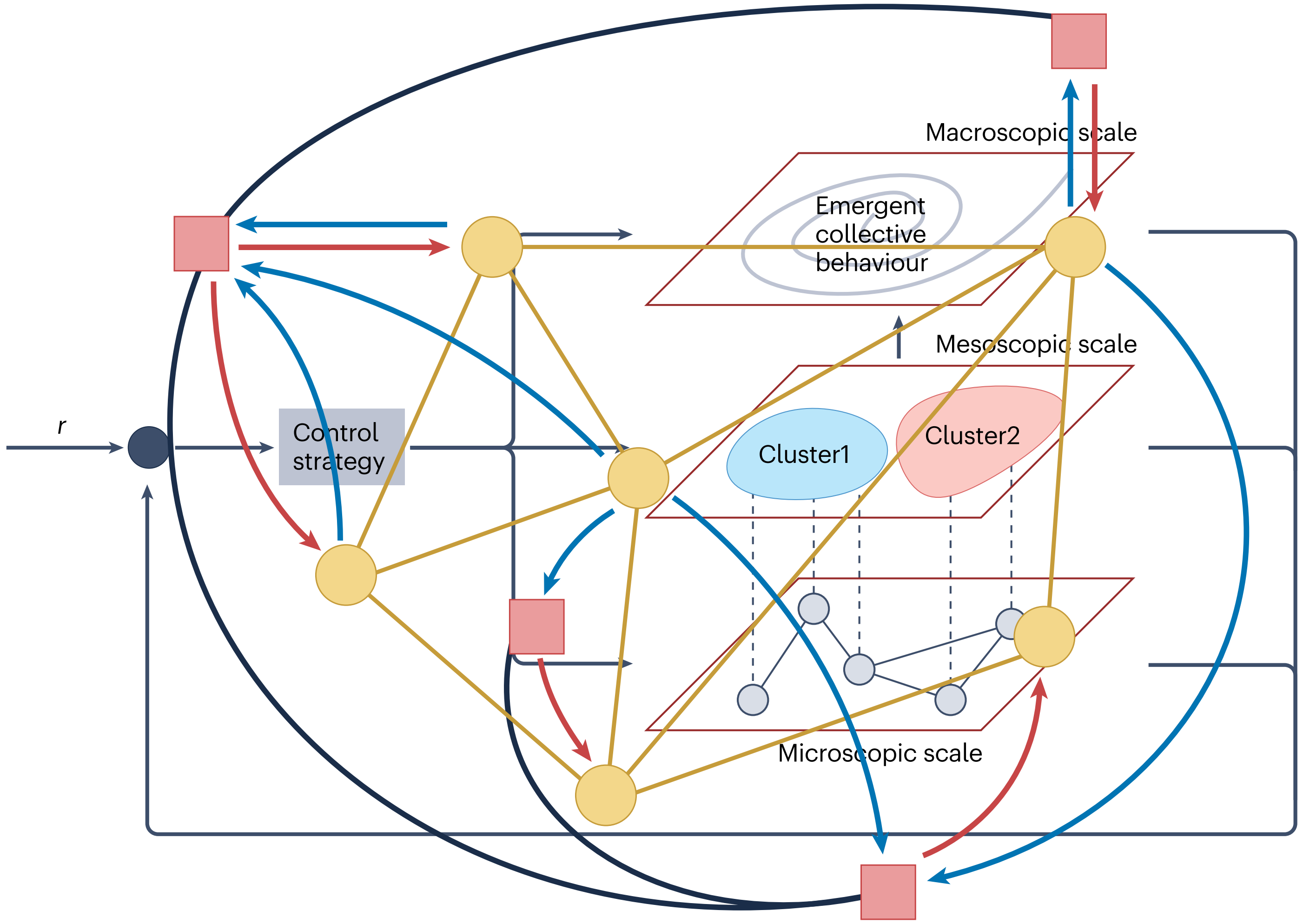
- Each of these approaches yields different types of problems but also opens up different opportunities for control (pinning control, adaptive control, network topological control..)

R. De Souza, MdB, YY Liu, "Controlling complex systems with complex nodes", Nature Reviews Physics, 2023

A multi-scale problem

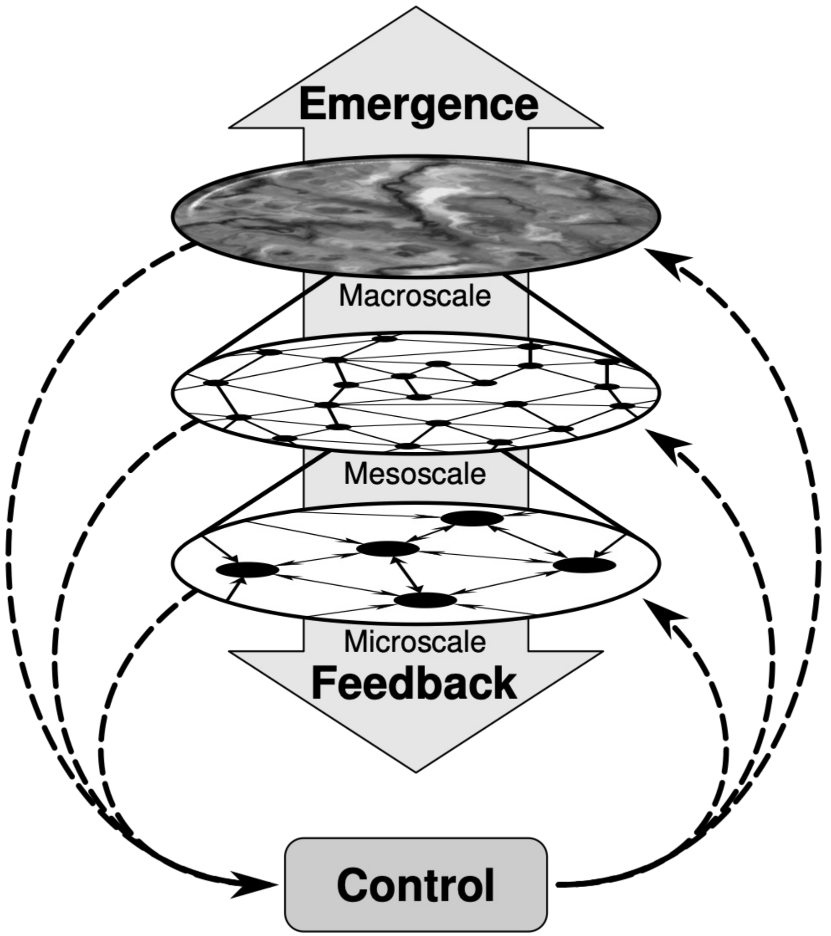
- We need to ”close the loop” *across different scales*

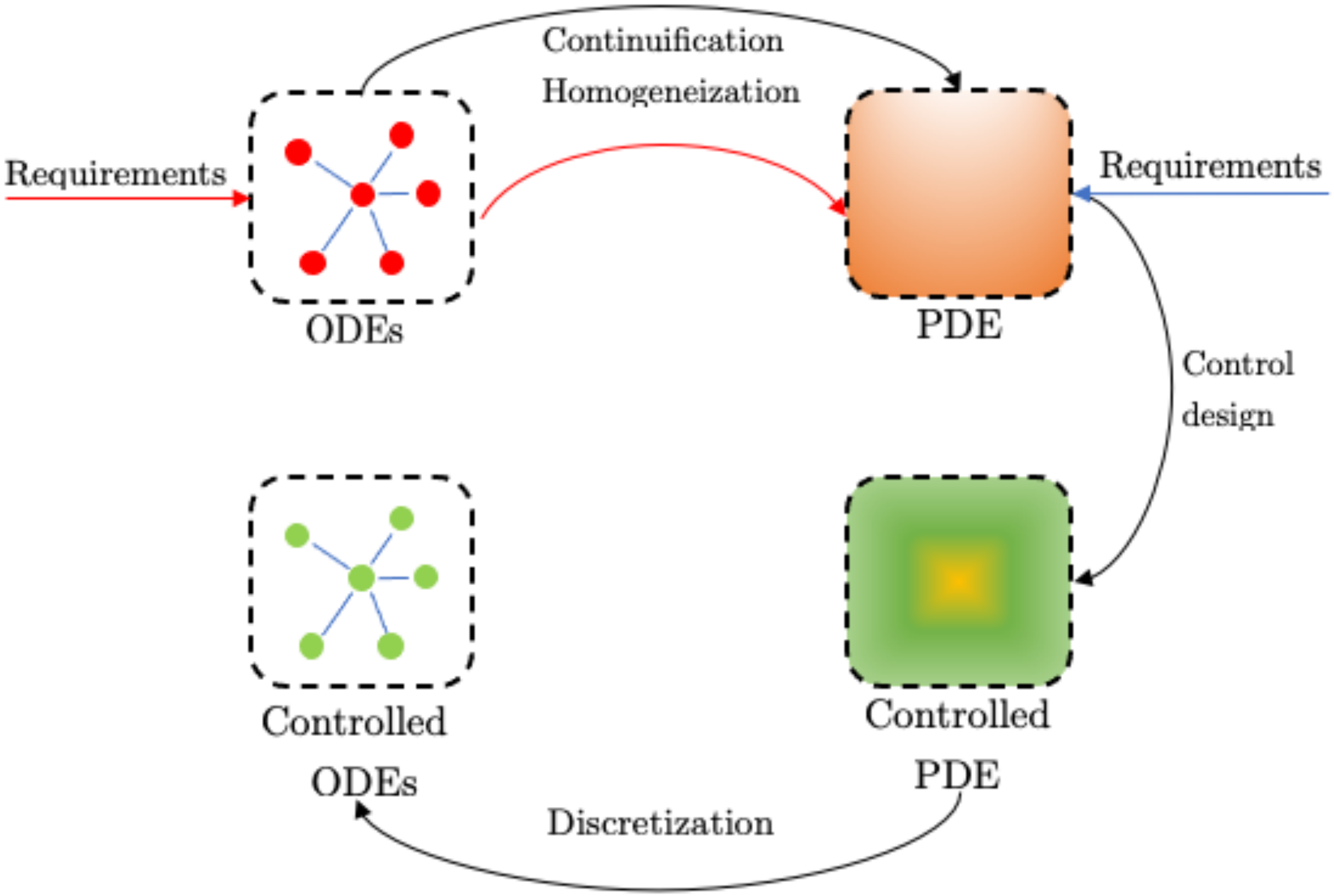




M. di Bernardo, “Controlling collective behaviour in complex systems”, Encyclopedia of Systems and Control, Springer, 2020

Continuification-based control





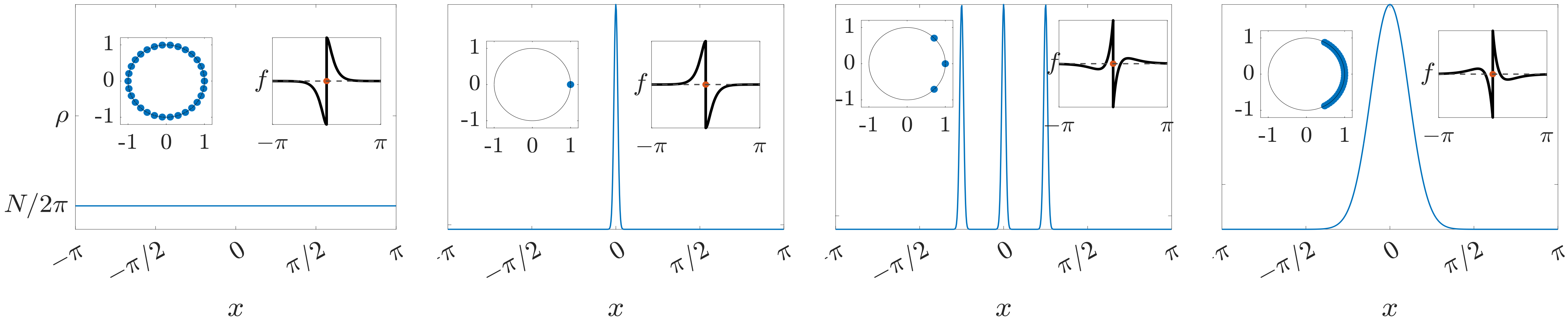
Nikitin et al. "A continuation method for large-scale modeling and control: from ODEs to PDE, a round trip." *IEEE Trans on Automatic Control*, 2021

Microscopic modeling and emerging behaviors

- Take *N* identical coupled dynamical systems swarming on a ring:

$$\dot{x}_i = \sum_{j=1}^{N} f\left(\{x_i, x_j\}_\pi\right) + u_i \qquad f : [-\pi, \pi] \to \mathbb{R}$$

- **Goal:** steer the agents towards a desired distribution



1. Continuification (micro -> macro)

- In the limit of infinitely many agents, we can find a macroscopic closure:

$$\dot{x}_i = \sum_{j=1}^{N} f\left(\{x_i, x_j\}_\pi\right)$$

Agents are moving on the circle of radius 1

Agents starts evenly distributed

Micro

$$\rho_t + (\rho V)_x = 0$$

$$V(x,t) = \int_{-\pi}^{\pi} f(\{x,y\}_\pi)\rho(y,t)dy = (f * \rho)(x,t)$$

$$\rho(\pi,t) = \rho(-\pi,t)$$

$$\rho(x,0) = N/2\pi$$

Macro

$$\rho \to \rho^d$$

Leverentz et al. "Asymptotic dynamics of attractive-repulsive swarms." *SIAM Journal on Applied Dynamical Systems* 8.3 (2009): 880-908

- Consider the addition of a macroscopic control input

$$\rho_t(x,t) + [\rho(x,t)V(x,t)]_x = q(x,t)$$

$$e(x,t) = \rho^{\mathrm{d}}(x,t) - \rho(x,t)$$

$$V^{\mathrm{d}}(x,t) = (f * \rho^{\mathrm{d}})(x,t)$$

$$V^{\mathrm{e}}(x,t) = (f * e)(x,t)$$

Theorem (Local macroscopic convergence)

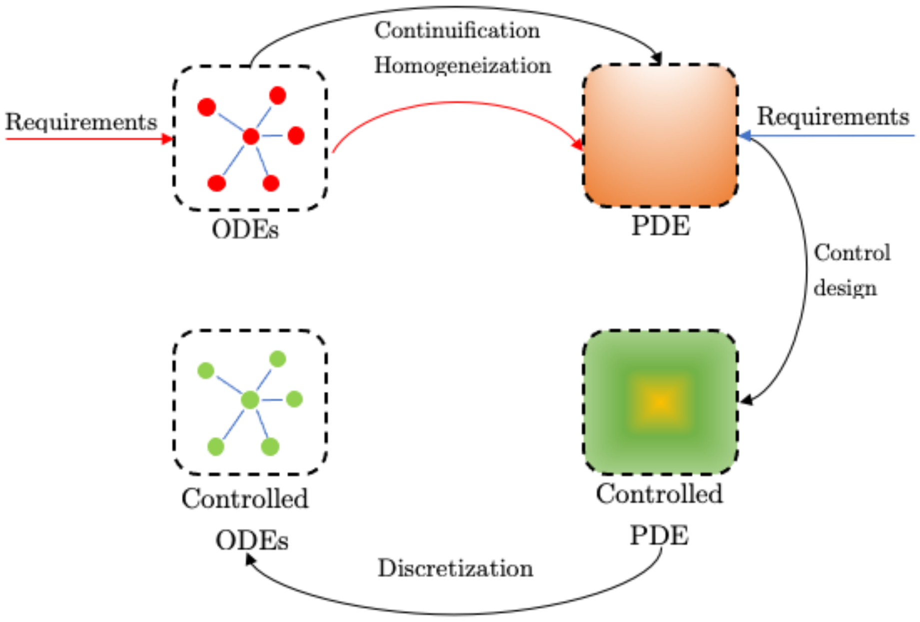
By choosing $q(x,t) = K_{\mathrm{p}} e(x,t) - \left[e(x,t)V^{\mathrm{d}}(x,t)\right]_x - \left[\rho^{\mathrm{d}}(x,t)V^{\mathrm{e}}(x,t)\right]_x$, the closed loop macroscopic dynamics converges globally asymptotically towards the desired density profile.

3. Discretization (macro -> micro)

- Recast the control action as an additional velocity field

$$\rho_t(x,t) + [\rho(x,t)(V(x,t)+U(x,t))]_x = 0$$

$$[\rho(x,t)U(x,t)]_x = -q(x,t)$$



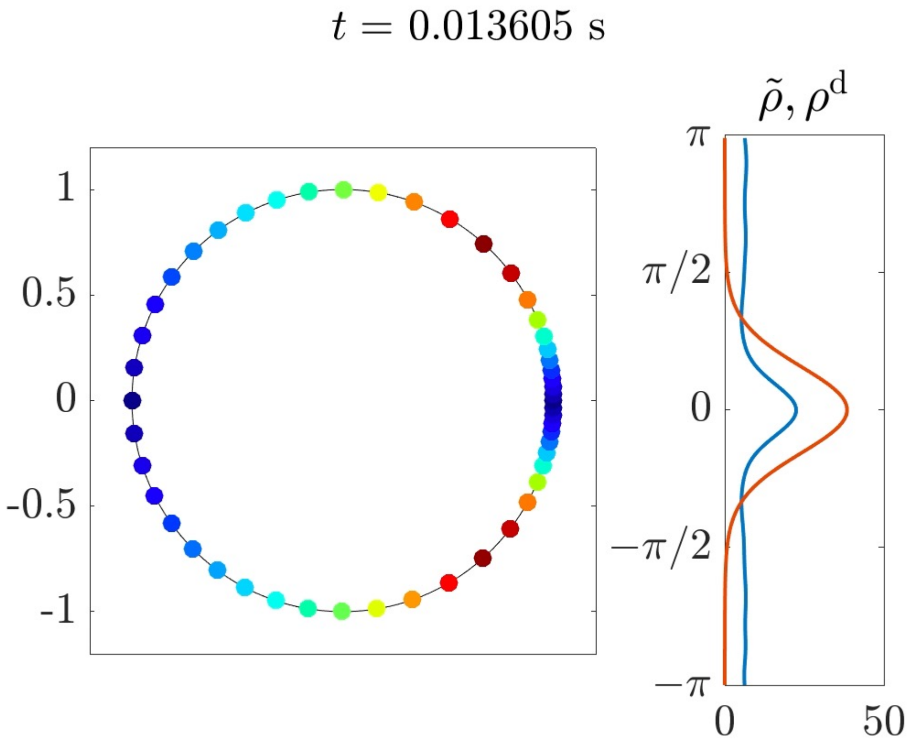
- Then discretize into microscopic control inputs

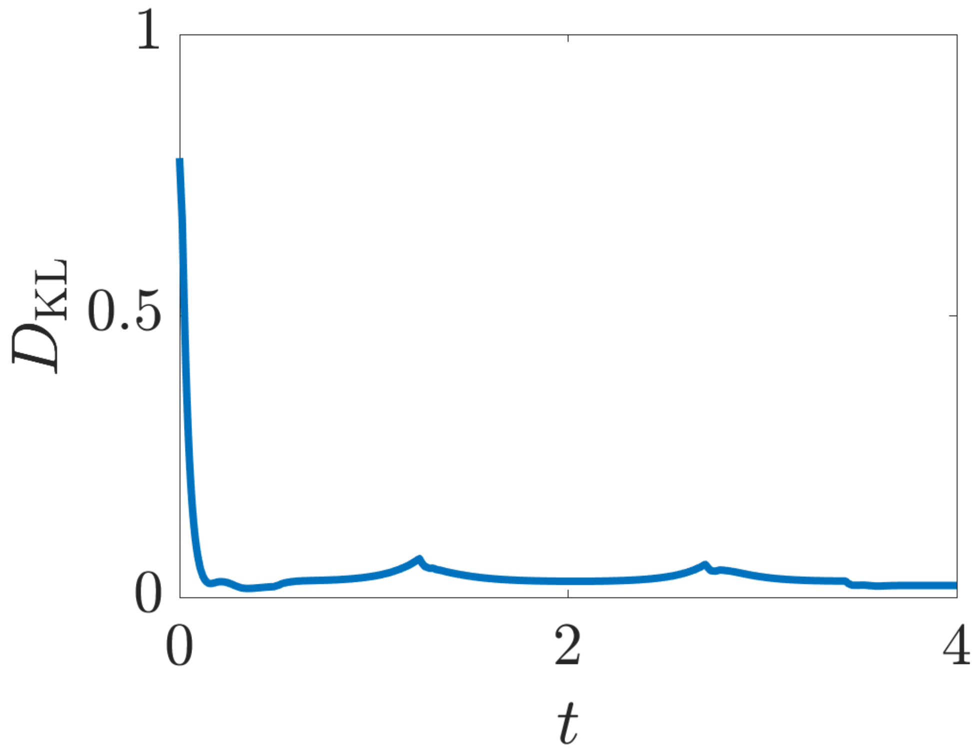
$$u_i(t) = U(x_i,t), \quad i = 1,2,\ldots N$$

Maffettone et al, "Continuification Control of Large-Scale Multiagent Systems in a Ring", IEEE Control Systems Letters 7 (2022), 841-846

Validation: tracking a time-varying distribution

Repulsive swarm





Overcoming a potential pitfall

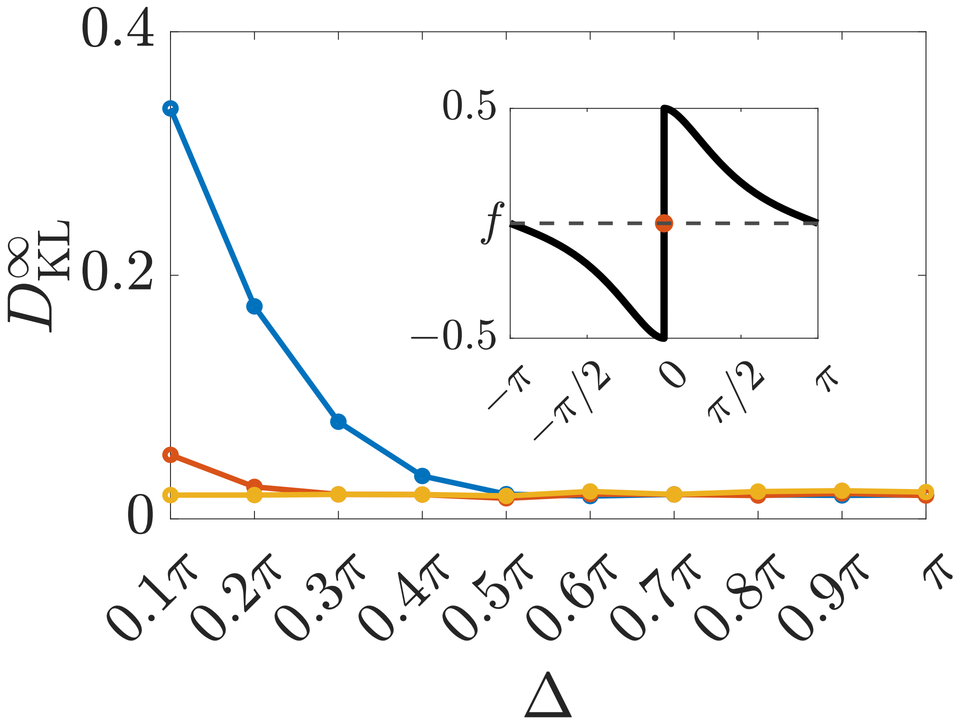
- **Problem:** control action is nonlocal (convolutions are involved)

$$q(x,t) = K_\mathrm{p} e(x,t) - \left[e(x,t)V^\mathrm{d}(x,t)\right]_x - \left[\rho^\mathrm{d}(x,t)V^\mathrm{e}(x,t)\right]_x$$

- What if we limit the spatial range of interaction?

$$\hat{f}(x) = \begin{cases} f(x) & \text{if } |x| < \Delta \\ 0 & \text{otherwise} \end{cases}$$

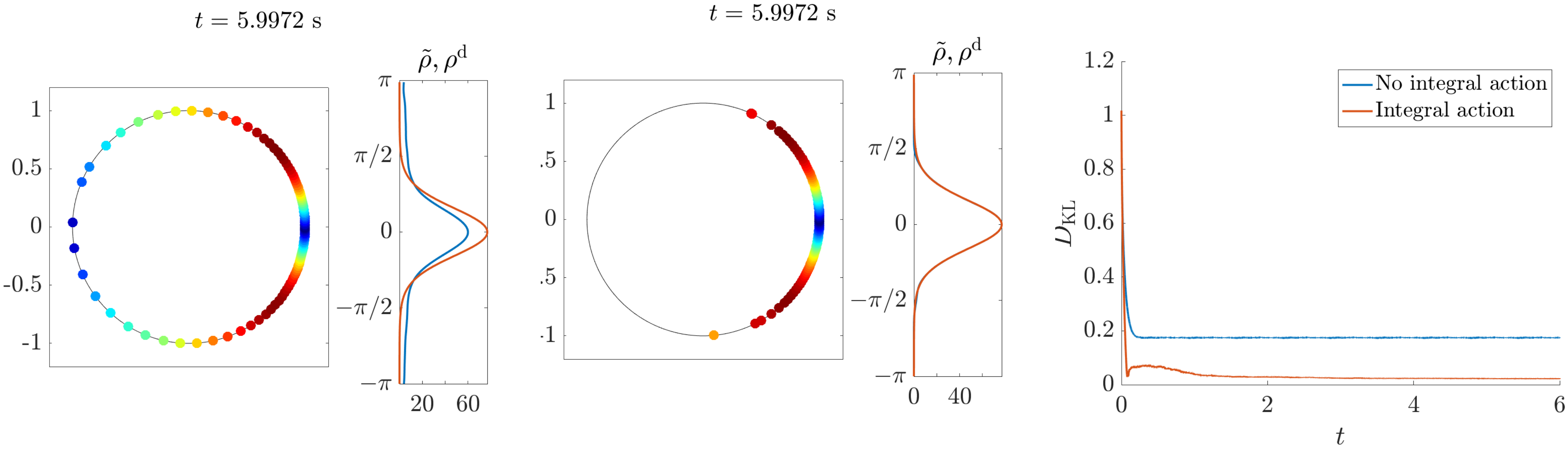
$$\hat{V}^\mathrm{e}(x,t) = \int_{x-\Delta}^{x+\Delta} f(\{x,y\}_\pi)e(y,t)\,\mathrm{d}y$$



Maffettone et al, "Continuification control of large-scale multiagent systems under limited sensing and structural perturbations, IEEE Control Systems Letters, 2023

Local control action

- In this case we get a residual control error (that decreases with the sensing radius)



$$e_t(x,t) = -K_{\mathrm{p}} e(x,t) - K_{\mathrm{i}} \int_0^t e(x,\tau)\,\mathrm{d}\tau$$

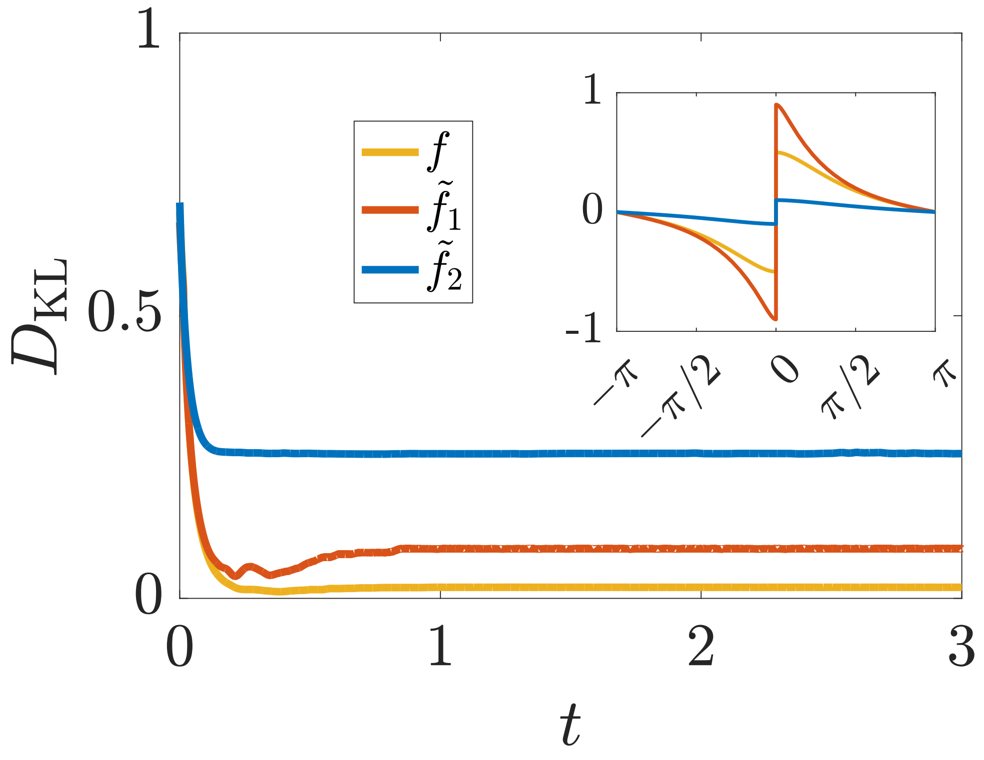
Robustness theorems

- We can prove robustness to
  - Limited sensing capabilities
  - Spatio-temporal perturbations of the velocity field $\rho_t(x,t) + [\rho(x,t)(V(x,t) + d(x,t))]_x = q(x,t),$
  - Perturbations of the interaction kernel itself

Theorem 2 (Bounded convergence in the presence of velocity perturbations) *There exists a threshold value $D_2 < \kappa < +\infty$ such that, if $2K_\mathrm{p} > \kappa$, the dynamics of the squared error norm is bounded and*

$$\limsup_{t\to\infty} \|e(\cdot,t)\|_2 \leq \frac{2LD_1 + 2MD_2}{\kappa - D_2}$$

Hence, the upper bound on the steady-state error can be made arbitrarily small by choosing κ sufficiently large.



Maffettone et al, "Continuification control of large-scale multiagent systems under limited sensing and structural perturbations, IEEE Control Systems Letters, 2023

Extension to higher dimensions

- The derivation can be applied almost as is to higher dimensions but..

$$\rho_t(\mathbf{x},t) + \nabla \cdot [\rho(\mathbf{x},t)\,(\mathbf{V}(\mathbf{x},t) + \mathbf{U}(\mathbf{x},t))] = 0$$

$$\nabla \cdot [\rho(\mathbf{x},t)\mathbf{U}(\mathbf{x},t)] = -q(\mathbf{x},t)$$

- To uniquely derive $\mathbf{U}(\mathbf{x},t)$ from $q(\mathbf{x},t)$, we need an extra condition

$$\mathbf{w}(\mathbf{x},t) = \rho(\mathbf{x},t)\mathbf{U}(\mathbf{x},t)$$

$$\mathbf{w}(\mathbf{x},t) \text{ periodic on } \partial\Omega$$

$$\begin{cases} \nabla \cdot \mathbf{w}(\mathbf{x},t) = -q(\mathbf{x},t) \\ \nabla \times \mathbf{w}(\mathbf{x},t) = 0 \end{cases}$$

Maffettone et al, Hybrid Platform for Swarm Robotics: Experiments and High-Dimensional Continuification Control, IEEE Trans. Robotics, in preparation, 2023

Discretization in higher dimensions

- The problem can be recast as a Poisson equation in terms of a scalar potential φ

$$\mathbf{w}(\mathbf{x},t) = -\nabla\varphi(\mathbf{x},t) \qquad \nabla^2\varphi(\mathbf{x},t) = -q(\mathbf{x},t) \qquad \nabla\varphi(\mathbf{x},t) \text{ periodic on } \partial\Omega$$

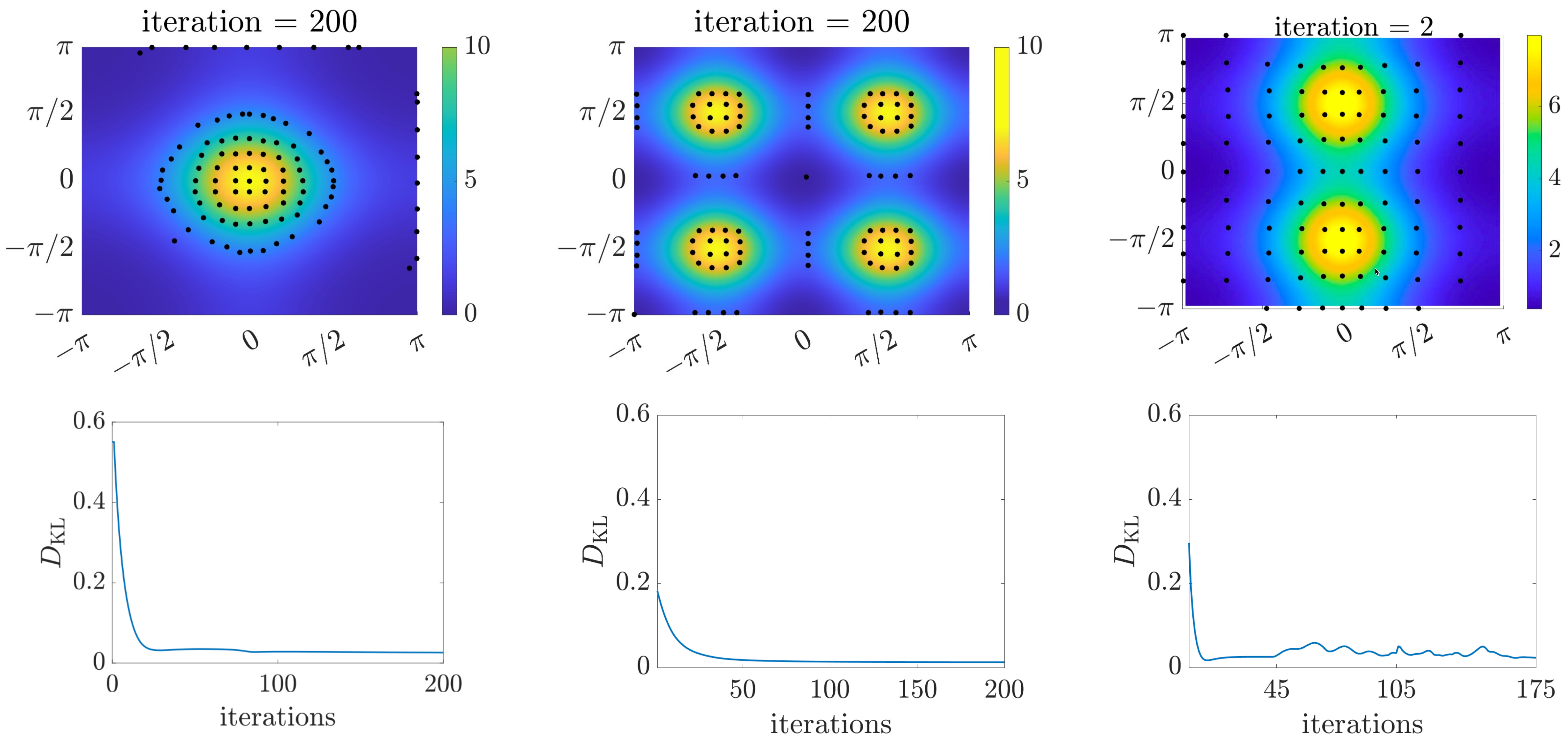
- This PDE can be solved using Fourier series expansion
- From the potential we can derive the flux $\mathbf{w}$, and

$$\mathbf{U}(\mathbf{x},t) = \mathbf{w}(\mathbf{x},t)/\rho(\mathbf{x},t)$$

- We then finalize the discretization by a spatial sampling

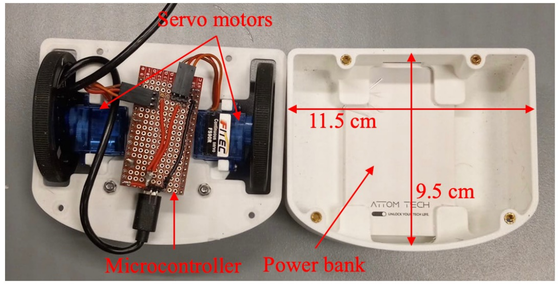
$$\mathbf{u}_i(t) = \mathbf{U}(\mathbf{x}_i,t), \quad i = 1,2,\ldots,N.$$

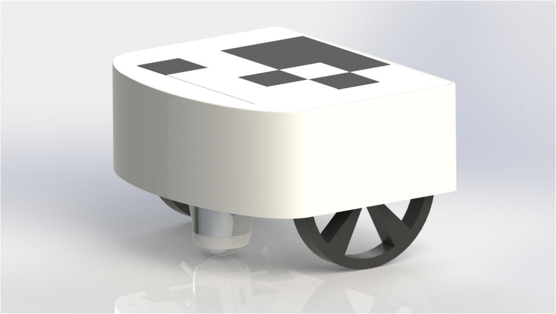
Numerical validation

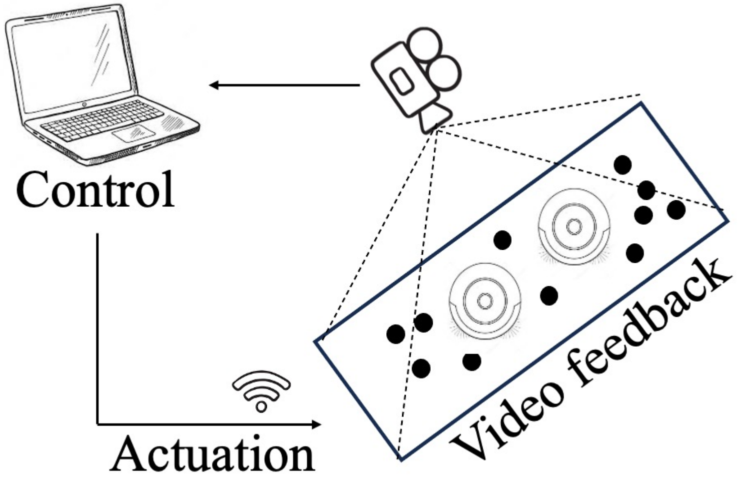


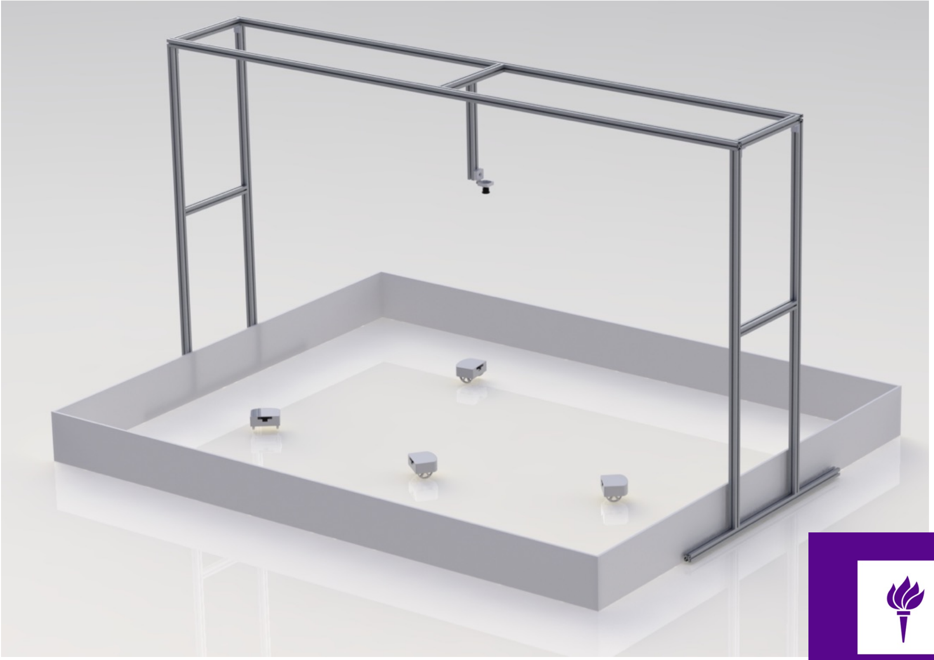
Hybrid experimental platform

- We built an experimental platform for validating control solutions for swarm robotics
- The platform is hybrid (part of the agents are virtualized) and allows to run full-scale experiments with large scale systems (time and costs)





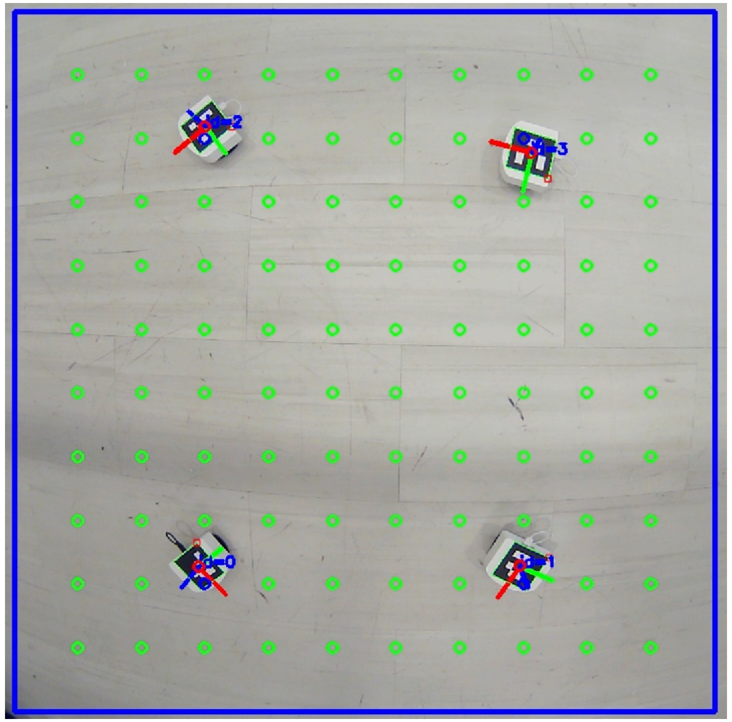




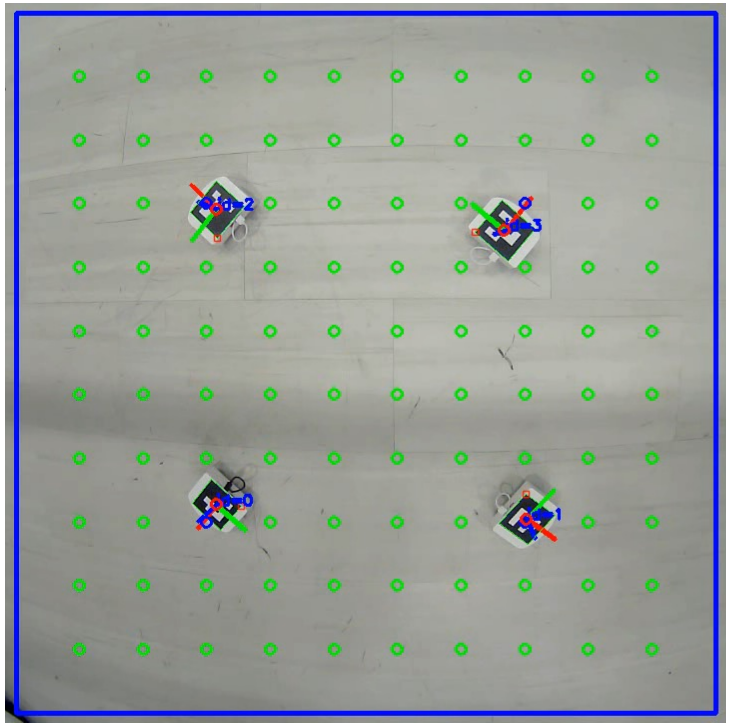
Experimental validation (4 robots, 96 virtual agents)

- An inner control loop is embedded on the robots to deal with their kinematic constraints
- The periodicity assumption is adapted considering a fictitiously extended domain

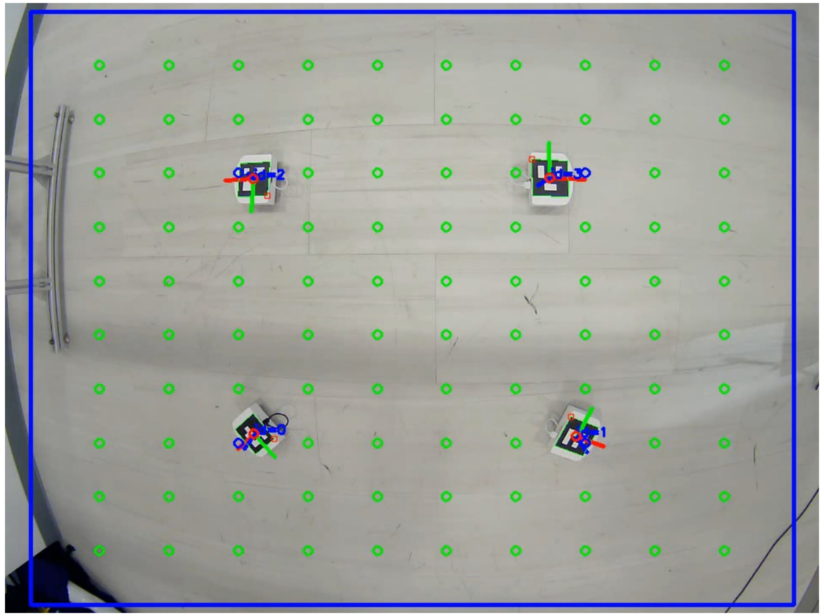
Monomodal regulation



Monomodal tracking



Multimodal tracking

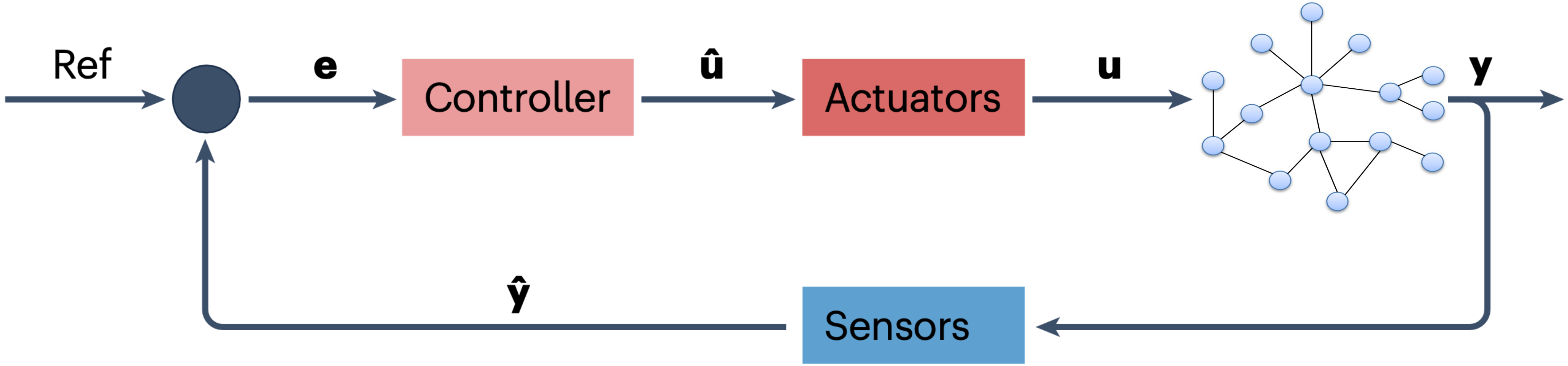


Remarks

- The approach seems to work well but there are many open problems:
- How do you find closures of the ABMs?
- Can you ensure control actions are local when discretized?
- How do you better characterize convergence?
- We are currently exploring the use of physics-informed learning methods to find closures (see e.g. work by Kevrekidis, Siettos et al)

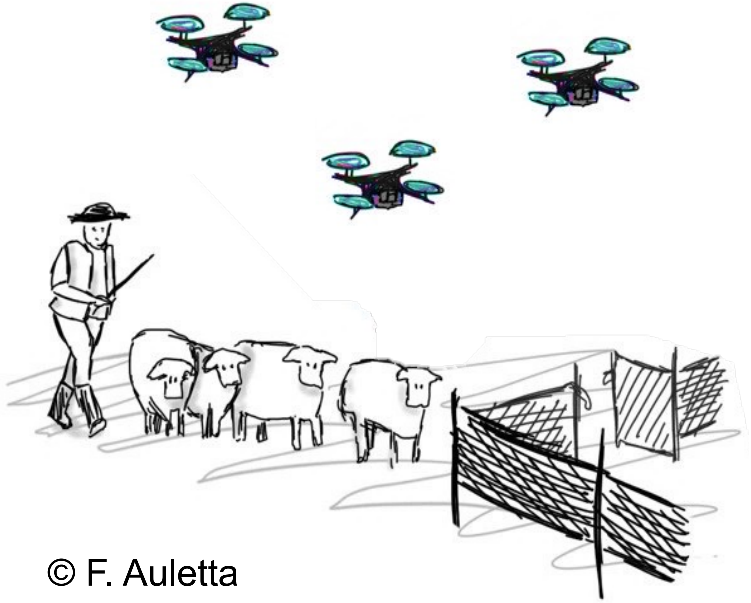
Complex systems for control

- So far we looked at *how to control* a complex system
- What if the complex system acts as the controller rather than being the system we wish to control?
- *Can we engineer the collective behaviour of a complex system to perform a control task?*



The shepherding problem

- A paradigmatic example is the shepherding problem
- Here a group of agents, *the herders*, need to steer the collective dynamics of another group of agents, *the targets,* in some desired way

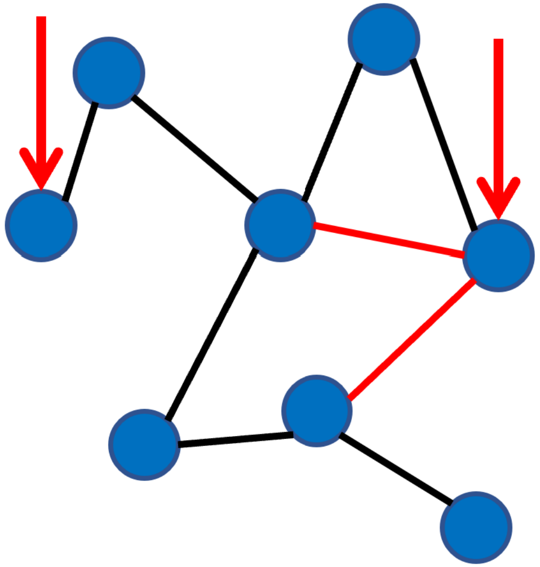


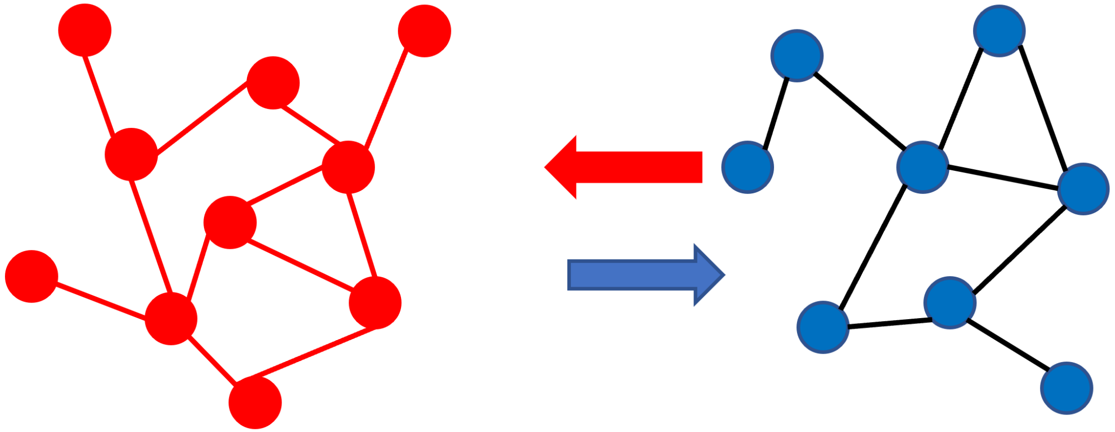
Relevance

- Observed in biological systems (e.g dolphins hunting fish [Haque et al, *2011,Int. J. Bio-Inspired Comp*], ants collecting aphids [Oliver et al,2007,Proc. R. Soc. B]
- Technological applications: search & rescue, crowd control, oil cleanup [Long et al, 2021, IEEE Emerging Comp applications]

A complex system performing a control task

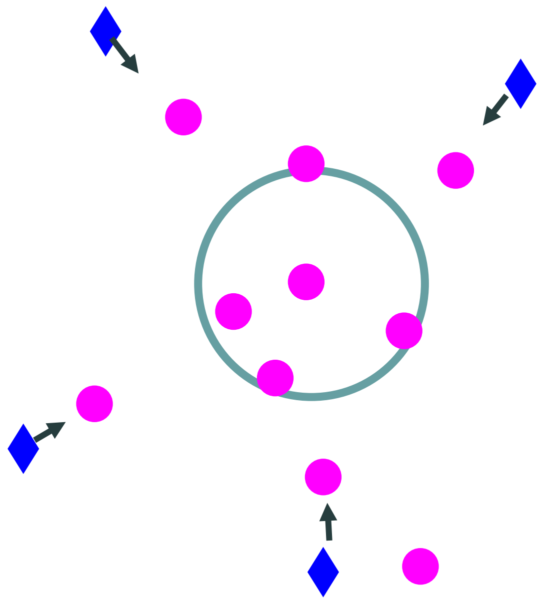
- In these situations the emerging collective behaviour of a complex system must be controlled by the driving the emerging behaviour of another complex system





Deciding the herding behaviour

- The crucial problem is the design of the herders’ dynamics so as to achieve the desired goal
- *Herders must cooperate with each other* and collectively implement decision-making strategies
- An intuitive solution is to rely on formation control or pre-computed optimal control solutions
- Or use virtual attractive/repulsive forces related to their relative positions with each other and the targets



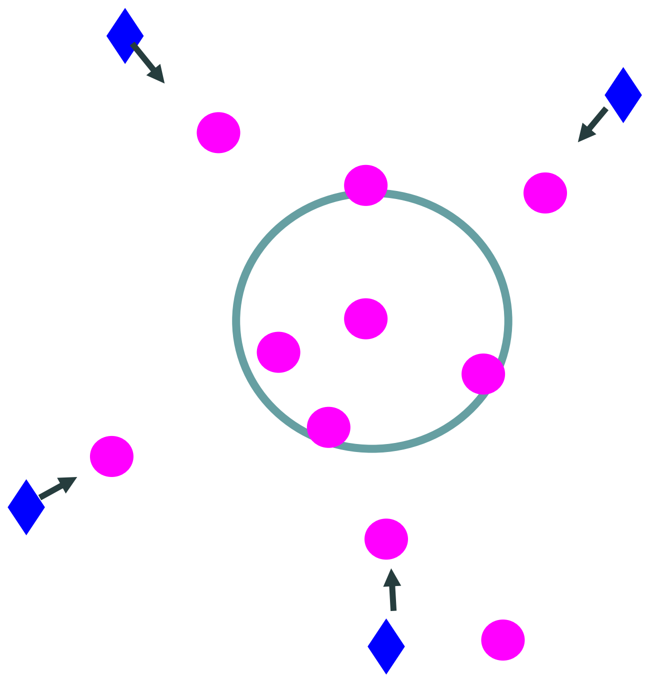
A Pierson, M Schwager, Controlling noncooperative herds with robotic herders, IEEE Trans Robotics 2018
R.A. Licitra, Z Bell, W Dixon, Single-agent indirect herding of mutliple targets with uncertain dynamics”, IEEE Trans Robotics, 2019
D. Ko, E. Zuazua, Asymptotic behaviour and control of “a guidance by repulsion model”, Math Models Methods Appl Sci, 2020

Two key assumptions

- All existing solutions are based on two key assumptions:

  1. Targets’ cohesiveness (e.g. flocking)
     [Pierson et al, IEEE T. Robotics, 2017; Licitra et al 2019]

  2. Herders’ Unlimited sensing
     [Auletta et al, Auton, Rob., 2022]



Key research question

- Also, current solutions do not exploit a crucial feature of complex systems..
- ..their ability of exhibiting emerging collective behaviour from simpler agent behaviour
- In this spirit, *shepherding solutions should not be engineered into the model* but could emerge out of the herders following simpler local engagement rules

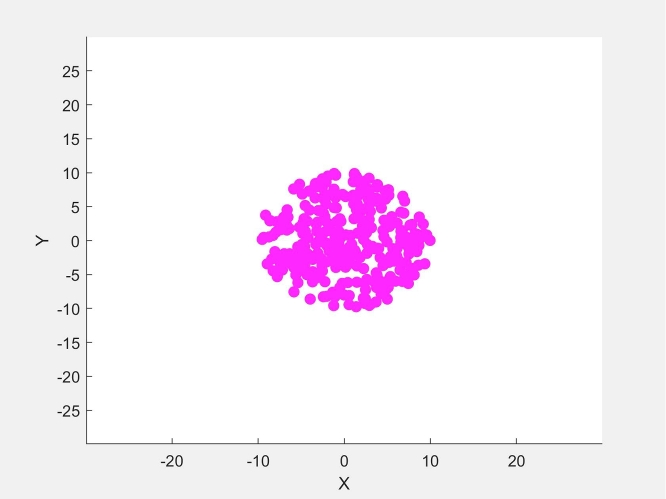
Can local simpler feedback rules solve the global herding control problem in the presence of limited sensing and non-flocking targets?

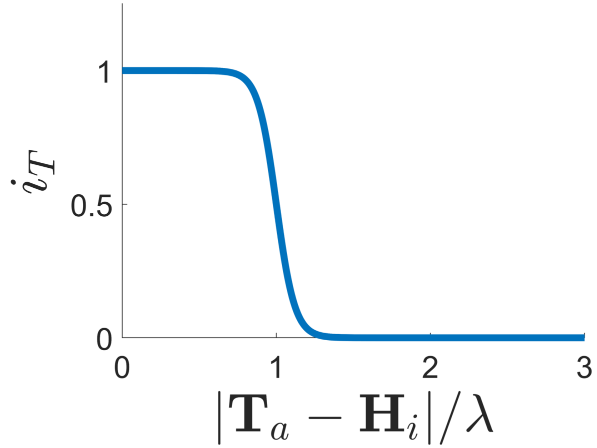
F. Auletta, D. Fiore et al , "Herding stochastic autonomous agents via local control rules and online global target selection strategies", *Autonomous Robots*, 2022

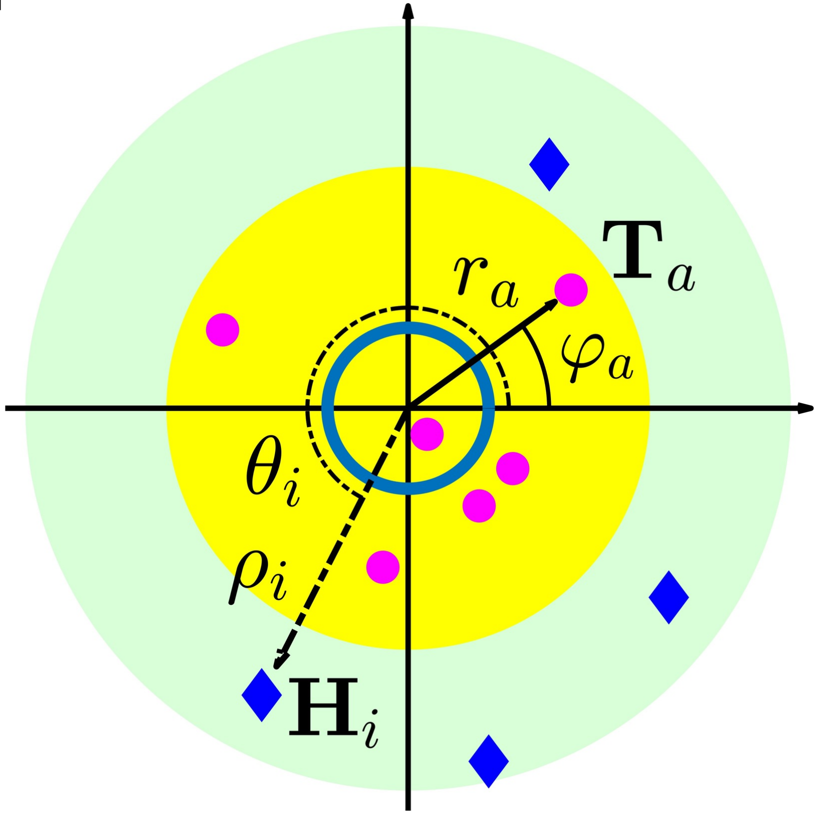
The planar shepherding problem

- A group of agents, *the herders*, is tasked with the goal of collecting and coralling another group of agents, *the targets* towards some goal region in the plane
- *M* targets, *N* herders initially distributed as shown

$$\dot{\mathbf{T}}_a = \sigma \mathbf{N}_a(t) + c\mathbf{I}_a^{TH}(\mathbf{H}, \mathbf{T}_a, \lambda)$$







A. Lama, MdiB, "Shepherding and herdability in complex multiagent systems", PRL, *under review*, 2023

Herders' local dynamics

$$\begin{cases} \dot{\rho}_i = -\alpha I_{i,\rho}^{HH}(\mathbf{H}, r^*) - \beta I_i^{HT}(\mathbf{H}, \mathbf{T}, \xi) + \Theta(\dot{\mathbf{H}}_i, v_H) \\ \dot{\theta}_i = -\alpha I_{i,\theta}^{HH}(\mathbf{H}) - \beta I_i^{HT}(\mathbf{H}, \mathbf{T}, \xi) + \Theta(\dot{\mathbf{H}}_i, v_H) \end{cases}$$

$$\begin{cases} I_{i,\rho}^{HH}(\mathbf{H}, r^*) = \sum_{j=i\pm1} (\rho_i - \rho_j) + (\rho_i - r^*) \\ I_{i,\theta}^{HH}(\mathbf{H}) = \sum_{j=i\pm1} (\theta_i - \theta_j) \end{cases}$$

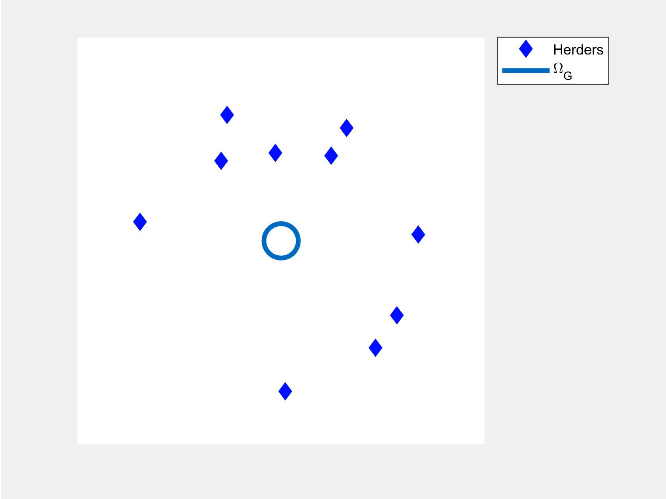
$$\begin{cases} I_{i,\rho}^{HT} = \rho_i - (\hat{r}_i + \lambda) \\ I_{i,\theta}^{HT} = \theta_i - \hat{\varphi}_i \end{cases}$$

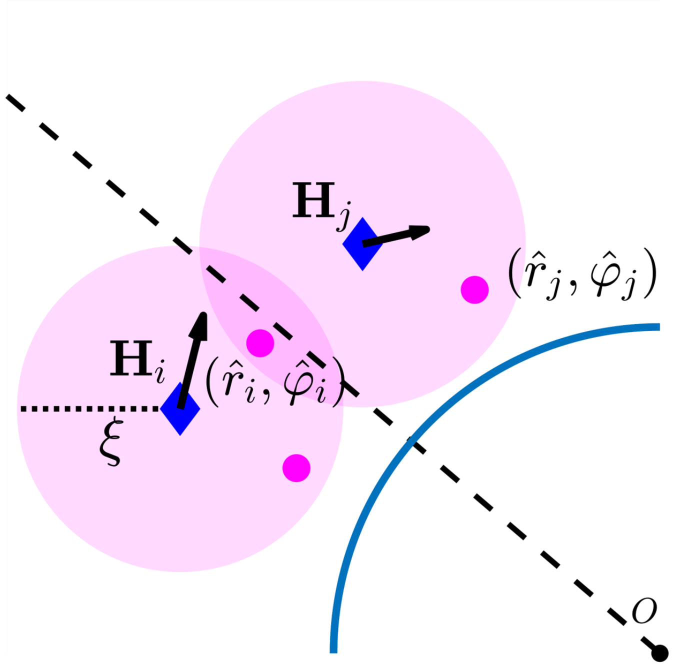
Selection rule

Herder i selects the furthest target from Ω_G $(\hat{r}_i, \hat{\varphi}_i)$

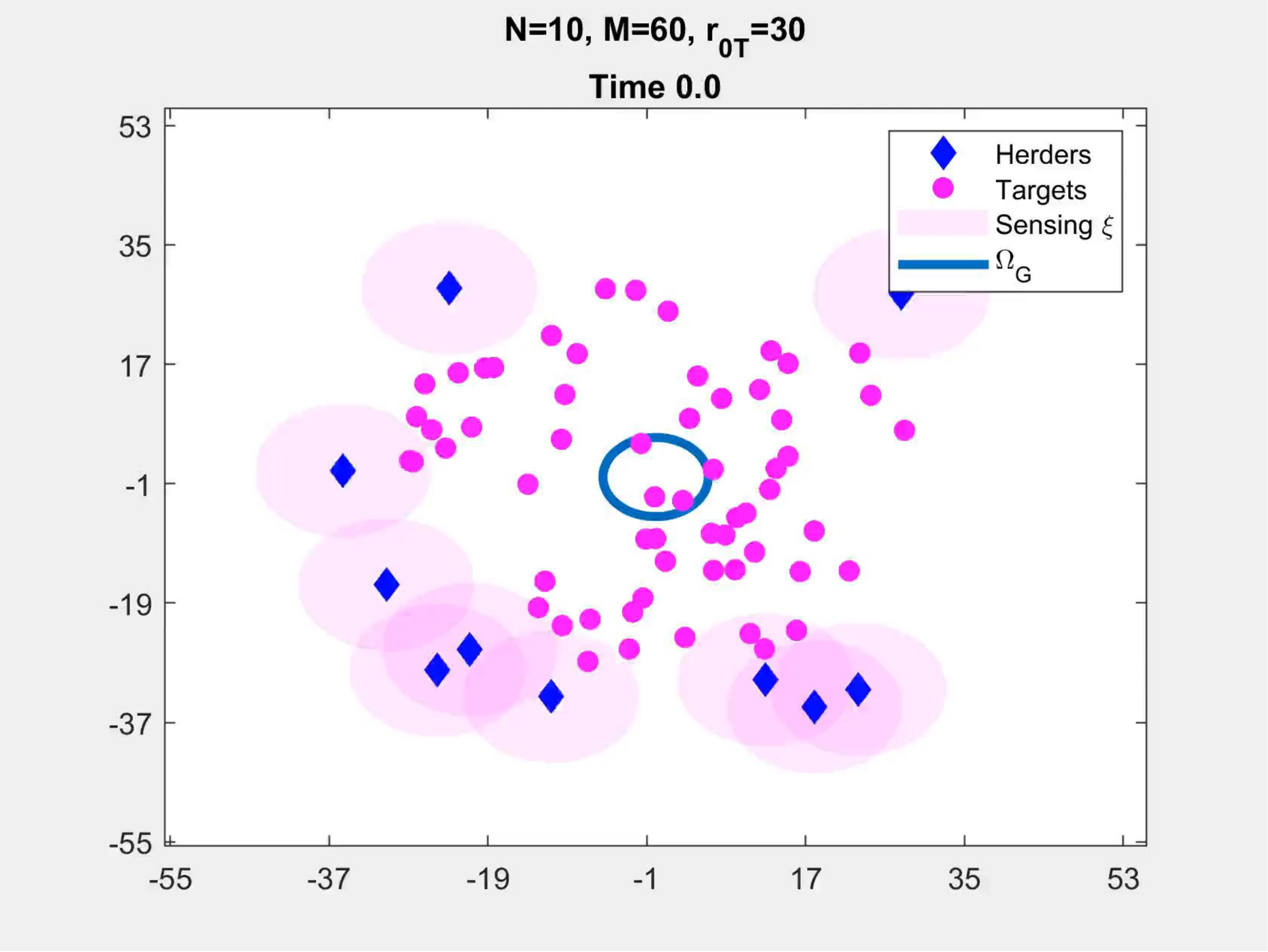
1. Within sensing radius ξ
2. With $\varphi \in \left[\frac{\theta_i+\theta_{i-1}}{2}, \frac{\theta_i+\theta_{i+1}}{2}\right]$

   Outside Ω_G



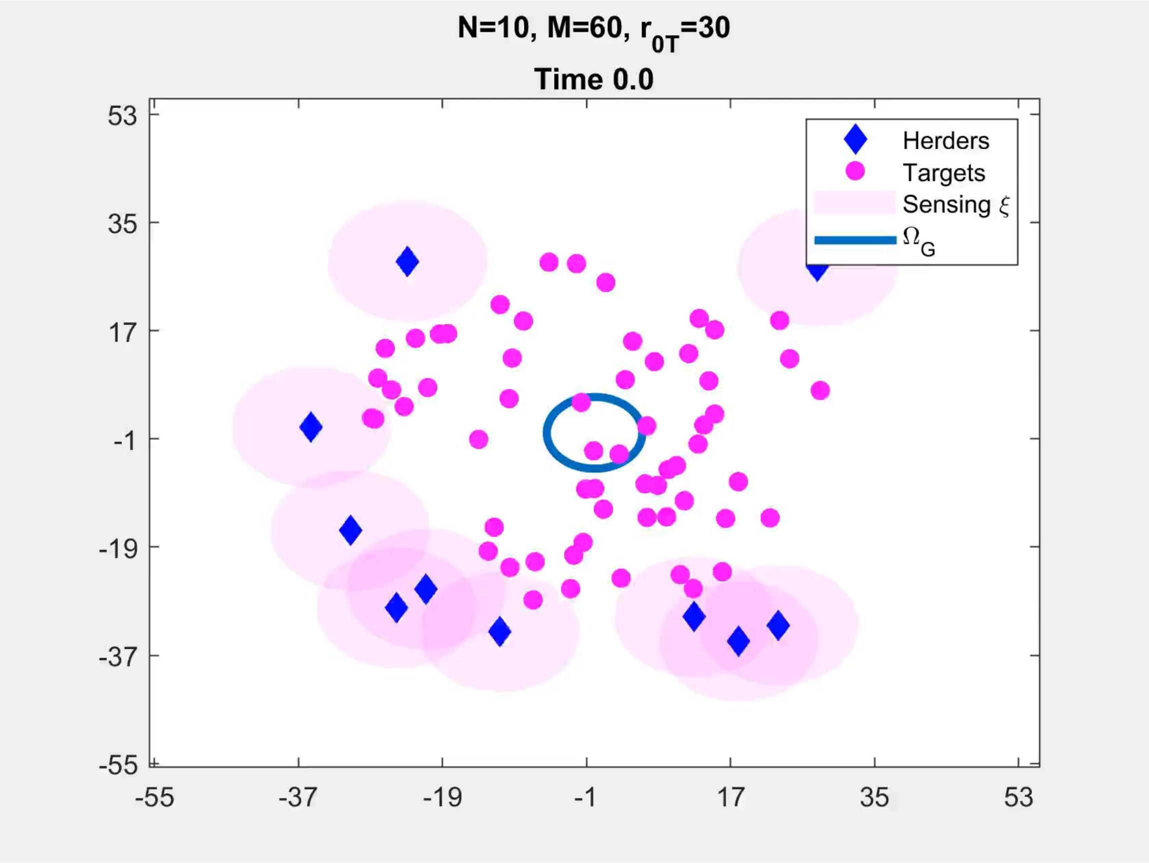


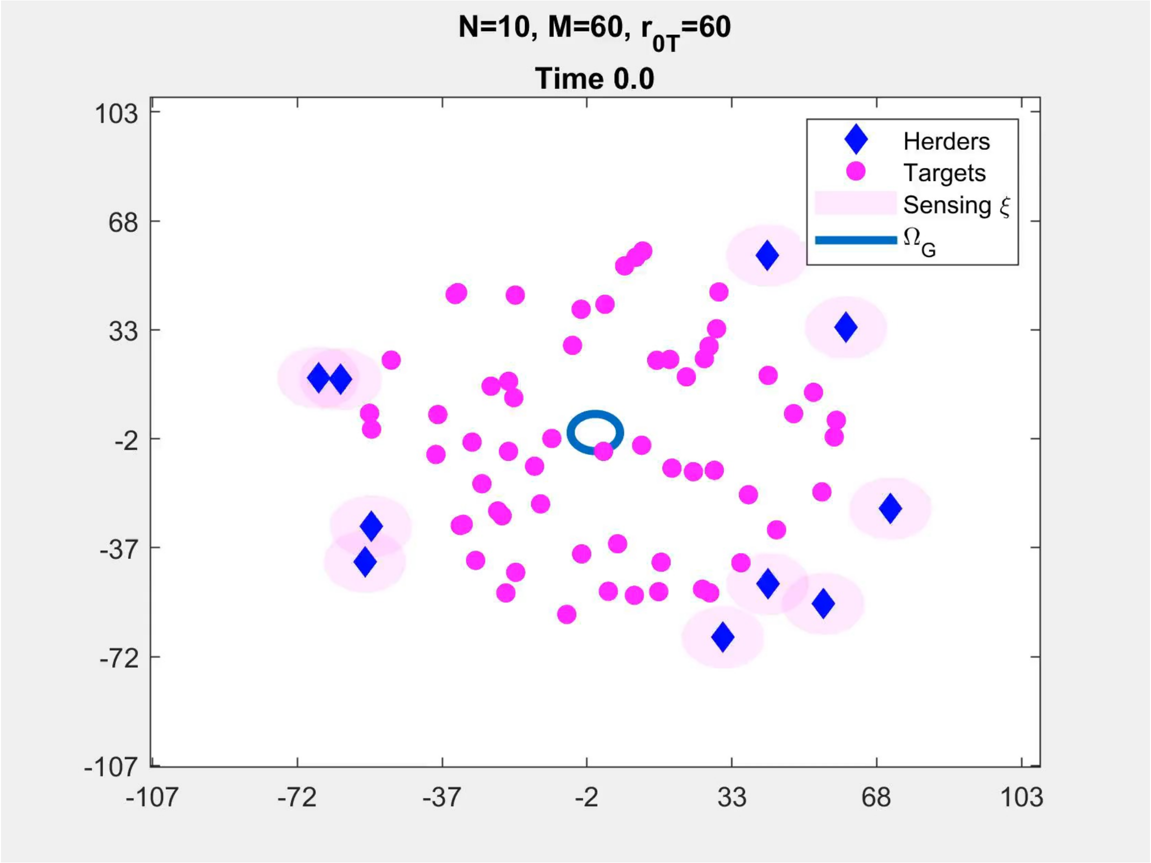
Shepherding can be successful



The herdability problem

- Under what conditions on the repulsion zone, the sensing area and the density of the targets can we achieve herdability of a given number of targets?

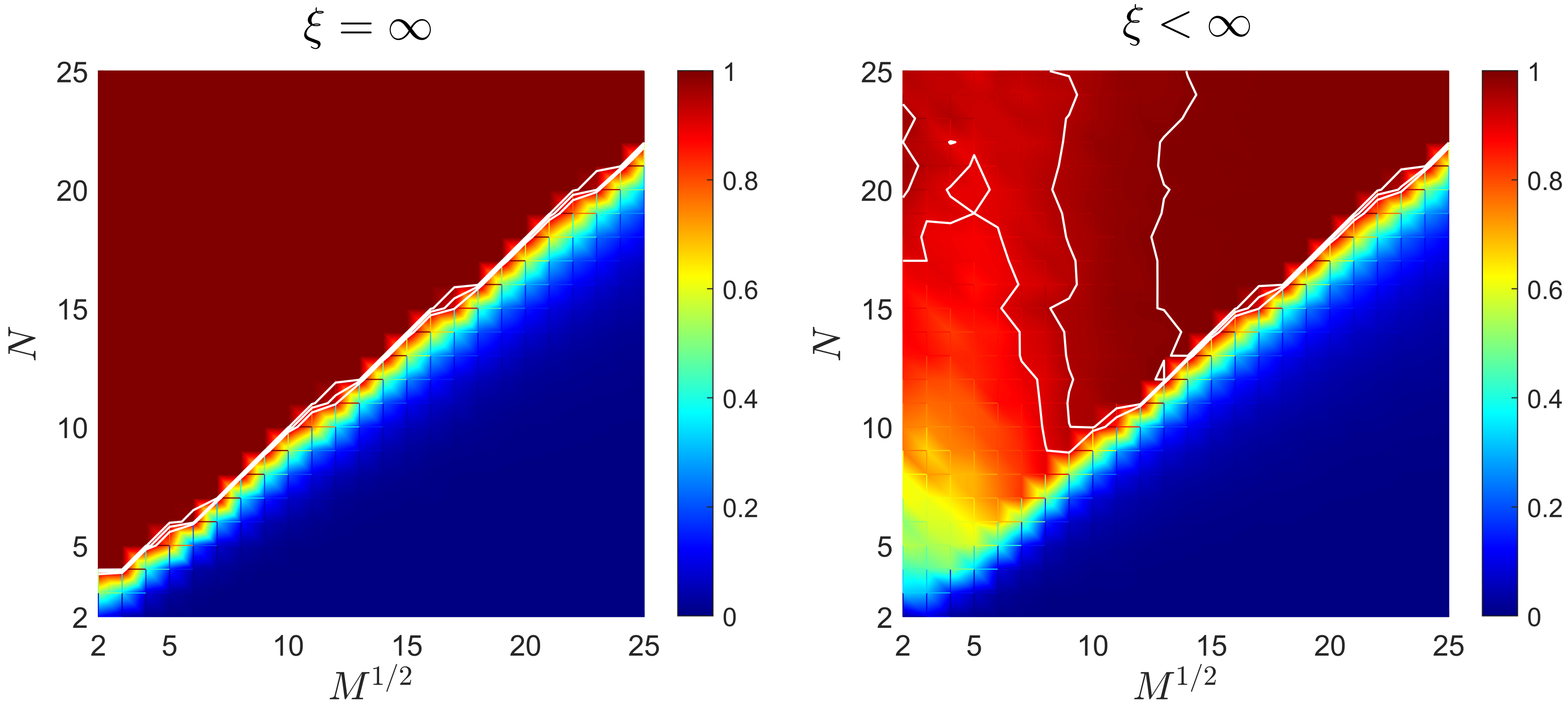




S. Ruf, M. Egerstedt, J Shamma, ,"Herding complex networks", online communication; Di Pasquale, Valcher, Automatica, 2023

Herdability conditions

- We look for the minimum number of herders $N^*(M)$ necessary to herd M targets



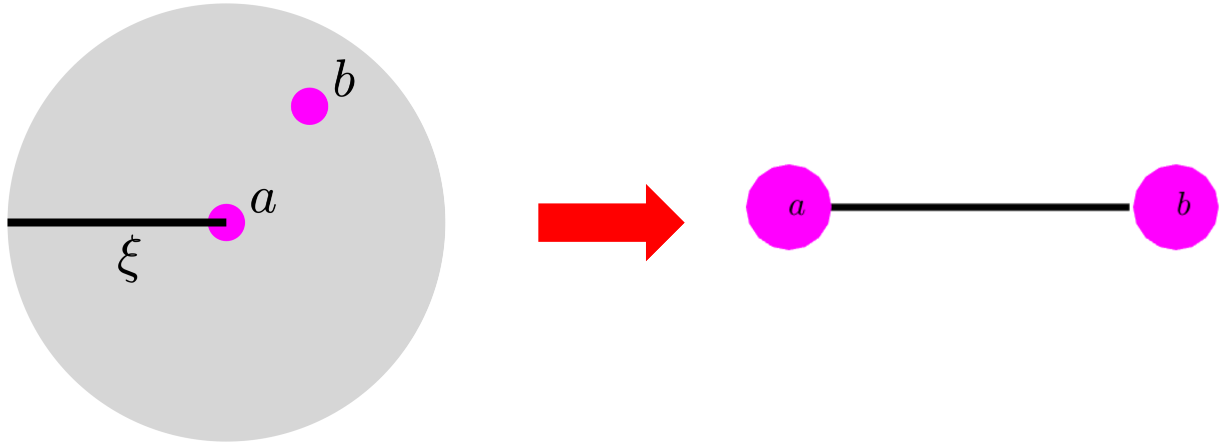
The herdability graph

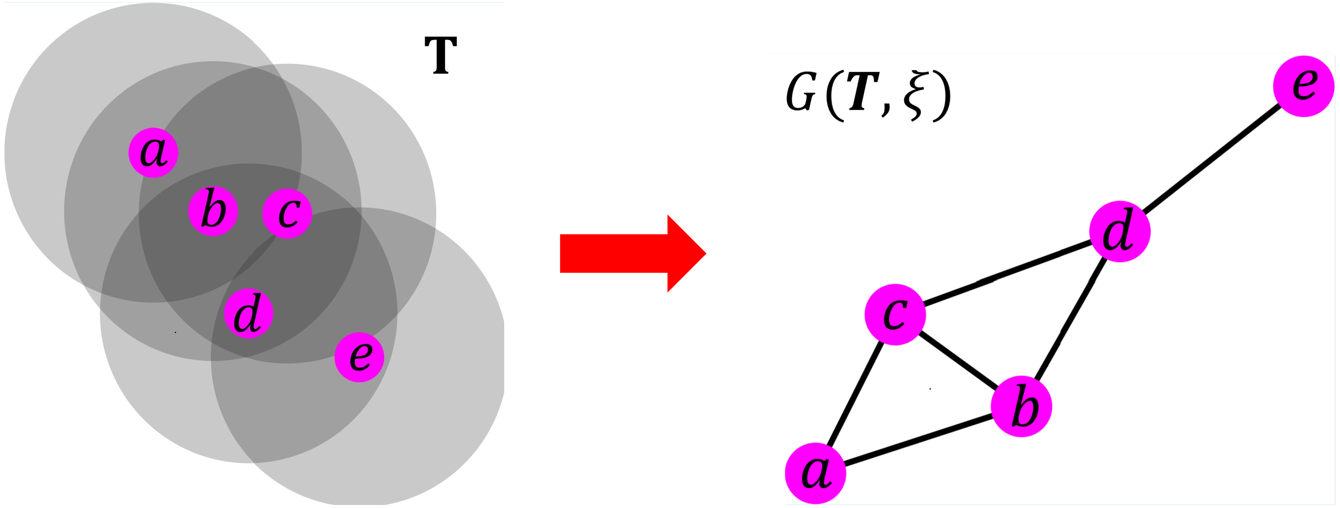
- We define the herdability graph

$$G_{ab}(\boldsymbol{T}, \xi) = 1$$

$$\text{if } |T_a - T_b| \leq \xi$$

- Assume that if there is a path on *G* between *a* and *e* the herder is theoretically able to switch from *a* to *e*
- Then study herdability in term of its *percolation*

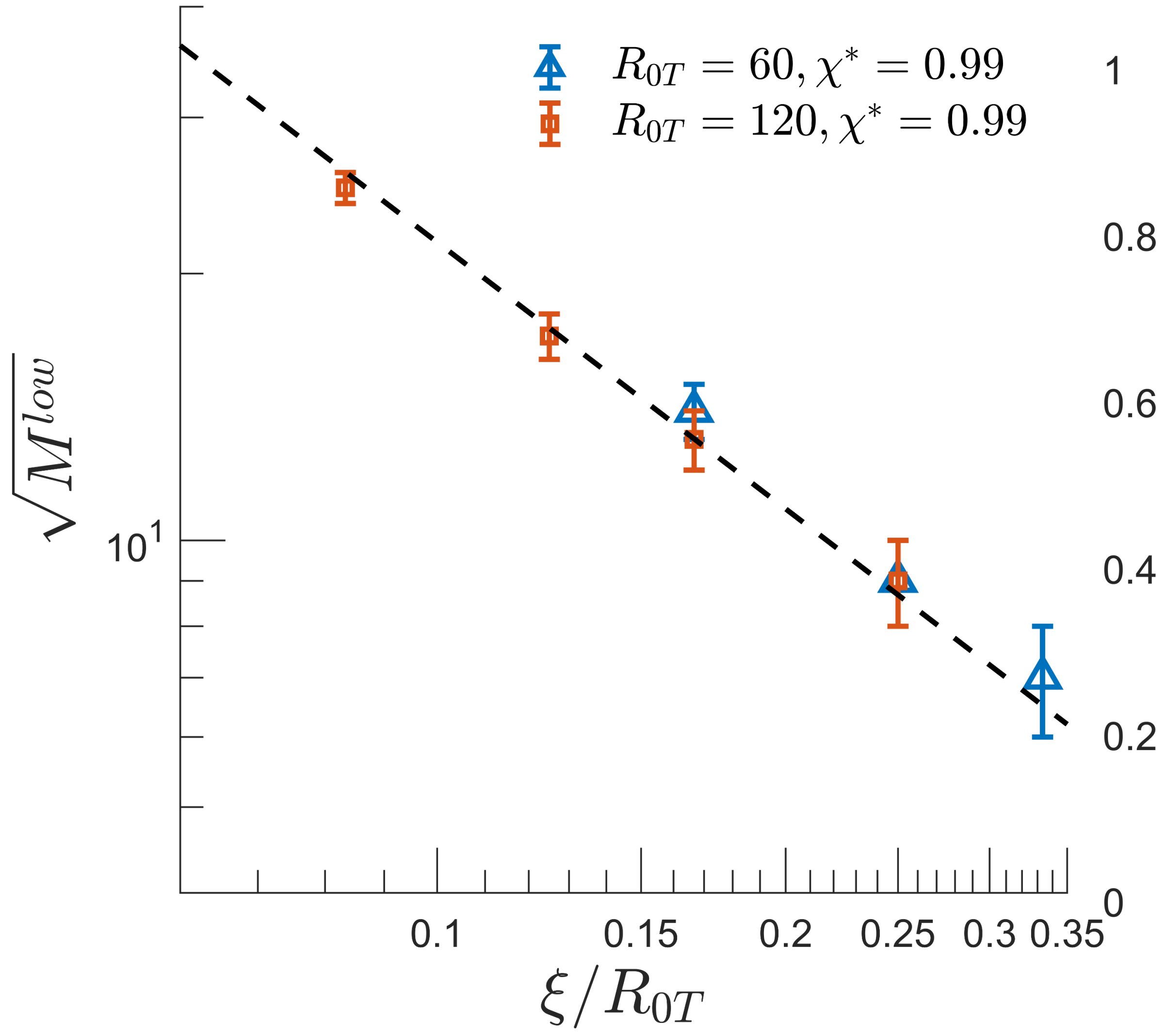




A. Lama, MdiB, ”Shepherding and herdability in complex multiagent systems”, PRL, *under review*, 2023

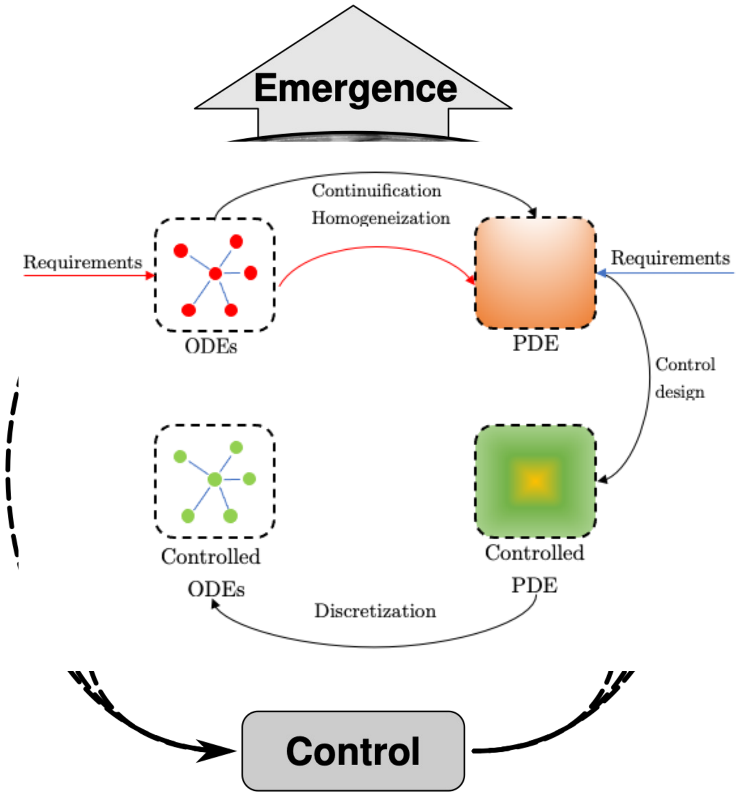
Percolation analysis

- We study when *G* becomes initially connected
- Percolation at $M = \widehat{M^{low}} \sim \frac{1}{\xi^2}$
- We use this as an estimate of the transition observed in the herdability diagrams
- Also, we can capture the scaling of the threshold wrt to ξ and R_{0T} ..
- ..and explain the observations and the scaling observed in the numerical experiments



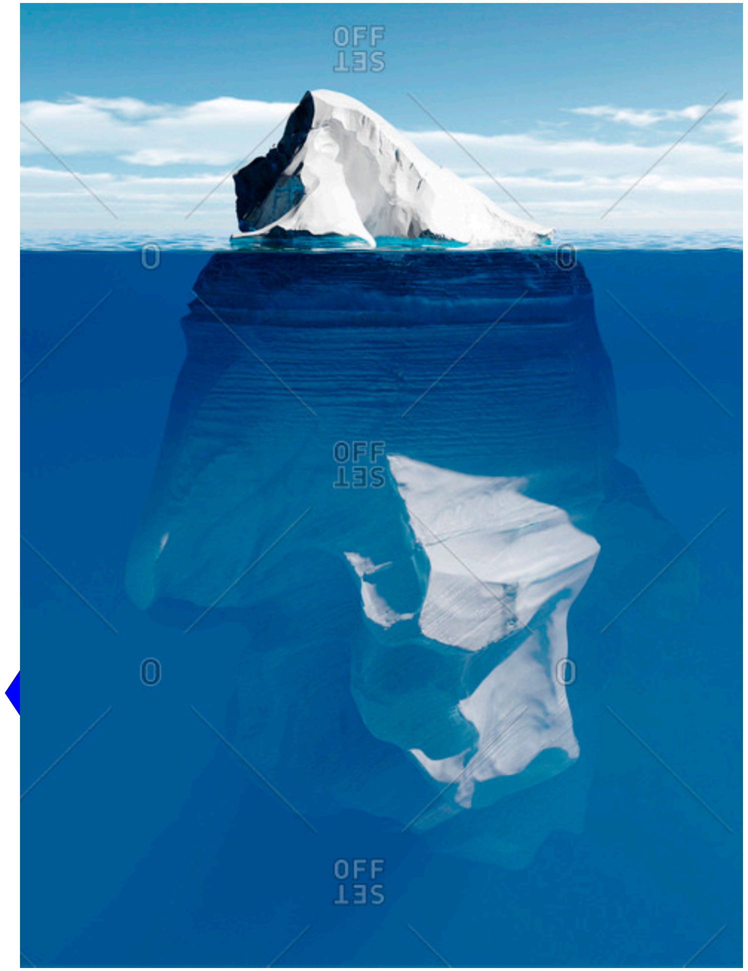
Conclusions

- Complex systems can be controlled by devising *multi-scale* feedback control strategies comprising sensing, computing and actuation
- *Continuification-based* approaches might be a solution
- Also, complex systems can solve control tasks where the control strategy emerges out of simple local rules of interaction
- *Shepherding* is a great paradigmatic example..
- ..where emerging behaviour needs to arise from a complex system in order to solve a control task

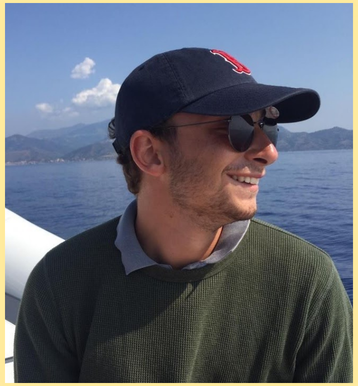


Challenges and open problems

- How do we engineer local rules of interaction for more complex tasks than shepherding?
- What if the targets actively escape from herders?
- What if the herders actively seek targets?
- Can we use multi-agent reinforcement learning?
- Can we use a continuification approach?
- Can we prove convergence?
- Lots of opportunities for further research!



Acknowledgements



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__Funding sources__: EU, Italian Ministry of Research and University, UK EPSRC

Thank you for your attention.

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