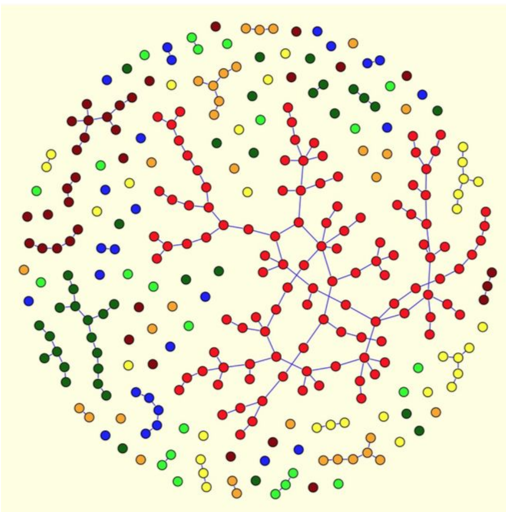
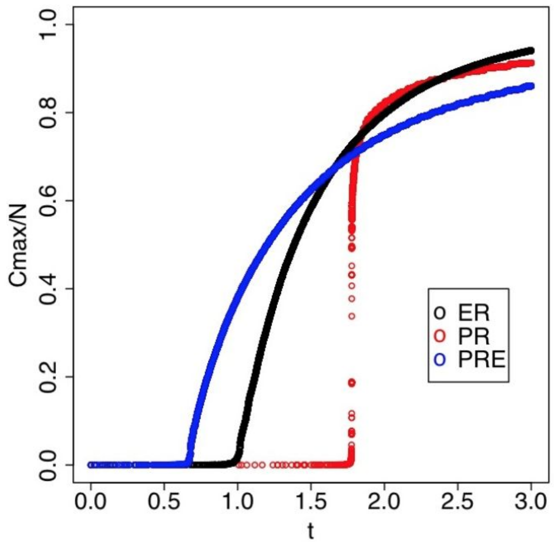
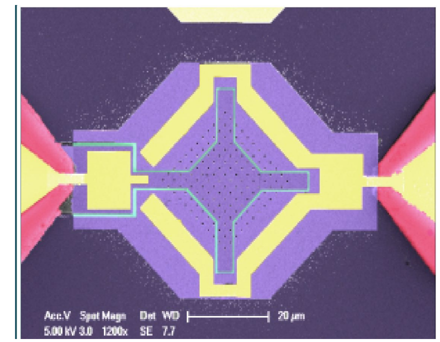
Complex networks with complex nodes







Raissa M. D’Souza

University of California, Davis

COMPLEXITY SCIENCE HUB VIENNA

UCDAVIS

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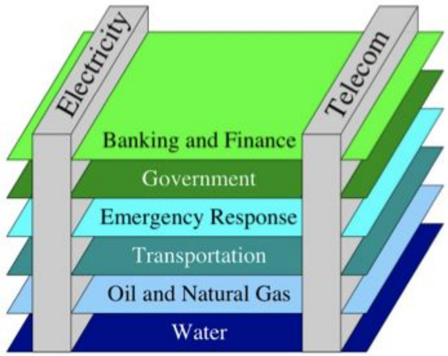
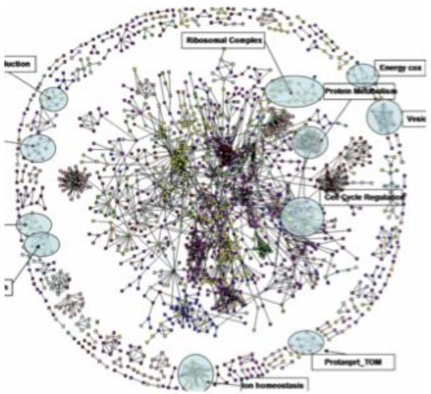
Structure and function of interdependent networks



Critical Infrastructure



Biological & Ecological networks

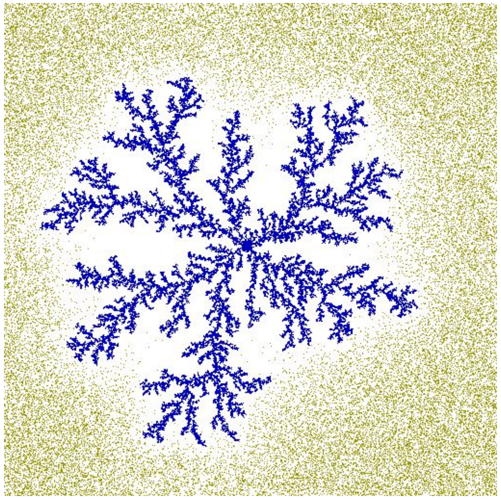
Social networks: Economics & Epidemics

Information and Communication technology

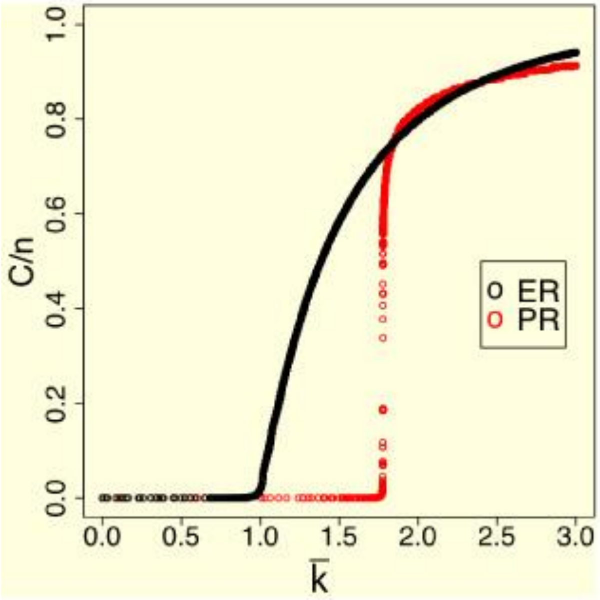
Each network is a complex system with emergent behaviors

What are emergent collective behaviors?

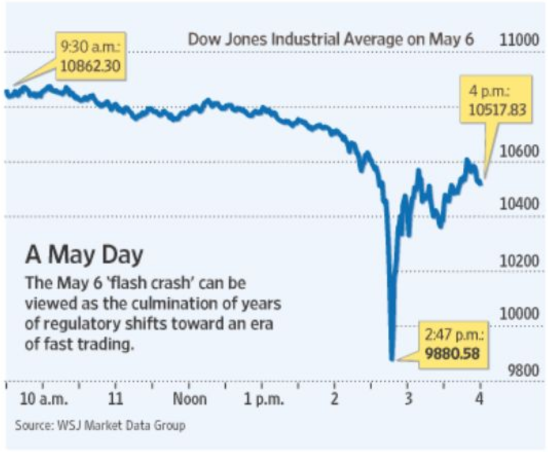
Behaviors not predicted a priori from the constituent equations of motion.



Synchronization and pattern formation.



Phase transitions
"Tipping points"



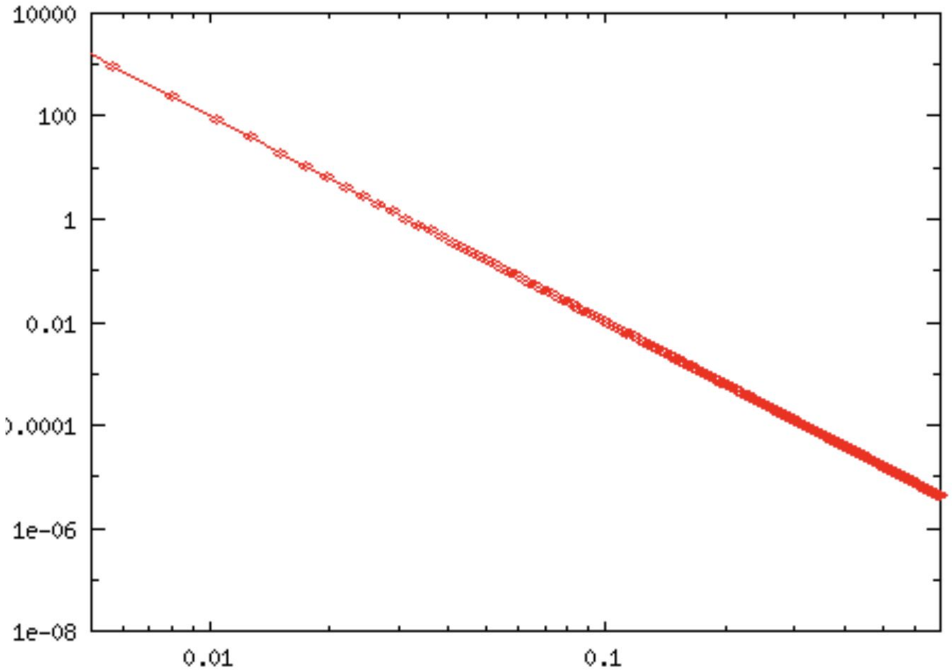
Cascading failures.

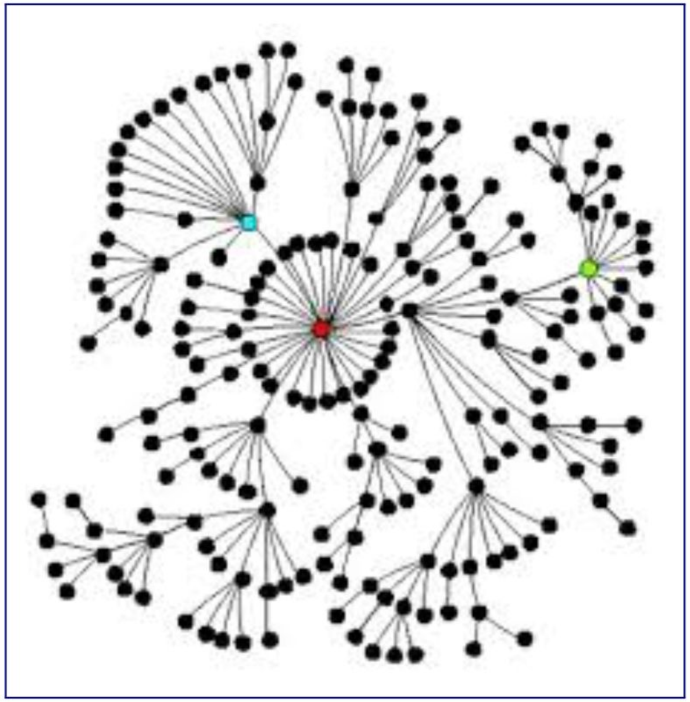
Statistical physics approach to networks

Start with a "random graph"

The Degree distribution, p_k

(Fraction of nodes of degree k)



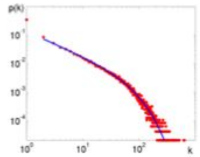


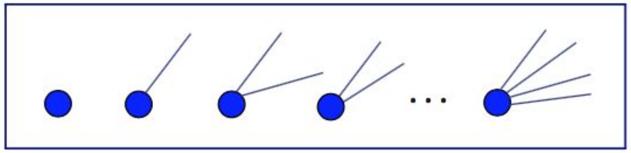
A power-law distribution is a "scale-free" network

Probabilistic properties of the **ensemble** of graphs.

Calculating properties of the random graph

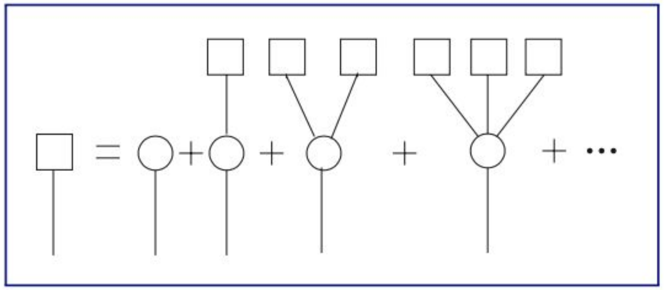
- **Configuration models** (Bollobás 1980, Molloy and Reed *RSA* 1995).
  Enumerating over all networks with specified degree distribution $\{p_k\}$.





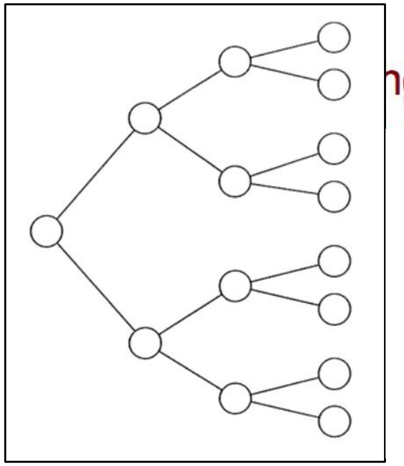
Start with half-edges.
Assign a random matching
to create an instance.

- **Generating functions :** $G_0(x) = \sum_k p_k \, x^k$



$$p_c = \frac{1}{g_1'(1)} = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}.$$

Criteria for giant component



ng properties →

Caveat:
Requires
locally
tree-like
structure

Properties of the random graph

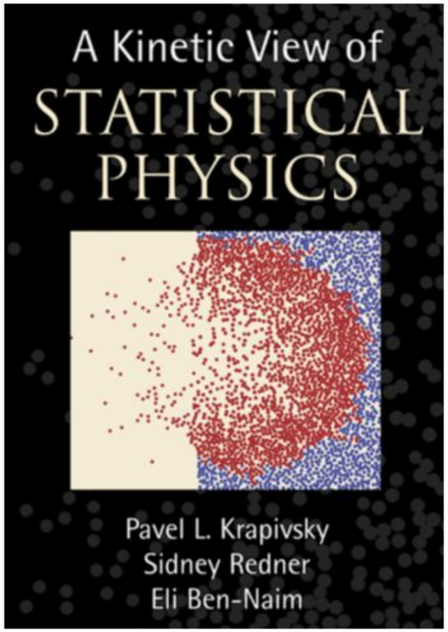
- **Rate equations / Kinetic theory** :
  Mean-field evolution of clusters/ graph structures.

$$x_k(t+1) = F\left(\vec{x}(t)\right)$$

e.g., "preferential attachment":

$x_k \equiv$ fraction of nodes of degree k:

$$x_k(t+1) = \tfrac{k-1}{n} x_{k-1}(t) - \tfrac{k}{n} x_k(t)$$



Cambridge U. Press, 2010

Analyzed for asymptotic properties: $N \to \infty$ and $t \to \infty$.

Achievements of random graphs

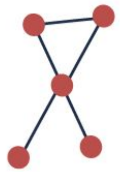
- **Vulnerability** to "hub" removal / **resilience** to random removal for broad-scale degree distributions.
- **Epidemic spreading**
- **Epidemic threshold can approach zero!**
- **Percolation and extent of connectivity**
- **Critical thresholds for cascades**
- **Diffusion and spreading**
- **Opinion dynamics, voter models, etc**

Statistical physics of networks

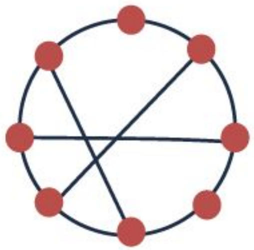
- Thermodynamic limit $N \to \infty$
- Equilibrium / Steady-state behavior
- Ensemble properties

- Simple nodes/edges (e.g., often binary state)

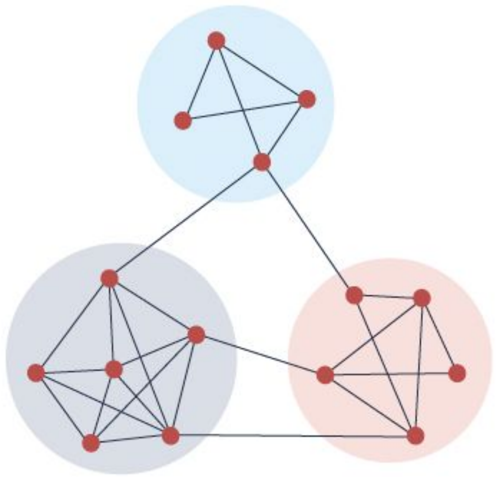
- Non-trivial network structure, “complex networks”:
  - Broad-scale degree distributions,



- Clustering/ triangular closure



- Small-worlds



- Community structure

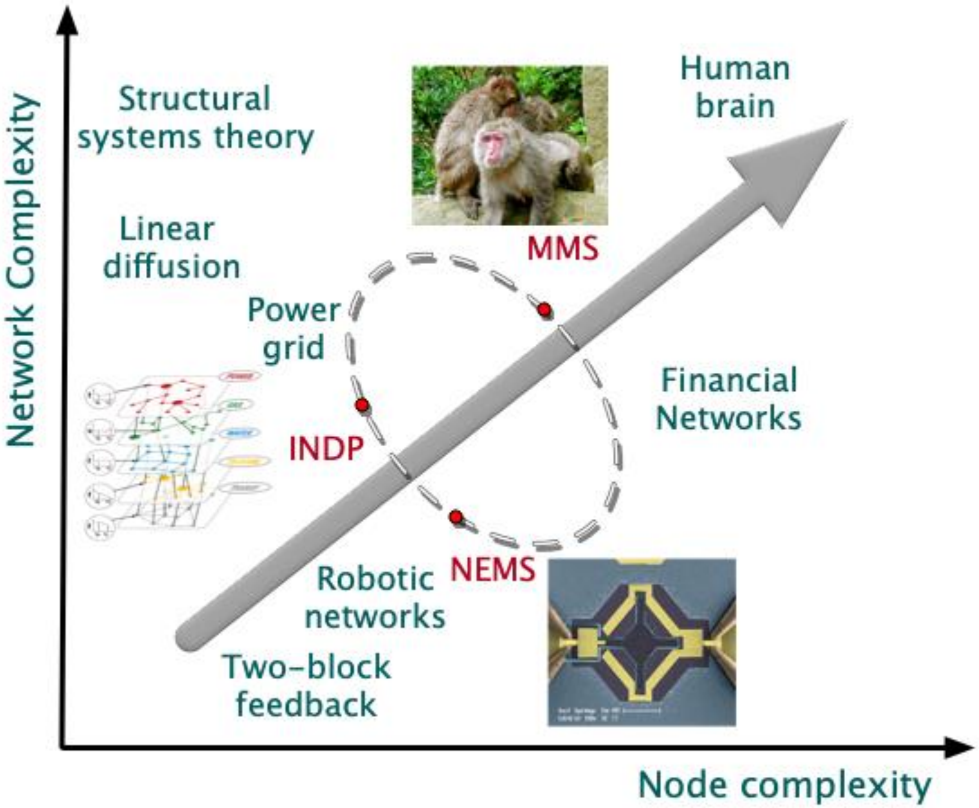
Origin of emergent collective behaviors?

Where is the complexity?

In the network structure?

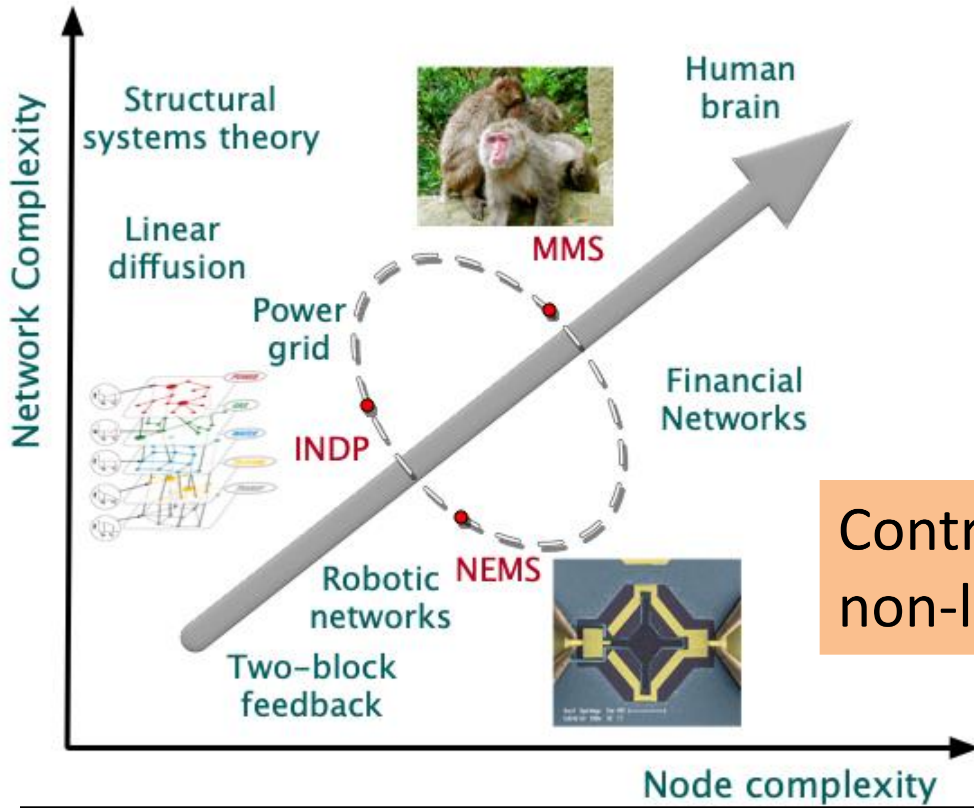
In the nodal dynamics?

In both?



Complex networks with complex nodes

Statistical Physics



Control theory and non-linear dynamics

ARO FY2013 MURI TOPIC #7

Submit white papers and proposal to Army Research Office

Controlling Collective Phenomena in Complex Networks

Thank you to the MURI team W911NF-13-1-0340 nature reviews physics https://doi.org/10.1038/s42254-023-00566-3

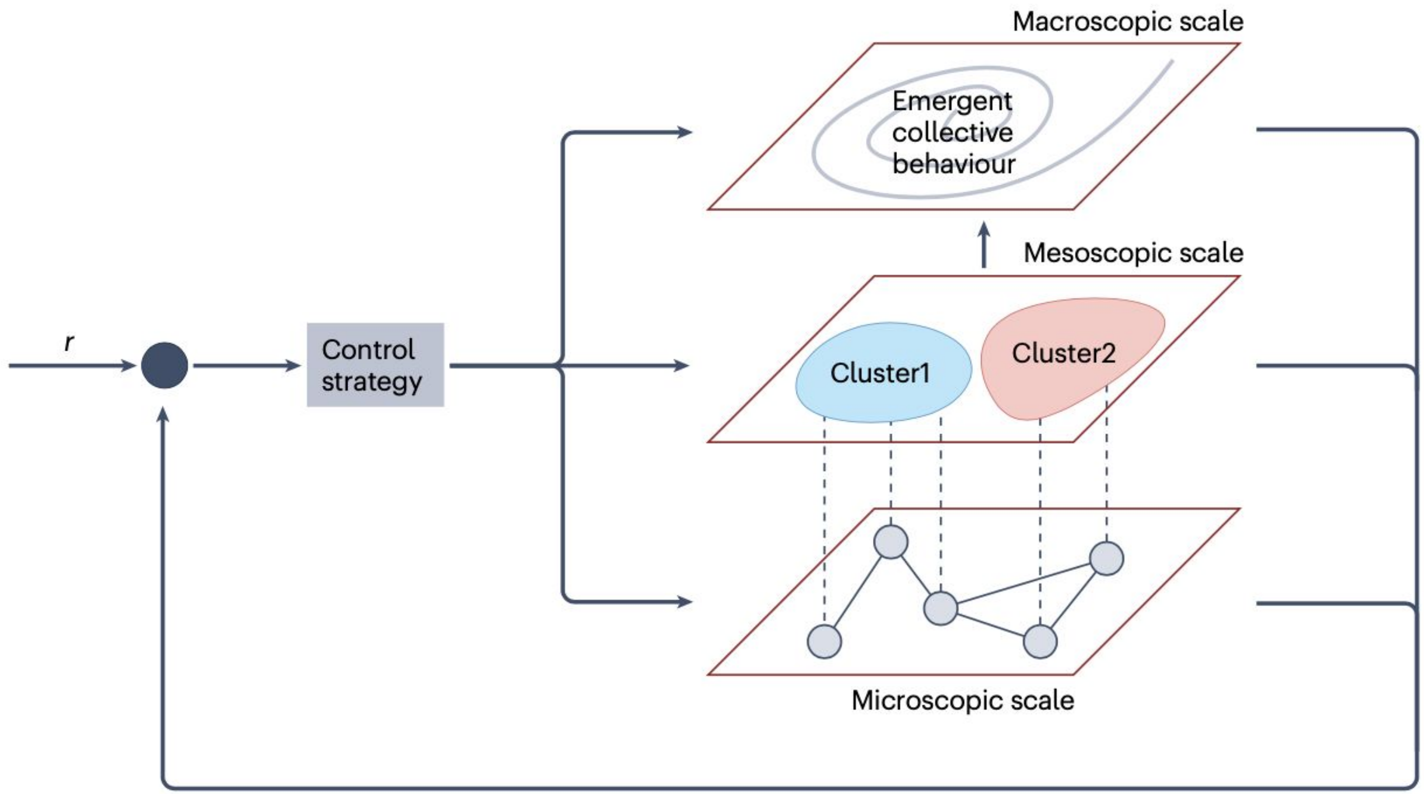
Perspective

Check for updates

Controlling complex networks with complex nodes

Published online: 24 March 2023

Raissa M. D'Souza[1,2,3], Mario di Bernardo[4,5] & Yang-Yu Liu[6,7]



Today's agenda: complex networks with complex nodes

Emergent interactions:

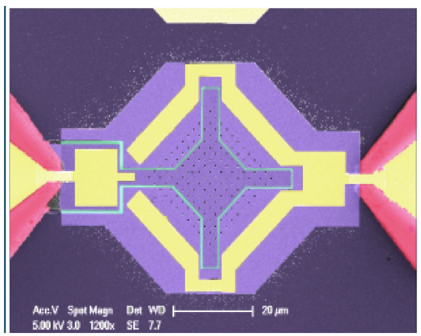
- Decoupled states – interactions of nodal dynamics & network structure
- BTW meets Kuramoto – sandpile cascades on oscillator networks

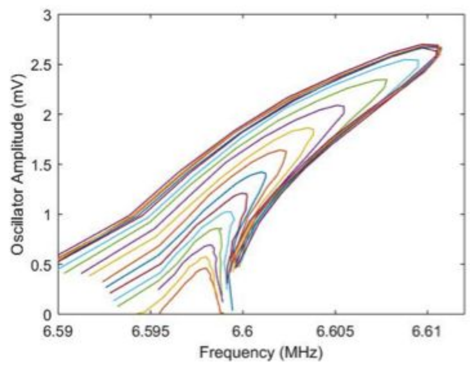
Controlling complex networks

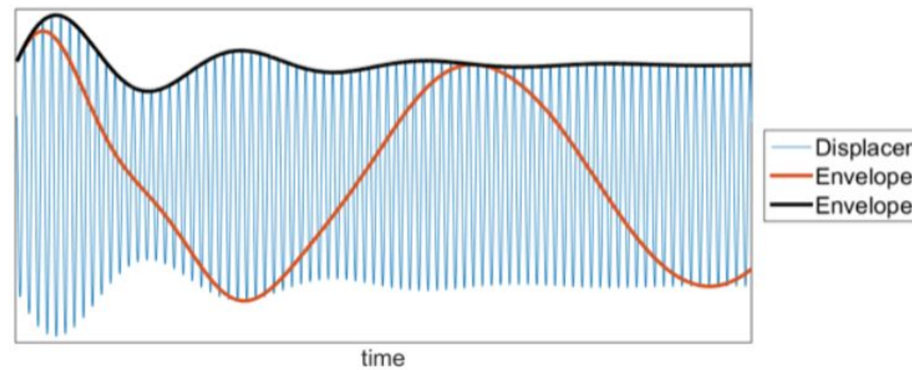
A partnership between Statistical Physics and Control Theory

Decoupled states: Phase-amplitude oscillators

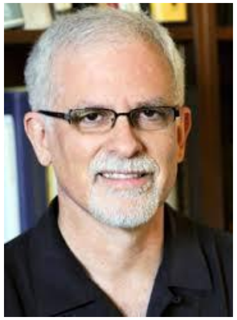
Nanoelectromechanical membrane, with a “Duffing”-like non-linearity







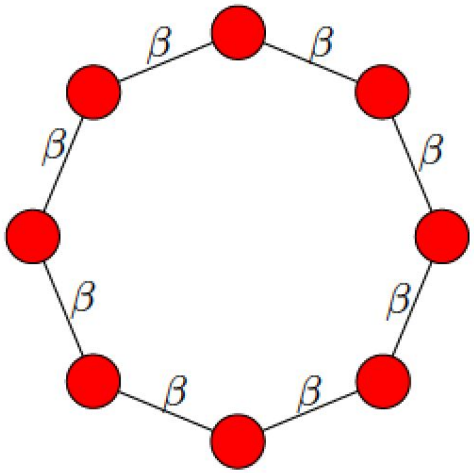
Described by slow-time envelope dynamics $A_i(t)$



Experimental collaboration with Micheal Roukes and Matt Matheny at Caltech

ARO MURI No. W911NF-13-1-0340

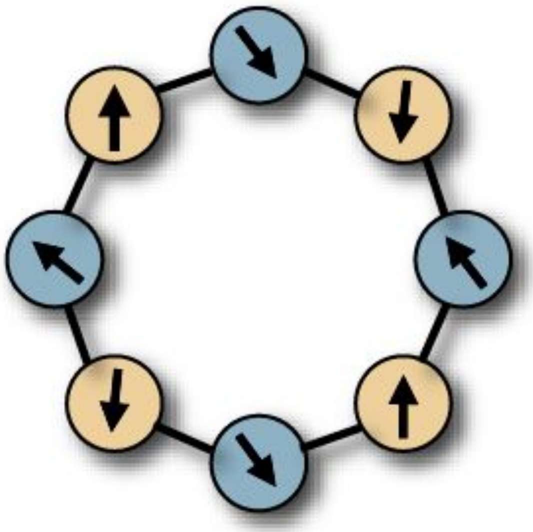
8-node of ring NEMs oscillators



$$\frac{dA_i}{dT} = \underbrace{-\frac{1}{2}A_i}_{\text{Dissipation}} + \overbrace{i\alpha|A_i|^2 A_i}^{\text{Duffing Nonlinearity}} + \underbrace{\frac{A_i}{2|A_i|}}_{\text{Self Feedback}} + \overbrace{\frac{i\beta}{2}\left(A_{i+1} - 2A_i + A_{i-1}\right)}^{\text{Coupling Feedback}}$$

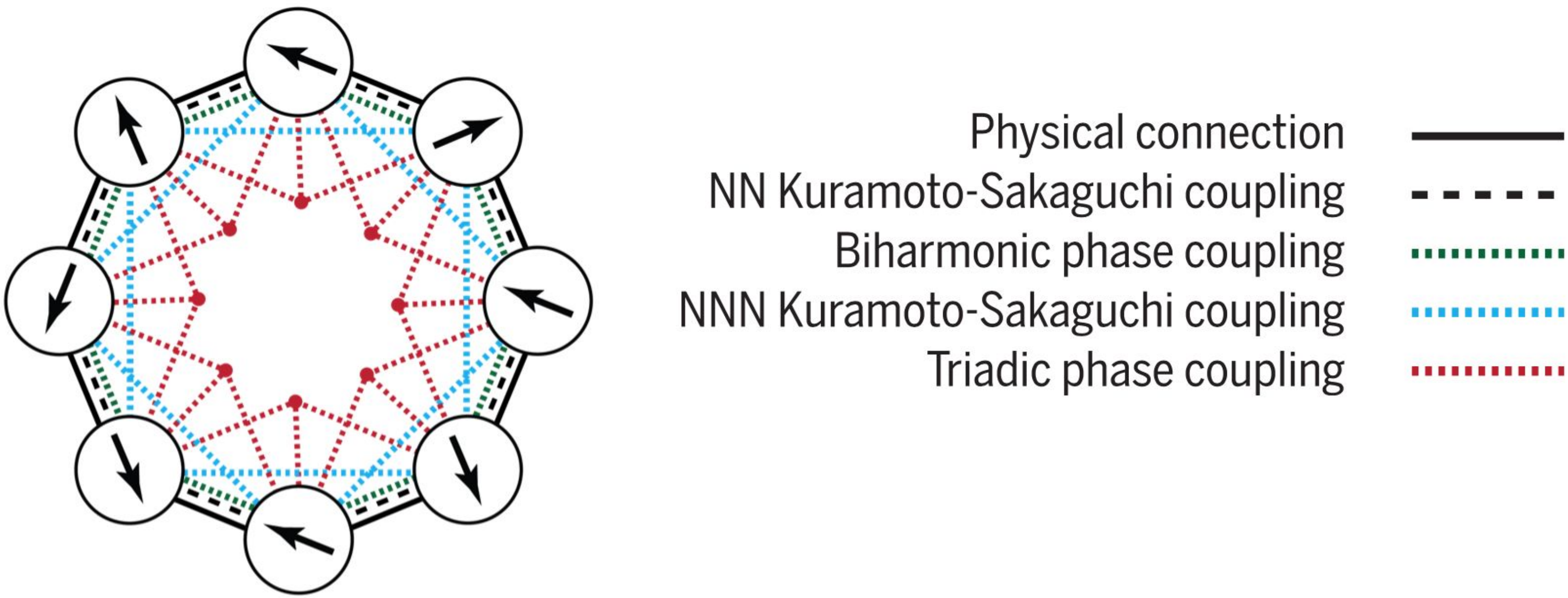
Decoupled NN with emergent NNN order

Interplay of nodal dynamics and coupling structure lead to decoupled states on ring of $N=4m$, $m \in \mathbb{Z}$.



Average $|A_i|=1$

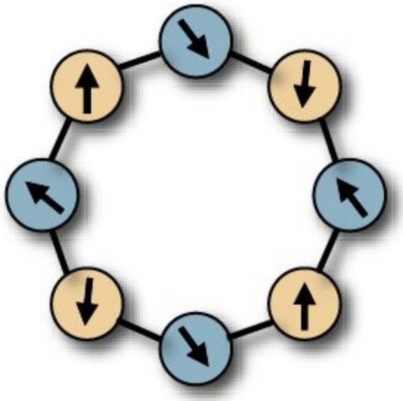
Emergent couplings of higher order



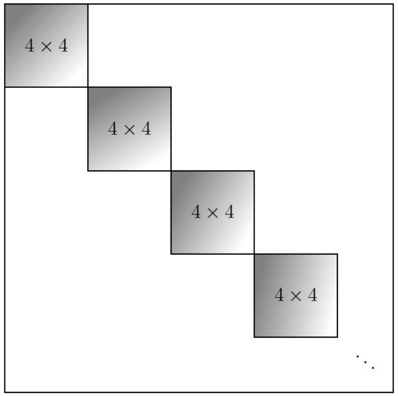
Matheny et al., "Exotic states in a simple network of nanoelectromechanical oscillators", ***Science***, 363**,** March 8, 2019.

Linear stability calculations – amplitude dynamics matter

Symmetry subgroups of nodal dynamics and coupling structure constrain the Jacobian:



Ring of 4m, $m \in \mathbb{Z}$



(a) Our method

$$D_k = \frac{1}{2}\begin{bmatrix} -1 & -\beta(1-\zeta^{-k})\sin\psi & 0 & \beta(1-\zeta^{-k})\cos\psi \\ \beta(1-\zeta^{k})\sin\psi & -1 & \beta(1-\zeta^{k})\cos\psi & 0 \\ 4\alpha & -\beta(1-\zeta^{-k})\cos\psi & 0 & -\beta(1-\zeta^{-k})\sin\psi \\ -\beta(1-\zeta^{k})\cos\psi & 4\alpha & \beta(1-\zeta^{k})\sin\psi & 0 \end{bmatrix}$$

- Stable for phase-amplitude oscillators.
- Although average $|A_i| = 1$, fluctuations are necessary to stabilize the system!

Unstable for phase-only oscillators.

Emenheiser, et al., arXiv:2010.09131

Admissibility and stability of decoupled states in general

PHYSICAL REVIEW RESEARCH **2**, 043261 (2020)

Decoupled synchronized states in networks of linearly coupled limit cycle oscillators

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[2]*Complexity Sciences Center, University of California, Davis, California 95616, USA*
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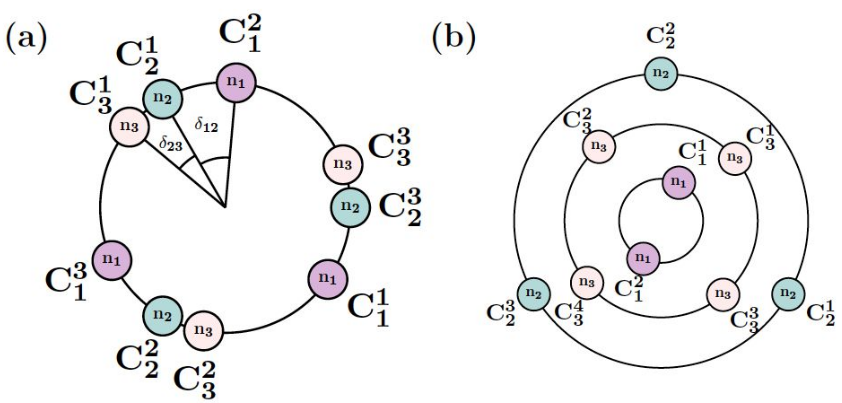
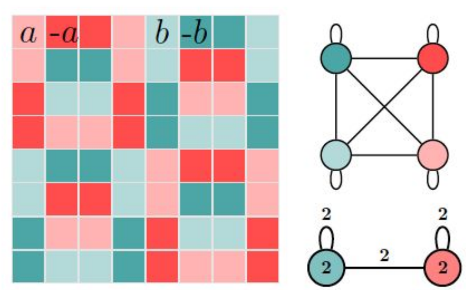
[4]*Santa Fe Institute, Santa Fe, New Mexico 87501, USA*

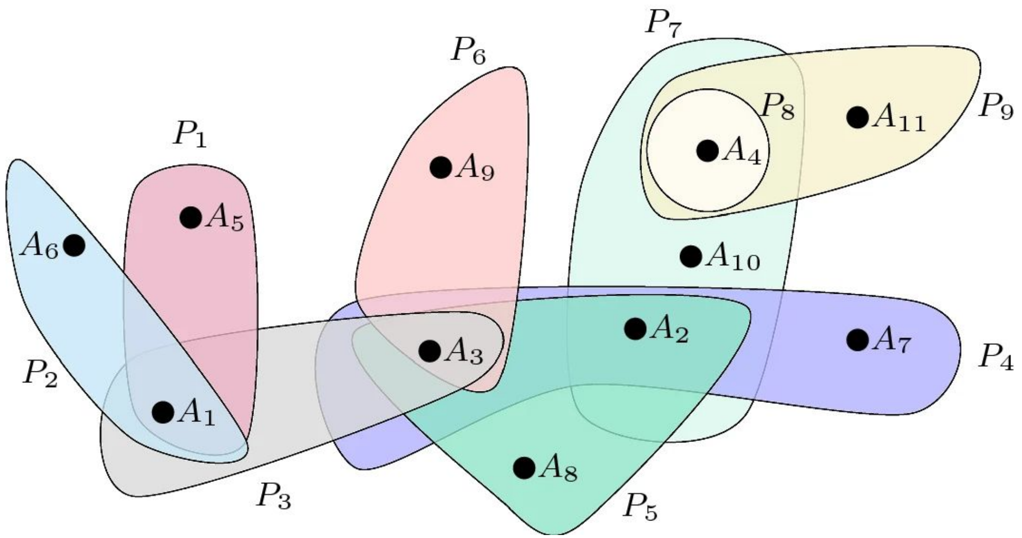
(Received 26 June 2020; accepted 27 October 2020; published 19 November 2020)





Increasing network complexity

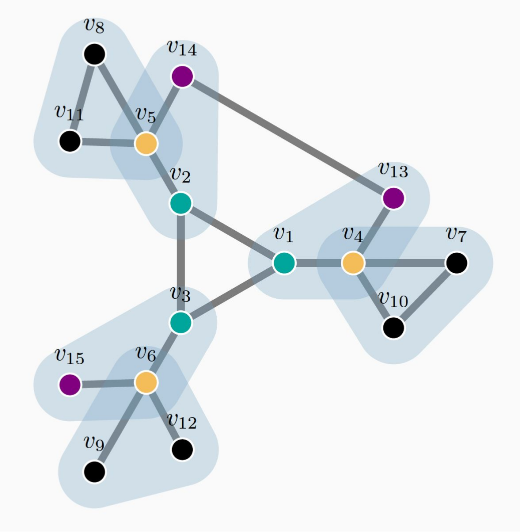
Hypergraphs – beyond dyadic coupling

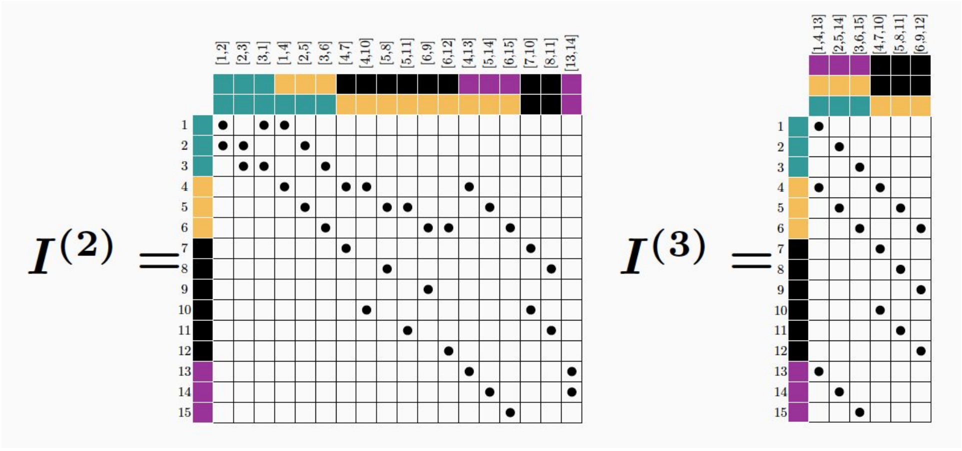


- **Challenge**: Hyperedges of all order contribute to the dynamics and the stability calculations.

Example higher-order interactions:
- Chemical reactions
- Co-authorship networks

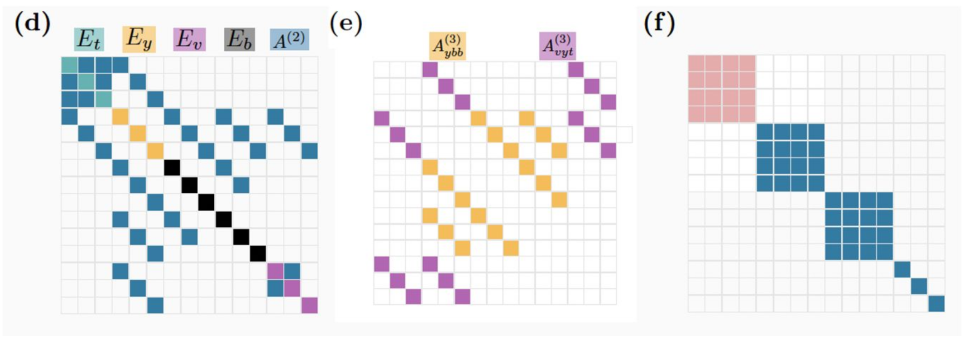
Cluster synchronization on hypergraphs





Simultaneously block-diagonalizing this set of matrices block-diagonalizes the Jacobian.

$$\{E_1, ..., E_K, \mathcal{A}^{(2)}, \mathcal{A}_1^{(3)}, ..., \mathcal{A}_{K_3}^{(3)}, ..., \mathcal{A}_1^{(d)}, ..., \mathcal{A}_{K_d}^{(d)}\}$$



Anastasiya Salova, R.D., arXiv:2101.05464

Anastasiya Salova, R.D., arXiv:2107.13712

https://github.com/asalova/hypergraph-cluster-sync

The role of nodal dynamics in cascading failures

Image © extremetech

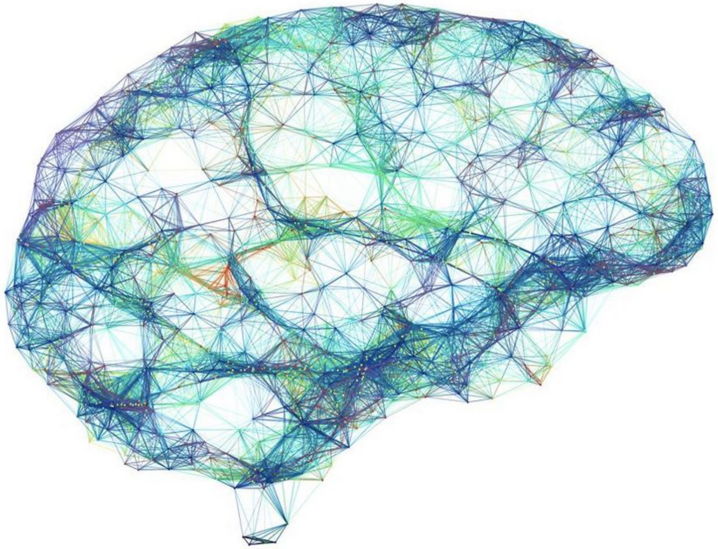


Image © Forbes

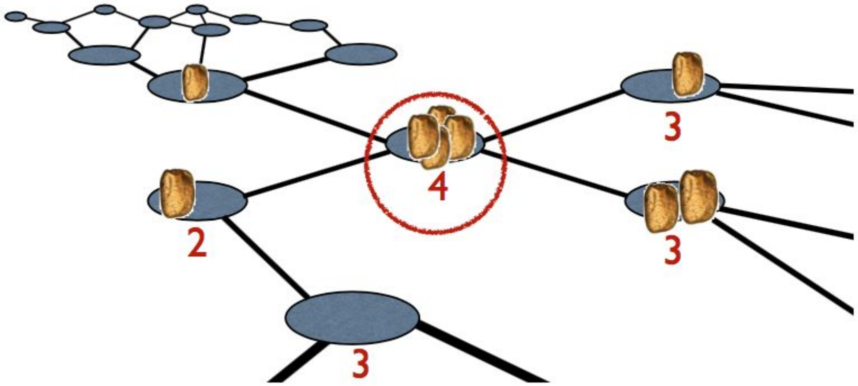
BTW sandpile model used to model power grid and brain networks

Self-organized criticality

Bak-Tang-Wiesenfeld *PRL* 1987: self-organized criticality

Sandpile model on networks

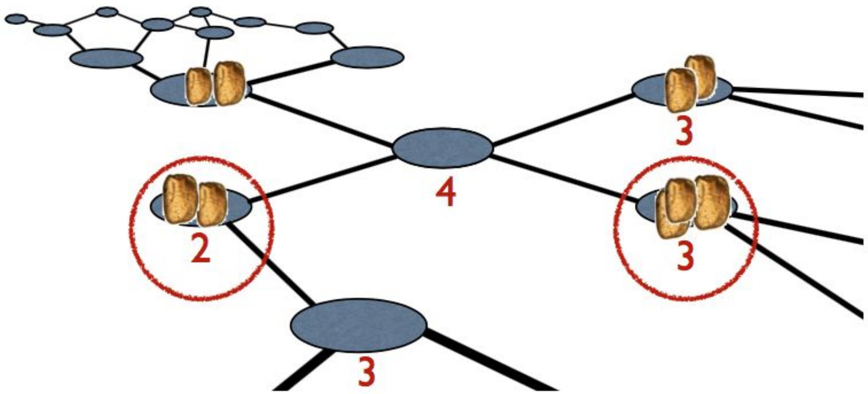
- Start with a network
- Drop <u>units of load</u> randomly on nodes
- Each node has a threshold.
  Here = degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves load to neighbors



Sandpile models on networks

- Start with a network
- Drop units of load randomly on nodes
- Each node has a threshold.
  Here = degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves load to neighbors
- Neighbors may topple. Etc.
  Cascade (or avalanche) of topplings.

Dissipation, ε



Power-law distribution of avalanche sizes, $P(s) \sim s^{-3/2}$

Self-organized criticality

Power law tails (Universal behavior)

Extreme events often referred to as "Black Swans"

This scaling behavior is robust on networks. (Goh et al. PRL 03, Phys. A 2004/2005, PRE 2005. PLRGs with $2 < \gamma < 3$ not mean-field.)

Power law tails seem to characterize the sizes of electrical blackouts, financial fluctuations, neuronal avalanches, earthquakes, landslides, overspill in water reservoirs, forest fires and solar flares.

[1] I. Dobson, B. A. Carreras, V. E. Lynch, and D. E. Newman, Chaos **17**, 026103 (2007).

[2] X. Gabaix, P. Gopikrishnan, V. Plerou, and H. E. Stanley, Nature **423**, 267 (2003).

[3] J. M. Beggs and D. Plenz, J, Neurosci. **23**, 11167 (2003).

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[5] T. Ribeiro, M. Copelli, F. Caixeta, and H. Belchior, PLoS ONE **5**, e14129 (2010).

[6] A. Saichev and D. Sornette, Phys. Rev. E **70**, 046123 (2004).

[7] S. Hergarten, Natural Hazards and Earth System Sciences **3**, 505 (2003).

[8] G. L. Mamede, N. A. M. Araujo, C. M. Schneider, J. C. de Araújo, and H. J. Herrmann, Proc. Natl. Acad. Sci. U.S.A. **109**, 7191 (2012).

[9] P. Sinha-Ray and H. J. Jensen, Phys. Rev. E **62**, 3216 (2000).

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[11] E. T. Lu and R. J. Hamilton, Astrophys. J. **380**, L89 (1991).

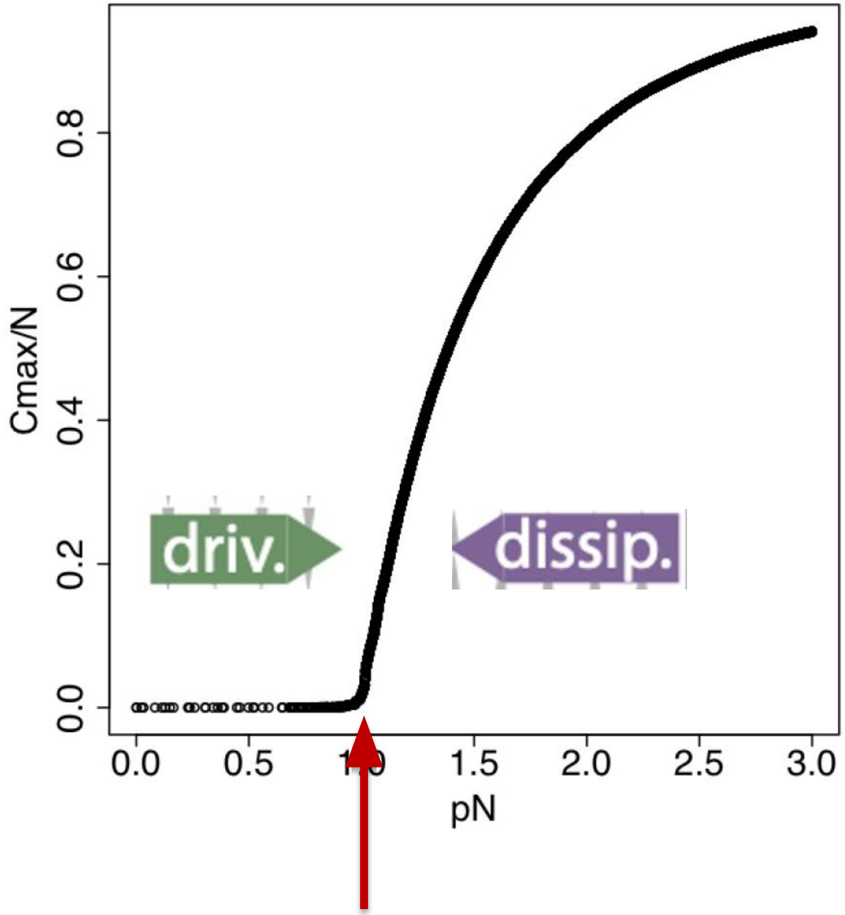
[12] M. Paczuski, S. Boettcher, and M. Baiesi, Phys. Rev. Lett. **95**, 181102 (2005).

[13] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

Why SOC? – ASPT

Absorbing state phase transition

R. Dickman, A. Vespignani, and S. Zapperi, Physical Review E 57, 5095 (1998).



The critical point (at $t=t_c$)

SOC in power grids and the brain?

Image © extremetech

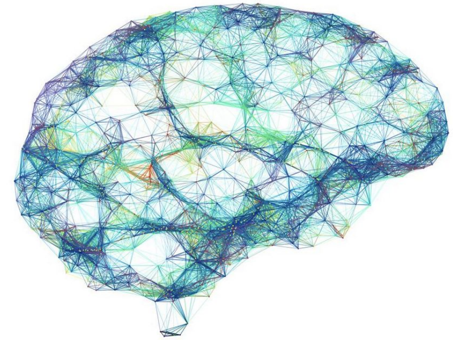


Image © Forbes

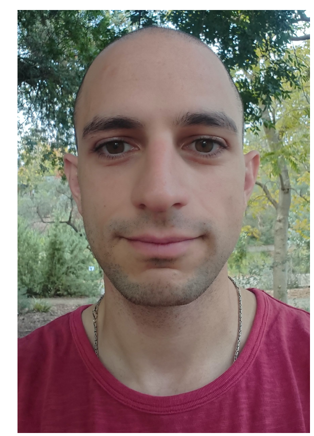
But this neglects the oscillatory nature of the nodes!

Sandpile cascades on oscillator networks: the BTW model meets Kuramoto

Guram Mikaberidze[1, a)] and Raissa M. D'Souza[2, 3]
[1)] *Department of Mathematics, University of California, Davis, CA, 95616, USA*
[2)] *University of California, Davis, CA, 95616, USA*
[3)] *Santa Fe Institute, Santa Fe, NM, 87501, USA*

Chaos **32**, 053121 (2022)

Initial goal: Leverage interaction to maximize synchronization and minimize large cascades.



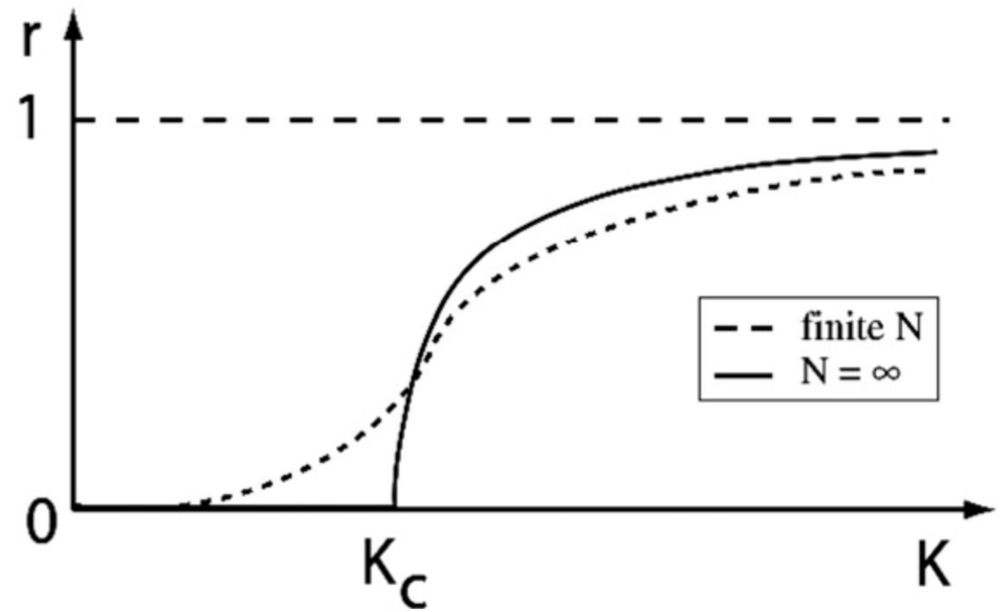
Guram Mikaberidze

Oscillator dynamics: The Kuramoto model

$$\dot{\phi}_i(t) = \omega_i + k \sum_{j \in \mathcal{N}_i} \sin(\phi_j(t) - \phi_i(t))$$

Time evolution of the phase of oscillator *i*

Synchronization phase transition at critical coupling

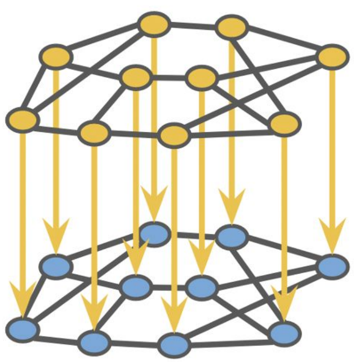


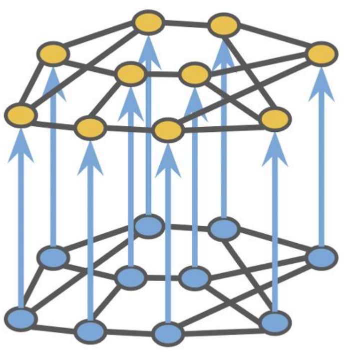
Coupled BTW-KM dynamics. Each node has:

- Phase ϕ_i (KM)
- Capacity c_i (BTW)
- Load s_i (BTW)

Coupled BTW-KM

Motivation: more out-of-phase is more vulnerable





BTW -> KM

- If a node topples during a cascade its phase is reset at random at the end of the cascade.
- Discrete dynamics
- Cascade dynamics happens “instantaneously” compared to Kuramoto

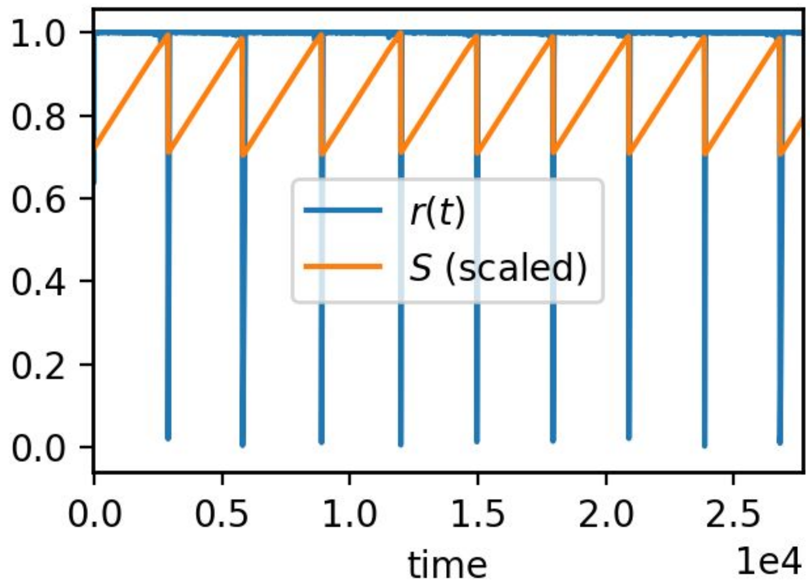
KM -> BTW

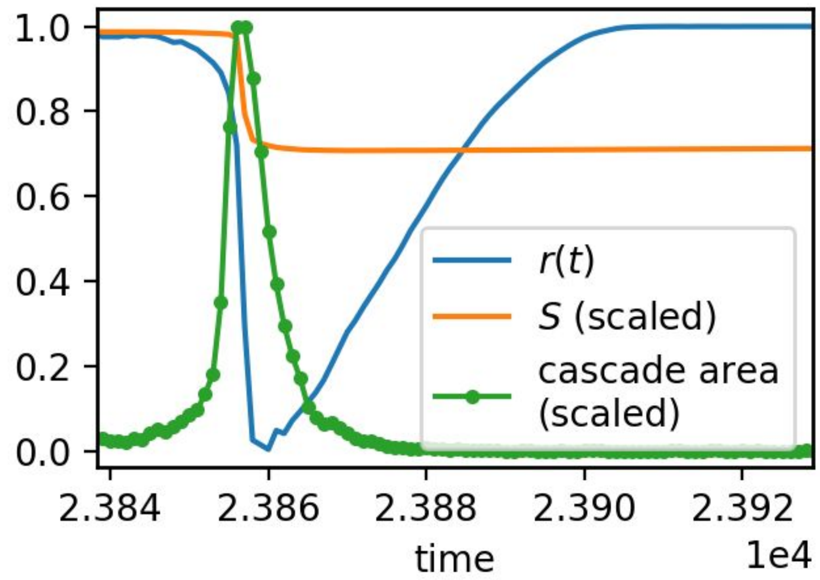
- Assume a node out-of-sync with its neighbors is more vulnerable so lower its capacity to hold load. This creates ***endogenous*** cascade seeds.
- Continuous dynamics
- Runs for time ΔT

Emergent periodic oscillations

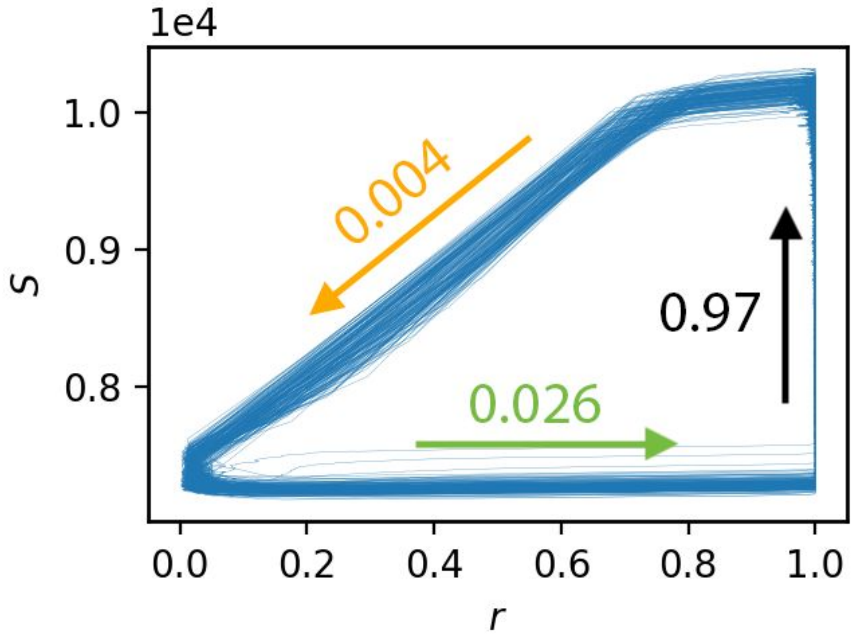
3-regular random graphs

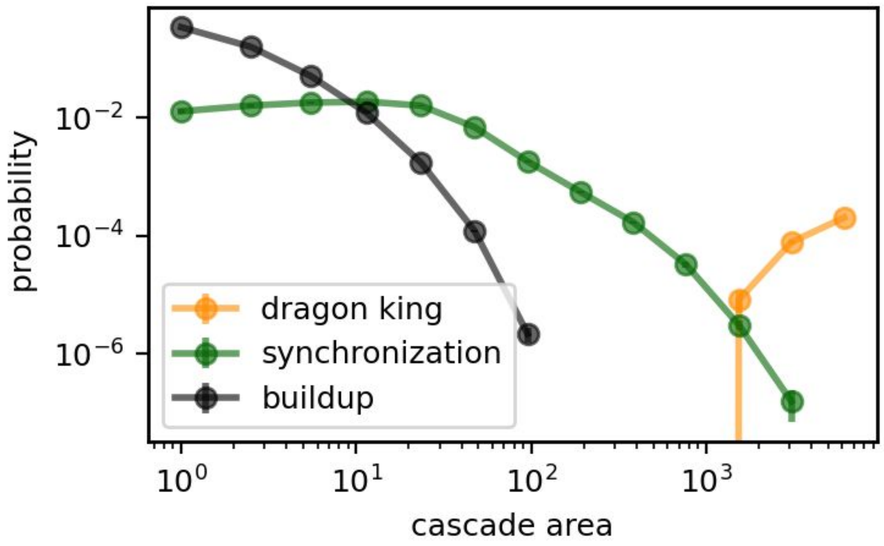
- *r(t)* is the Kuramoto order parameter
- *S* is the total load on the system





Emergent 3-phase oscillations

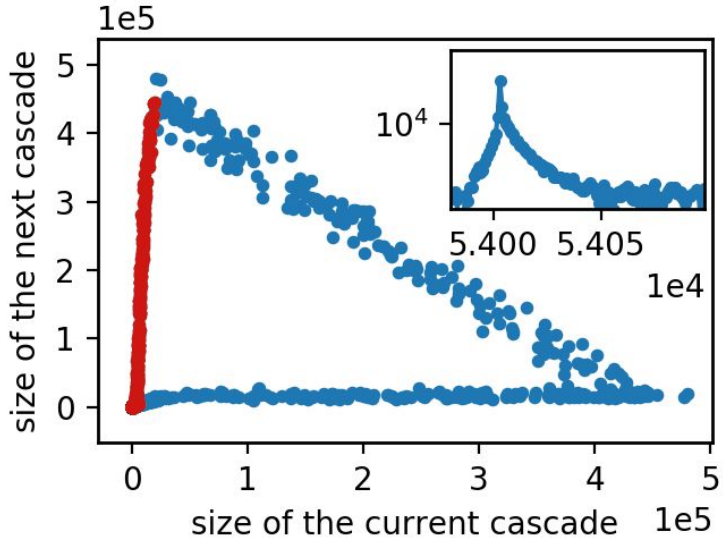


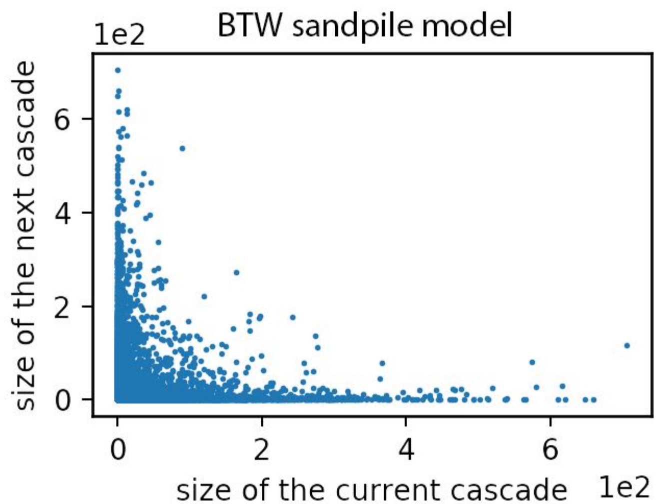


Cascade size distribution in each of the three phases

Self-amplifying cascades kick off a DK

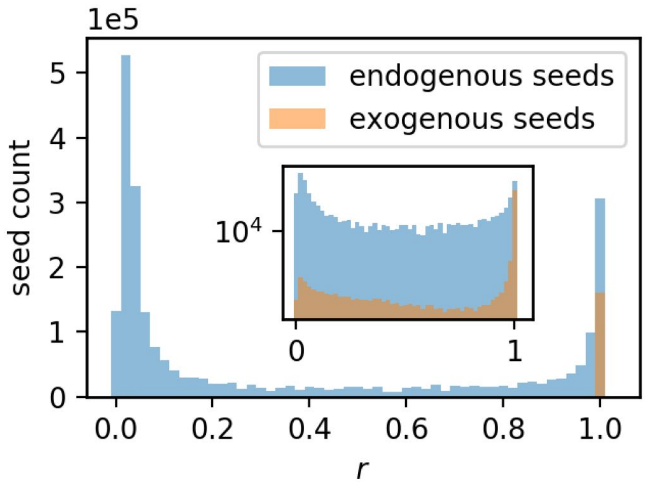
At the tipping point, a large cascade desynchronizes many nodes, causing an even larger cascade at the next step.



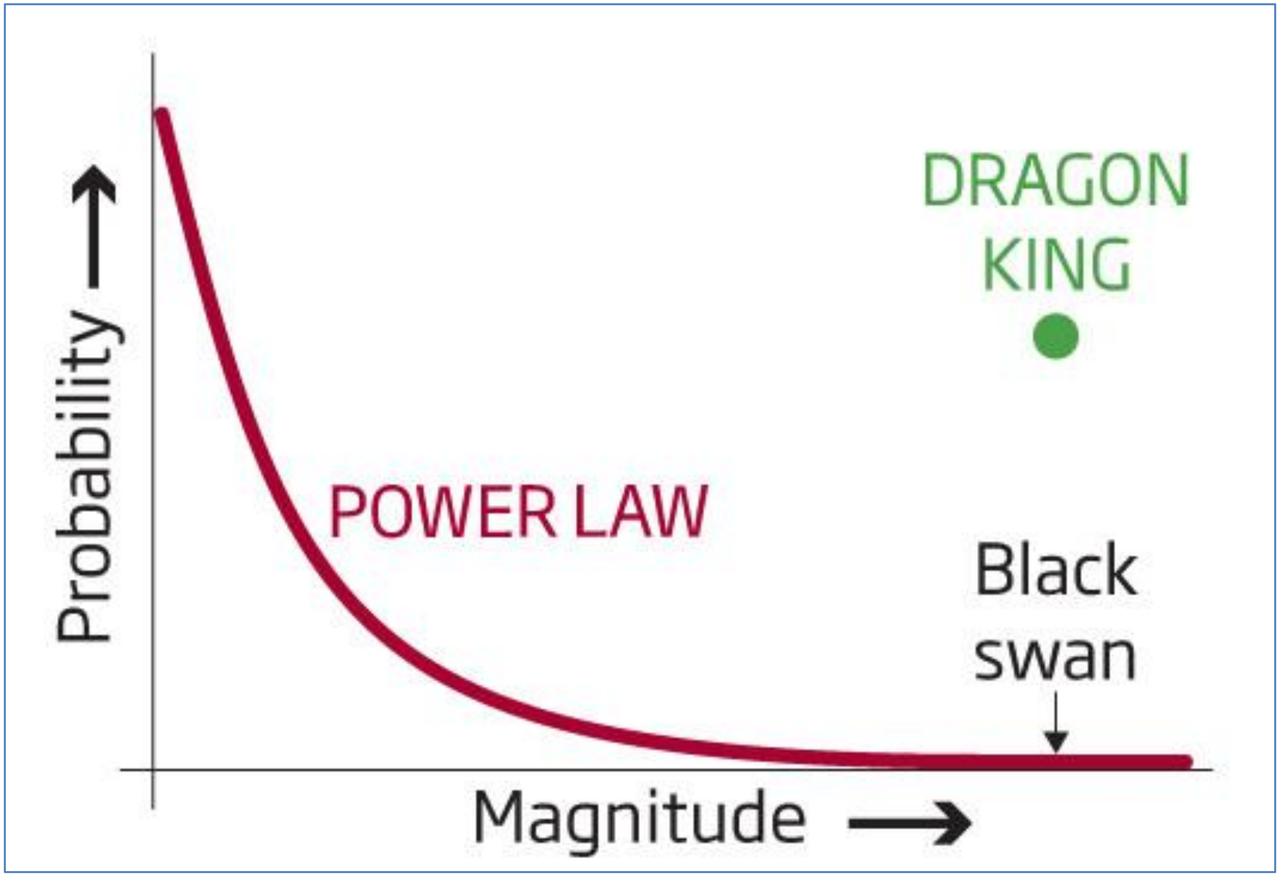


$$s(t) \propto \exp(\frac{a-1}{\Delta T}t)$$

Exponential growth in subsequent size:
"cascade of cascades"



Beyond “Black Swans” -> Dragon Kings



Poorly understood, massive events caused by nonlinear amplifying mechanisms. (Introduced by D. Sornette, 2009.)

Dragon Kings

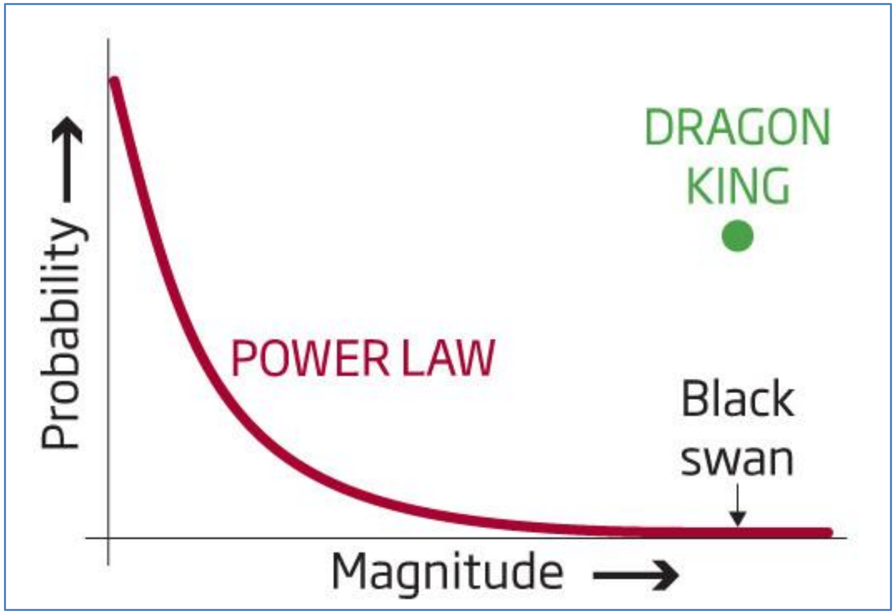
- Bubbles in financial markets; sizes of cities; failures in engineered systems & nuclear accidents, etc.

- Self-amplifying mechanism, endogenous nature

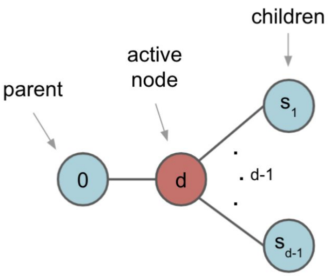
- Far more likely than Black Swans and equally massive

- Theory in its infancy:
  - Conjecture: needs homogeneous elements with large coupling

- Dragon kings have predictability

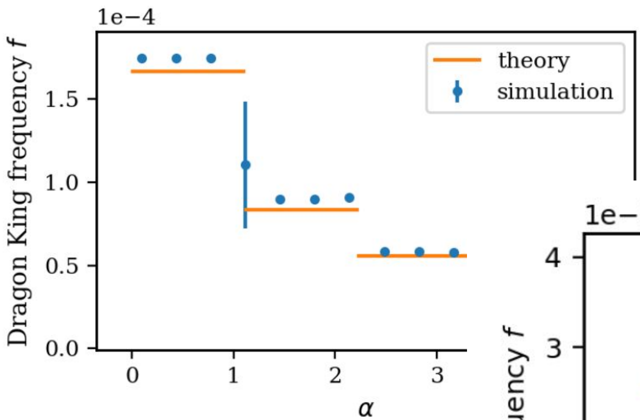


Analytic calculations

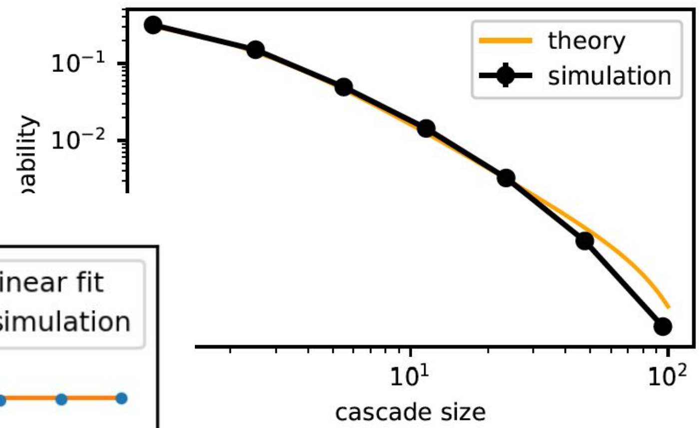


$$(S_{\mathrm{TP}}, r_{\mathrm{TP}}) = \left(\frac{\lfloor c^0 - \alpha(l-1) \rfloor N d}{2(d-1)}, 1 \right)$$

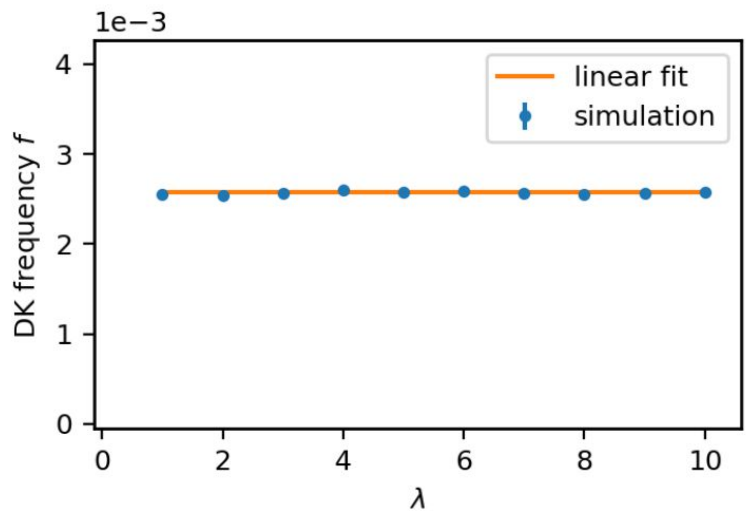
(1) The location of the tipping point



(2) The frequency of the



(3) The cascade distribution in the buildup phase

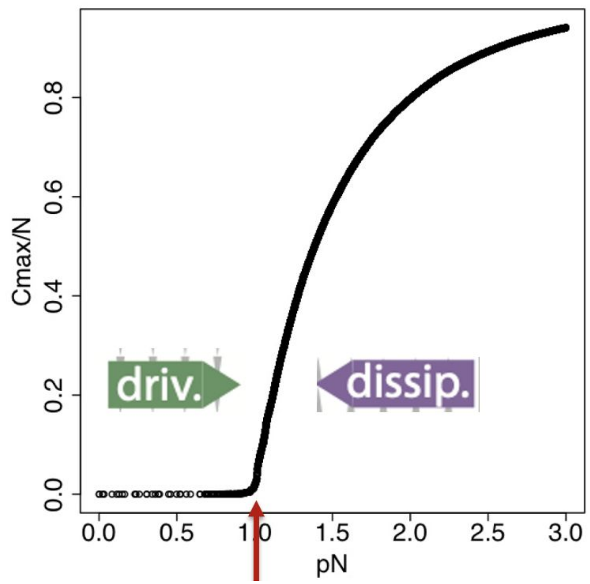


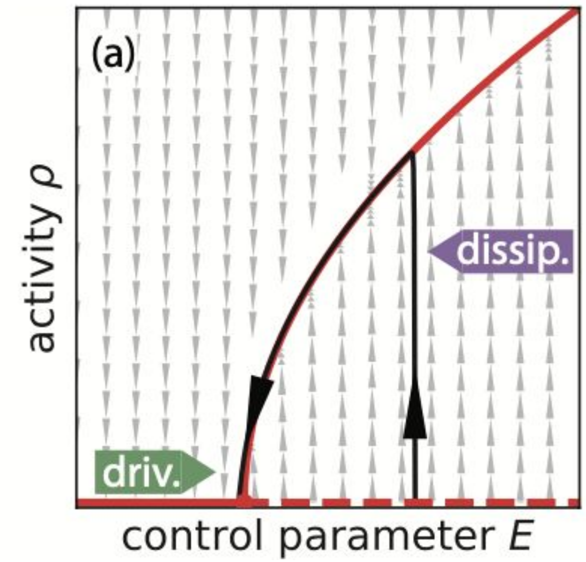
(4) The thermodynamic limit

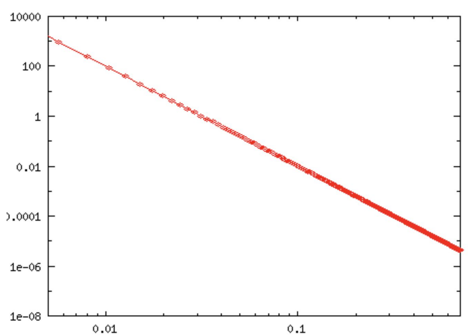
G. Mikaberidze & R.D. Chaos **32**, 053121 (2022);

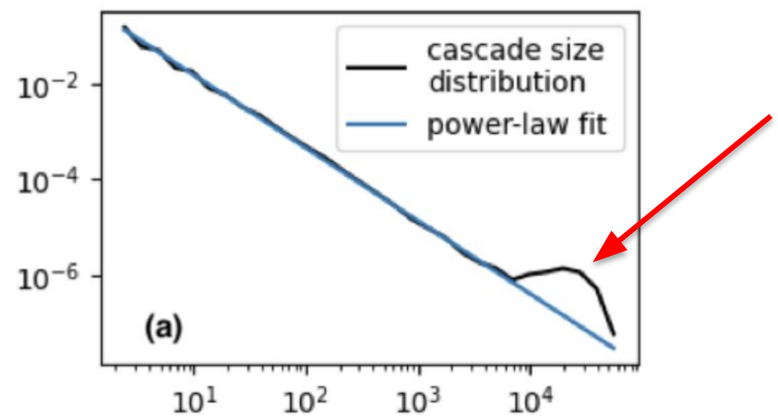
DKs in SOC!

Discrete nature of sandpile model allows driving into the supercritical regime.







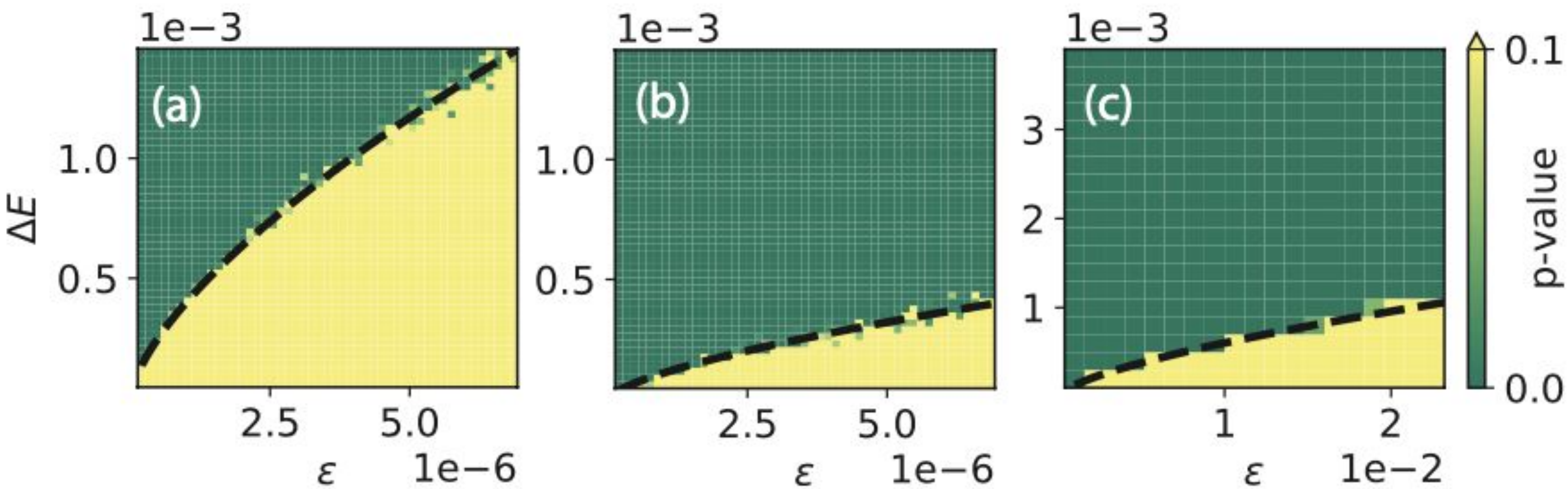


Enigmatic “peaks”/”bumps” observed in SOC studies, not explained by finite size

DKs in SOC!

DKs can exist even in the $\varepsilon \to 0$ limit.

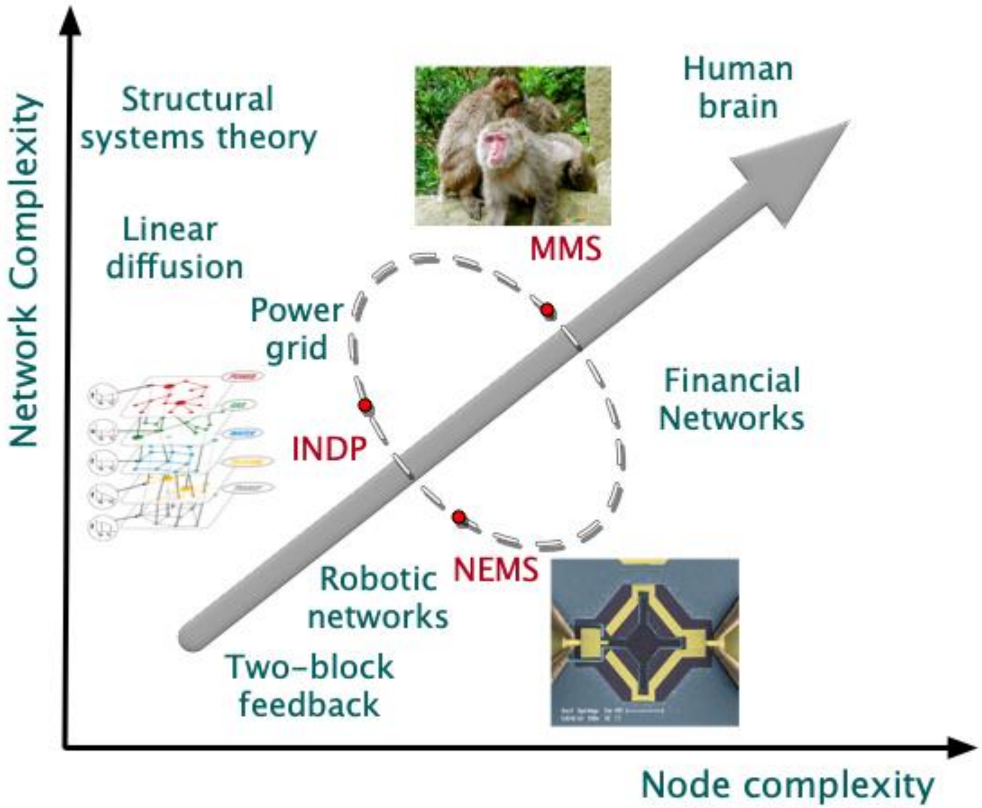
Tradeoff between driving impulse and dissipation determines if there is a DK.



Dragon King taxonomy

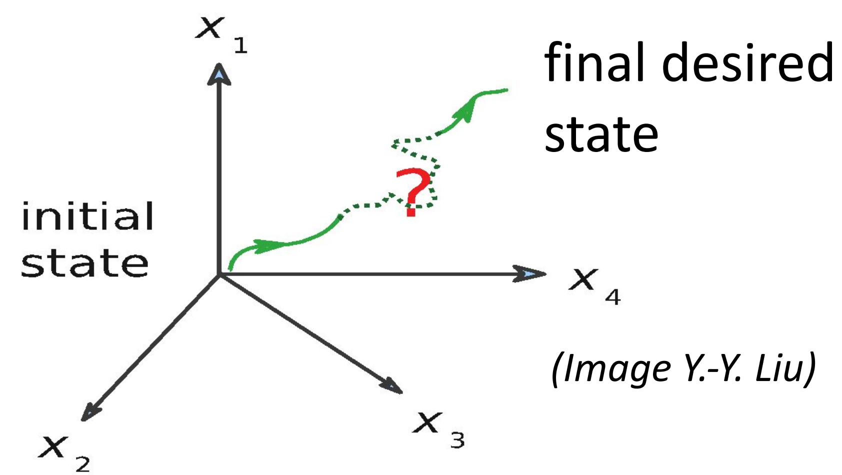
G. Mikaberidze & R.D.

Controlling complex networks with complex nodes



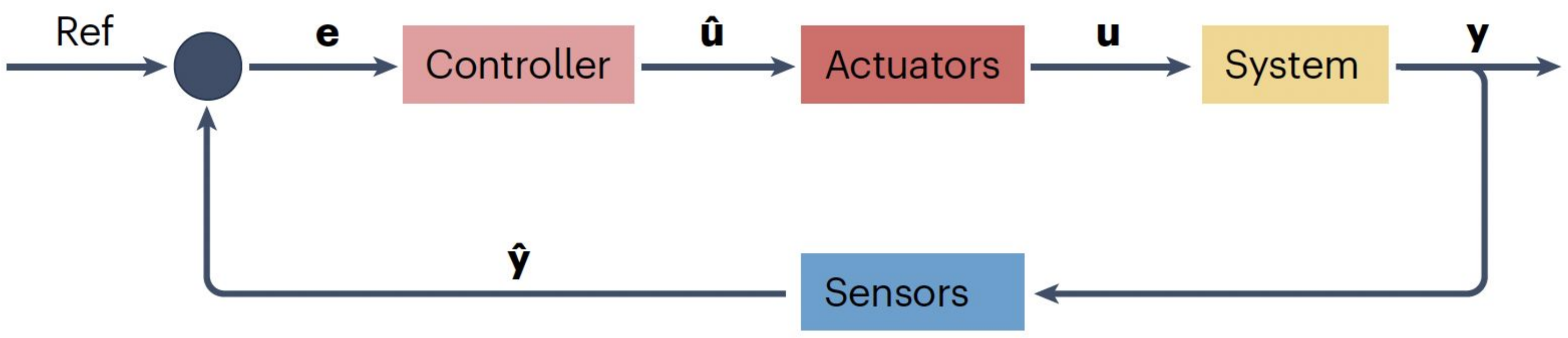
Classic feedback control theory

Control: Drive the system from any initial starting configuration to any specified end configuration in finite time.



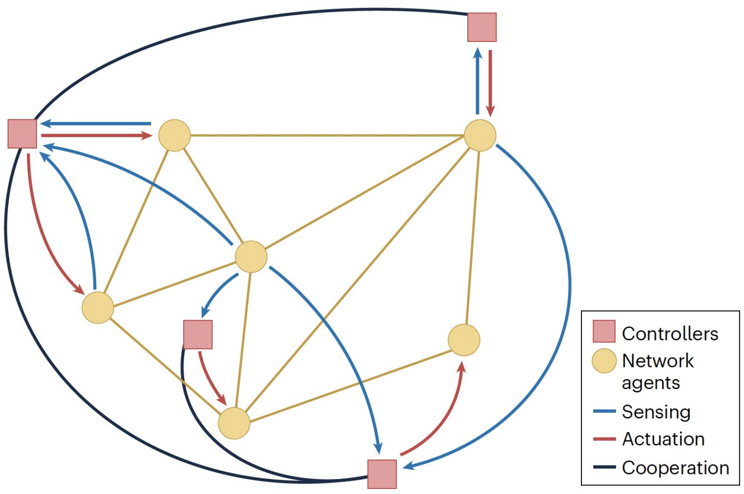
(Image Y.-Y. Liu)

Feedback control paradigm:



Classic control tools are not enough

Network control



- Millions of degrees of freedom
  - Full control is not feasible
  - Full knowledge may not be

- Self-organizing systems
  - Distributed control
  - Cascading failures – SOC and DKs are prevalent
  - Unanticipated response to interventions

e.g., Noel, Brummitt, RD., *Phys. Rev. Lett.* **111** (2013)

What do we want to control?

- Every degree of freedom?

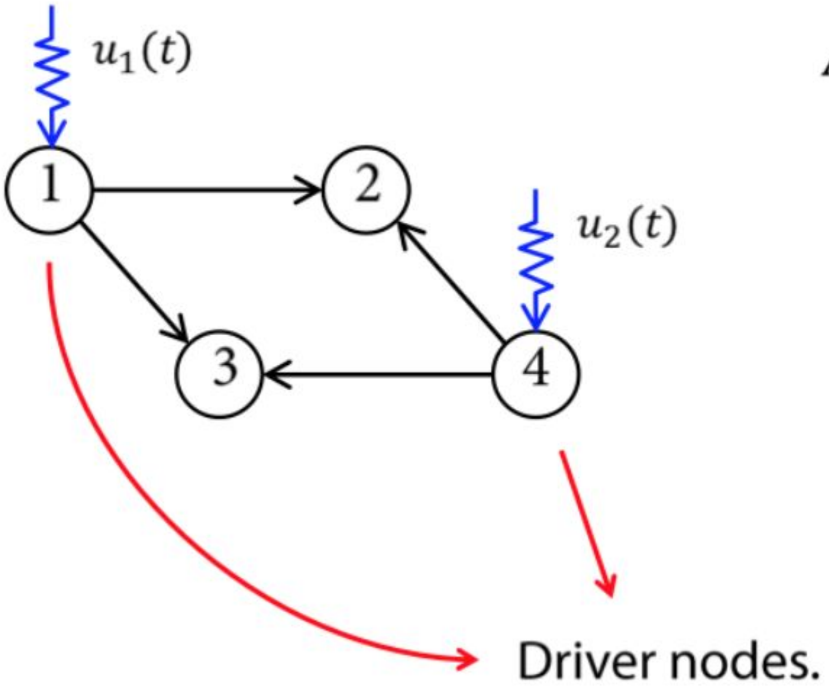
- Macroscopic or microscopic details? E.g.,
  - How many people are infected (macroscopic)
  - Which particular people are infected (microscopic).

- Steer towards some class of behaviors? Avoid certain attractors.

Starting point: Linear time invariant

Linear time-invariant system:

$$x(t+1) = \boldsymbol{A}\,x(t) + \boldsymbol{B}\,u(t)$$



$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} b_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & b_2 \end{pmatrix}$$

We want to find N_D the minimum number of driver nodes

B is a *MxN* matrix

- *N* nodes
- *M* control signals/driver nodes
- Columns are the unit vector of each driver node

Control of Linear time invariant (LTI) systems

Controllability matrix: $\mathcal{C} = = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$

Kalman's controllability criteria: $\text{rank}\,\mathcal{C} = N$

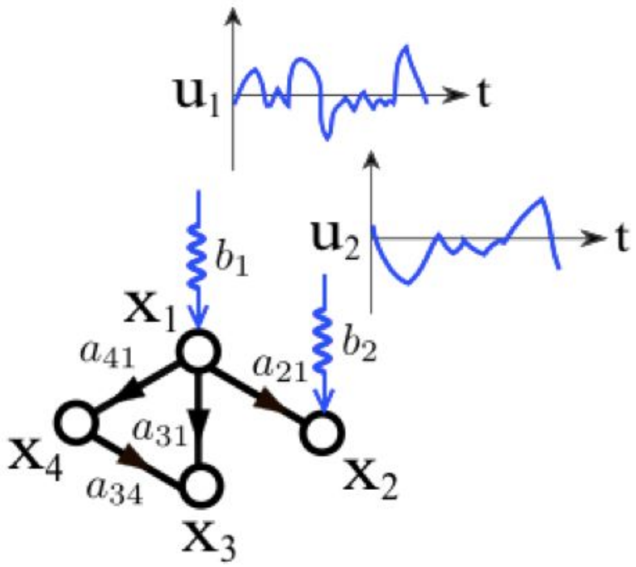
Controllable!

The matrix C is full-row rank (each row linearly independent)

Kalman, *J.S.I.A.M. Control* (1963)

The controllability Gramian

$$W_c(t) = \int_{t_0}^{t} e^{A(t-\tau)} BB^* e^{A^*(t-\tau)}\, d\tau.$$



If and only if the pair (A,B) is controllable, W_c is n
the minimum energy control signal:

$$u(t) = -B^* e^{A^*(t_1-t)} W_c^{-1}(t_1)[e^{A(t_1-t_0)} x_0 - x_1].$$

Structural control of LTI systems

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-19, NO. 3, JUNE 1974

Structural Controllability

CHING-TAI LIN, MEMBER, IEEE

Abstract—The new concepts of "structure" and "structural controllability" for a linear time-invariant control system (described by a pair (A,b)) are defined and studied. The physical justification of these concepts and examples are also given.

The graph of a pair (A,b) is also defined. This gives another way of describing the structure of this pair. The property of structural controllability is reduced to a property of the graph of the pair (A,b). To do this, the basic concept of a "cactus" and the related concept of a "precactus" are introduced. The main result of this paper states that the pair (A,b) is structurally controllable if an only if the graph of (A,b) is "spanned by a cactus." The result is also expressed in a more conventional way, in terms of some properties of the pair (A,b).

ing entry of (Ab) is also fixed (zero). Then one defines the pair (A_0,b_0) to be *structurally controllable* if and only if there exists a completely controllable pair (A,b) which has the same structure as (A_0,b_0).

The concept of "structural controllability" of a pair (A_0,b_0) makes the meaning of controllability (in the usual sense) more complete from the physical point of view. In fact, it is preferred whenever (A_0,b_0) represents an actual physical system (that involves parameters only approximately determined). Actually, the completely controllable pair (A,b) can be considered as "physically

C-T Lin, *IEEE Trans on Automatic Control*, 1974

We treat the nonzero elements in $\boldsymbol{A}$ and $\boldsymbol{B}$ as free parameters, and we keep the zero entries fixed.

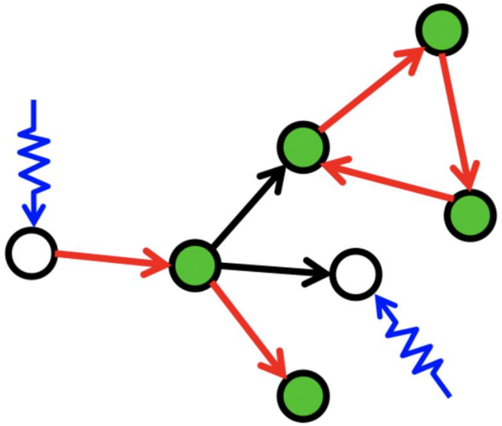
$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & 0 & 0 & a_{34} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \rightarrow \quad \boldsymbol{A}^* = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21}^* & 0 & a_{23}^* & a_{24}^* \\ a_{31}^* & 0 & 0 & a_{34}^* \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Structural control meets maximum matching

Y.-Y, Liu, A.L. Barabasi, J.-J. Slotine
"Controllability of complex networks"
Nature 2011.

Matching in Directed Networks

The unmatched nodes are the driver nodes.

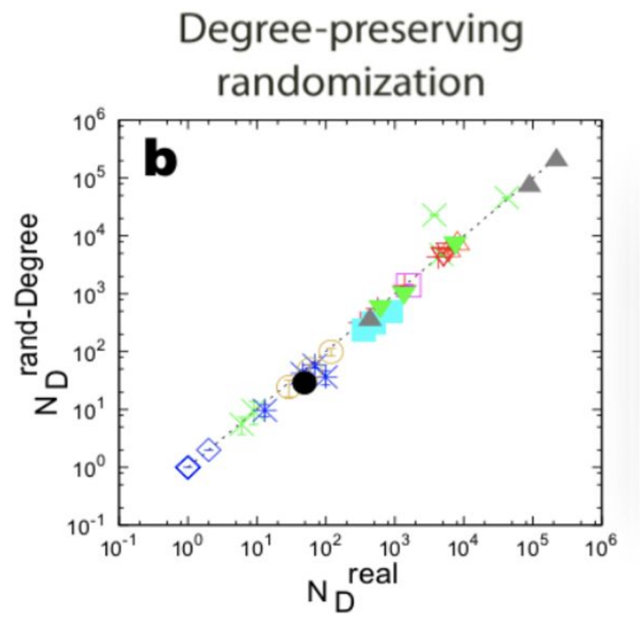


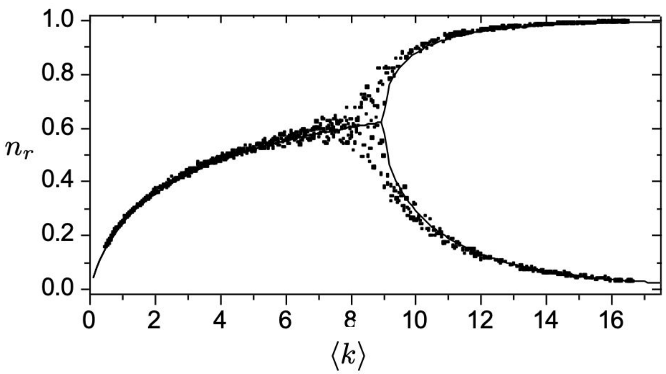
GOOD: algorithms and analytical tools!

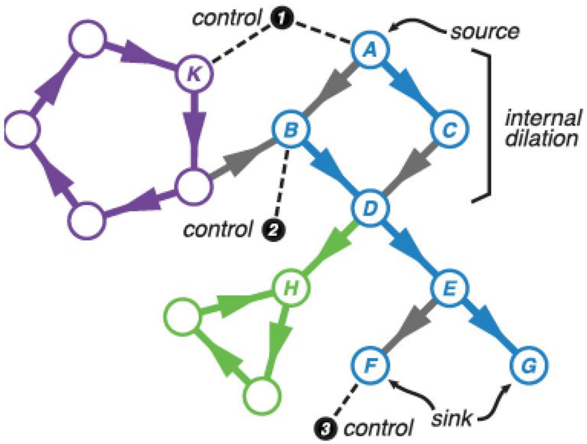
Cavity method from spin glasses

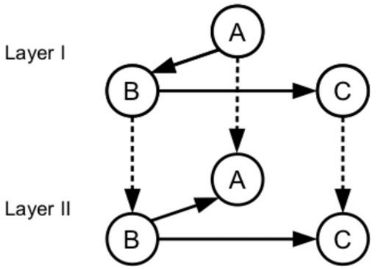
Insights from structural control:

- Role of degree distribution
- Control energy
- Exact controllability
- Connection to core percolation
- Control profiles: sources, sinks, dilations
- Control of multiplex LTI networks
- Target control

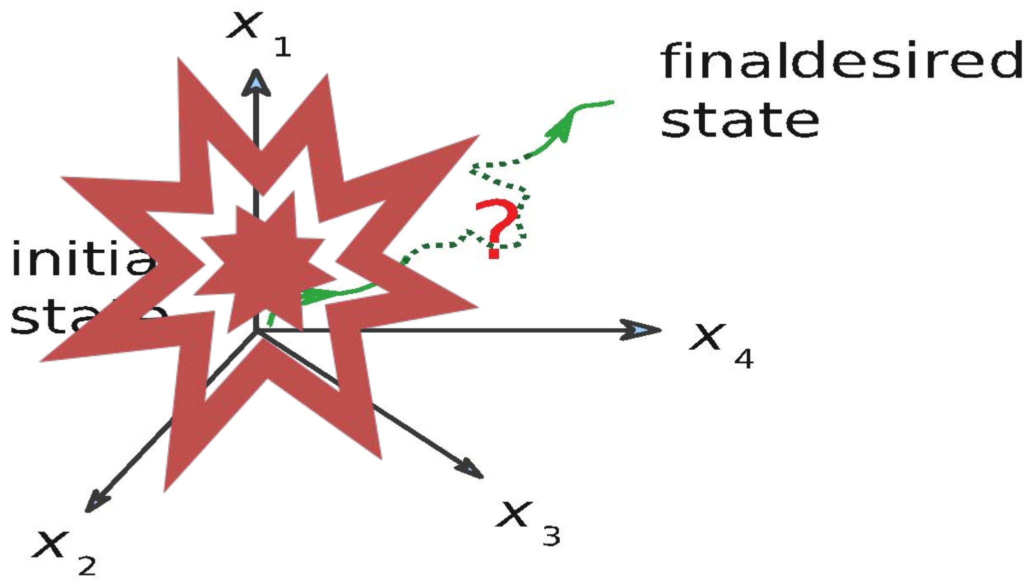








Real-world: Massive, non-linear systems



(Image Y.-Y. Liu)

- Possibly only partial knowledge
- Non-linear nodal dynamics
- Self-organization / far from equilibrium

Non-linear dynamics

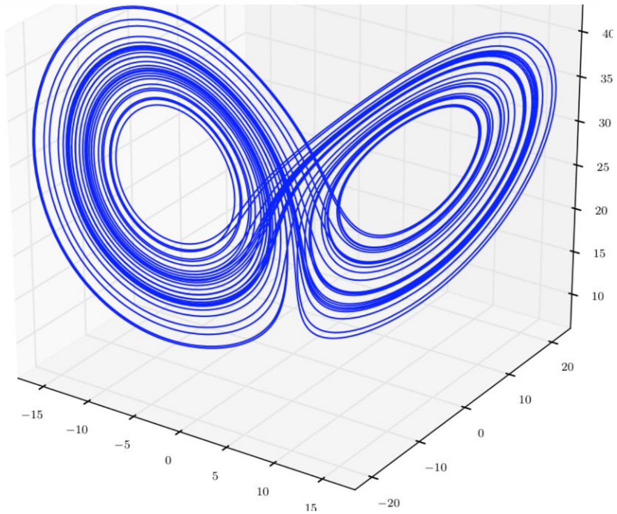
Start from the dynamical equations of motion – complex dynamics, low dimensional

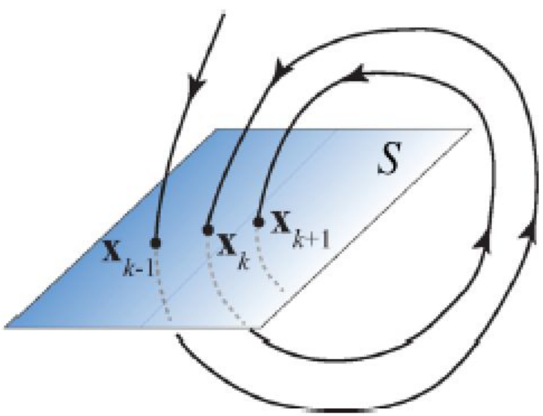
VOLUME 64, NUMBER 11 PHYSICAL REVIEW LETTERS 12 MARCH 1990

Controlling Chaos

Edward Ott,[a],[b] Celso Grebogi,[a] and James A. Yorke[c]
University of Maryland, College Park, Maryland 20742
(Received 22 December 1989)

- An infinite number of unstable periodic orbits typically embedded in a chaotic attractor.
  - d**x**/dt = F(**x**,p), where **x** is a **3D vector**



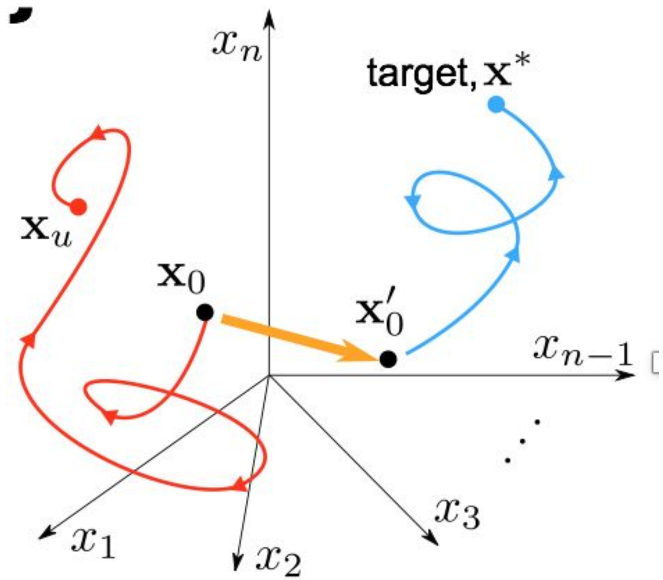


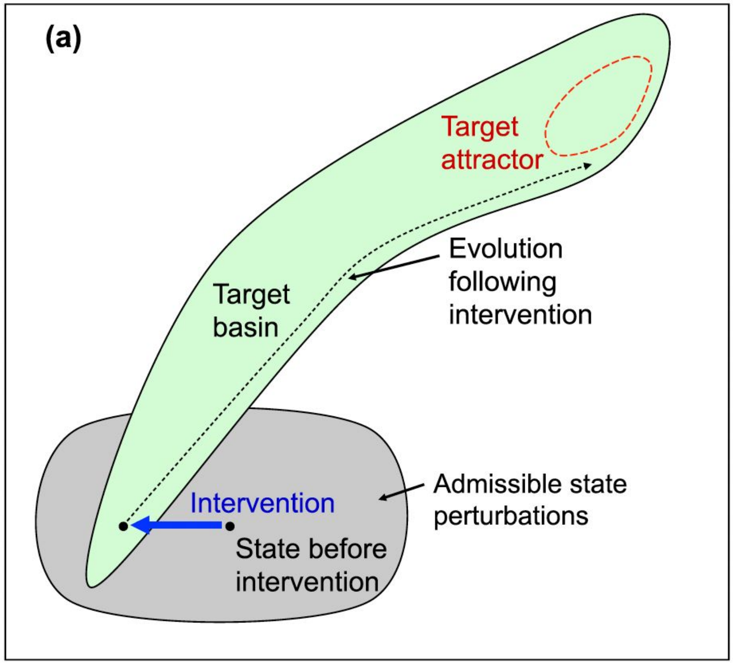
- Stabilize desired orbit by making only small time-dependent perturbations of a system parameter, p

Non-linear dynamics – basin structure key

“Kicking control”

Exploit basins of attraction and natural phase-space trajectories





S P Cornelius, WL Kath, and AE Motter. “Realistic control of network dynamics”. *Nature Communications*, 4, 2013.

AE Motter. “Networkcontrology”. *Chaos*, 25, 2015.

Data driven model discovery

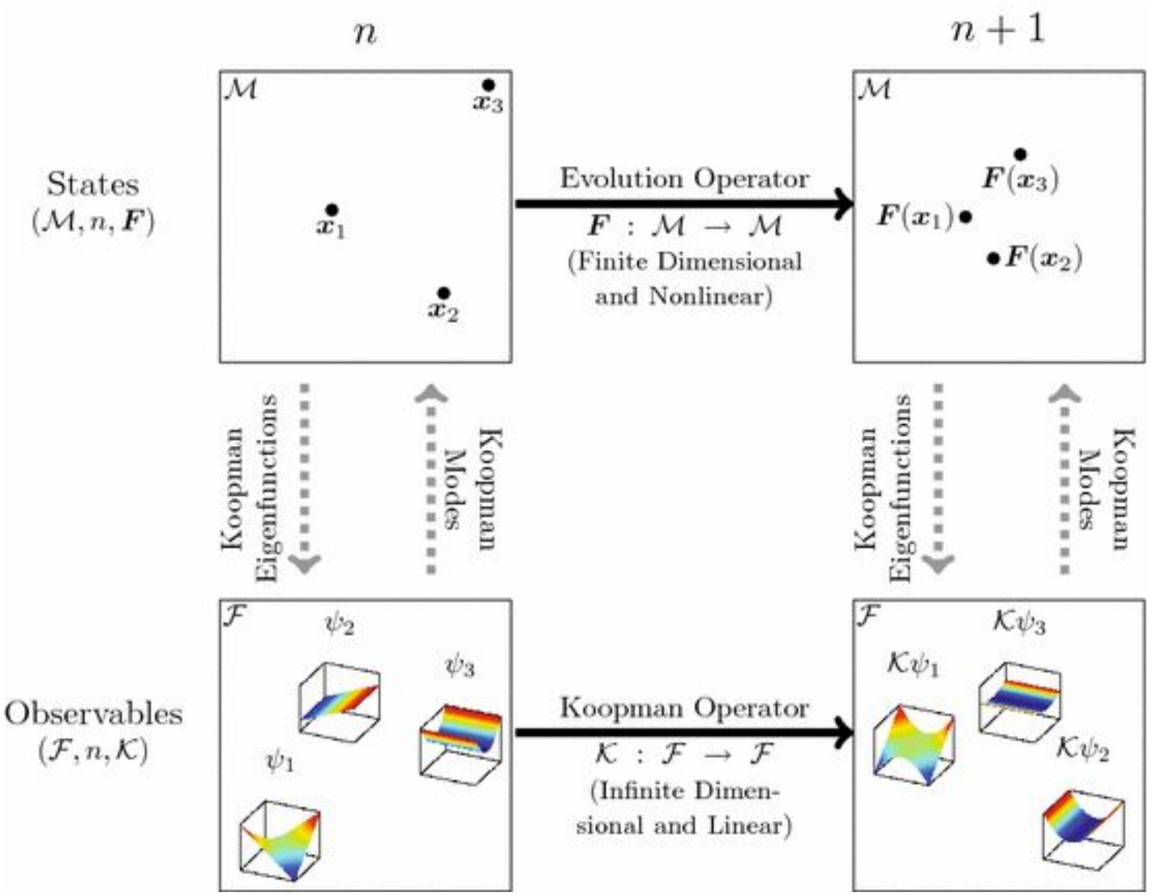
- **System identification** / network inference
  - Discover effective equations of motion from time series data

$$\phi(t+1) \approx \hat{A}\phi(t) \quad \text{where} \quad \hat{A} = \arg \min_{A \in \mathbb{R}^{|E|\times|E|}} \|\Phi_0 - A\Phi_1\|$$

- **Operator theory**
  (e.g. Koopman operators)

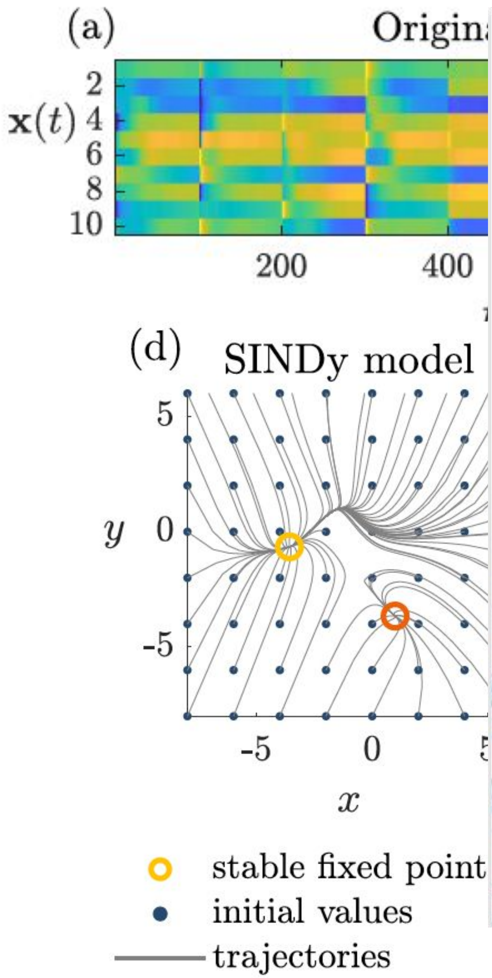
- Map a nonlinear dynamics to an infinite dimensional, linear dynamics

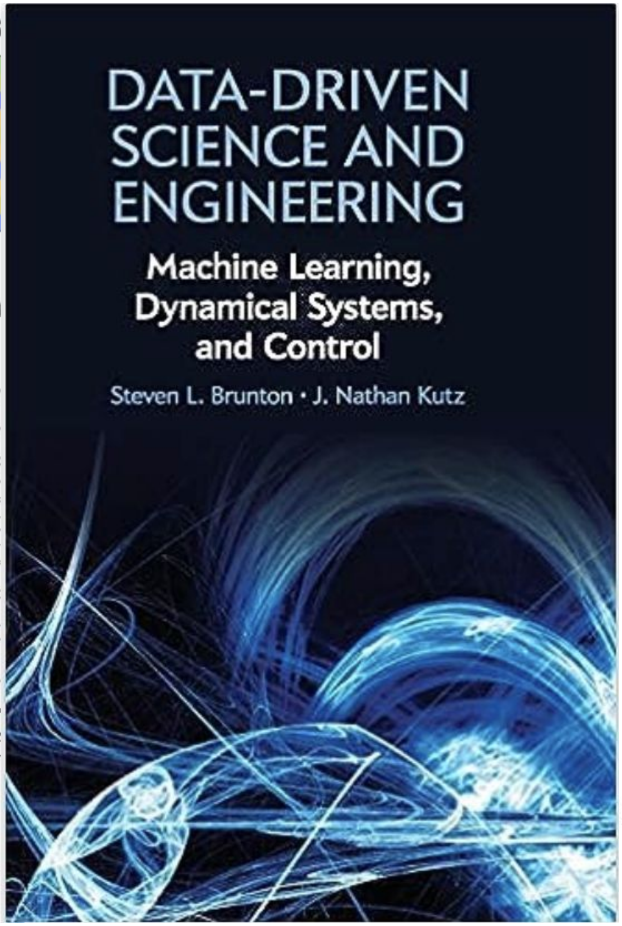
- Identify the key modes of interest



Data driven model discovery for control

- Find a **sparse (low dimensional) representation**, e.g., using SINDy*
- Map out **fixed points** in the low-dimensional space and develop strategic interventions



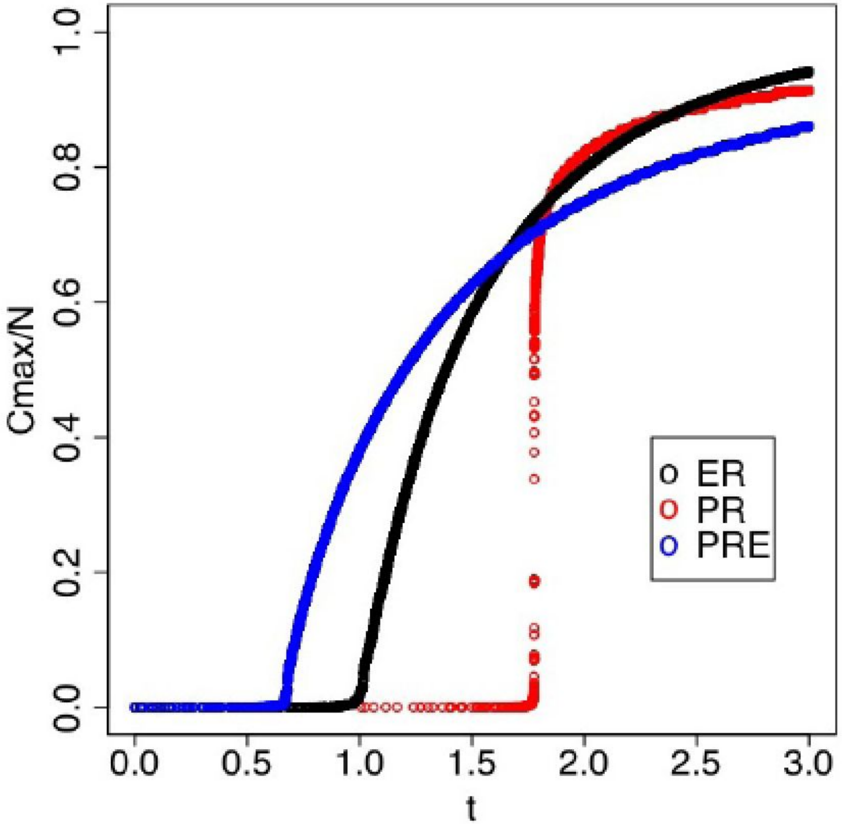


2019, Cambridge Univ. Press

Control of phase transitions

Design small interventions that enhance or delay the onset of phase transitions in a complex network.

- **Enhance** – similar to **ER** but with earlier onset.
- **Delay** – Extremely abrupt



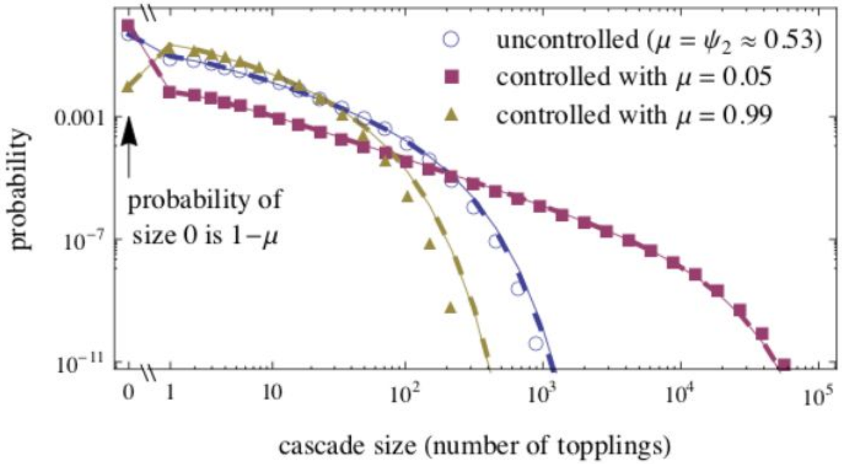
"Explosive percolation in random networks", D Achlioptas, RM D'Souza, J Spencer, *Science* 323 (5920), 1453-1455, 2009.

Control of SOC

Controlling the BTW model away from the SOC state

Noël, Brummitt, R.D., Phys. Rev. Lett. **111** *0780701, 2013*

Control parameter μ:
probability grain lands on a node at threshold*



- Avoid cascades, $\mu = 0.05 \rightarrow$ larger cascades when they do occur.
- Ignite cascades, $\mu = 0.99 \rightarrow$ smaller cascades, but more frequent.

- Tradeoffs and timescales!
  - Ignite cascades, no large cascades but also no profit
  - Suppress cascades, no failure for a long time, but massive when happens

Social networks and control interventions

Mathematical models of social behavior

Analyze extent of epidemic spreading, product adoption, etc:

- Thresholds models
- Voter models
- Opinion dynamics
  (e.g. The Naming game)
- Percolation
- Game theory
- Cascades

INSIDE SCIENCE NEWS SERVICE

Zealots Help Sway Popular Opinions

Image credit: Gabriel Saldana via Flickr | http://bit.ly/1E9IjCQ
Rights information: http://bit.ly/1dWcOPS

Enthusiasts can greatly influence the adoption of new ideas.

Originally published: Feb 19 2015 - 10:45am

By: Ker Than, Contributor

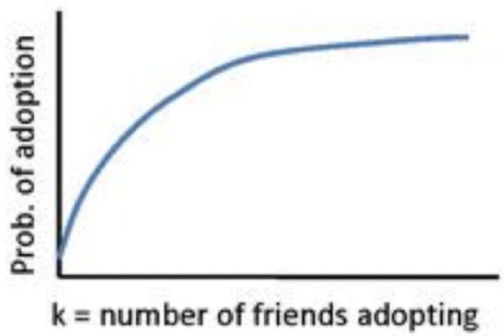
A. Waagen, G. Verma, K. Chan, A. Swami, R. D. *PRE*, 2015.

What mechanism makes an individual change their mind?

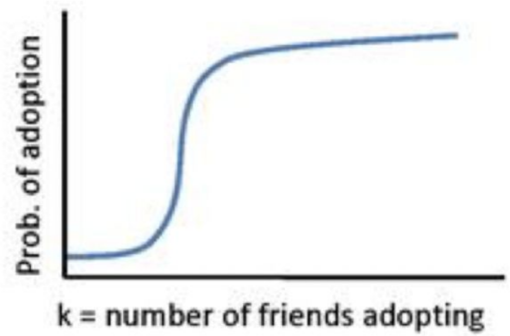
Basic question: What mechanism drives opinion dynamics?

Many competing hypothesis!

- Diminishing returns vs threshold models?



Diminishing returns?



Critical mass?

- Simple contagion versus complex contagion?
- Role of influence and attention? (cusp catastrophes)
- Perhaps people cannot be described by simple equations?

Opportunities

Feedback into physics; statistical treatments in control

- Increase network complexity – hypergraphs and multiplex networks
- Driven, far-from-equilibrium models in statistical physics (SOC, KPZ, ASEP)
- Activity driven temporal networks
- ML/AI and data driven model discovery
- Tradeoffs and timescales
- Hybrid treatments (continuous parts/discrete parts; continuafication…)
- Identify a set of paradigmatic problems or benchmark case studies that could be used to validate and contrast different approaches to control complex systems

Learn more:

nature reviews physics
https://doi.org/10.1038/s42254-023-00566-3

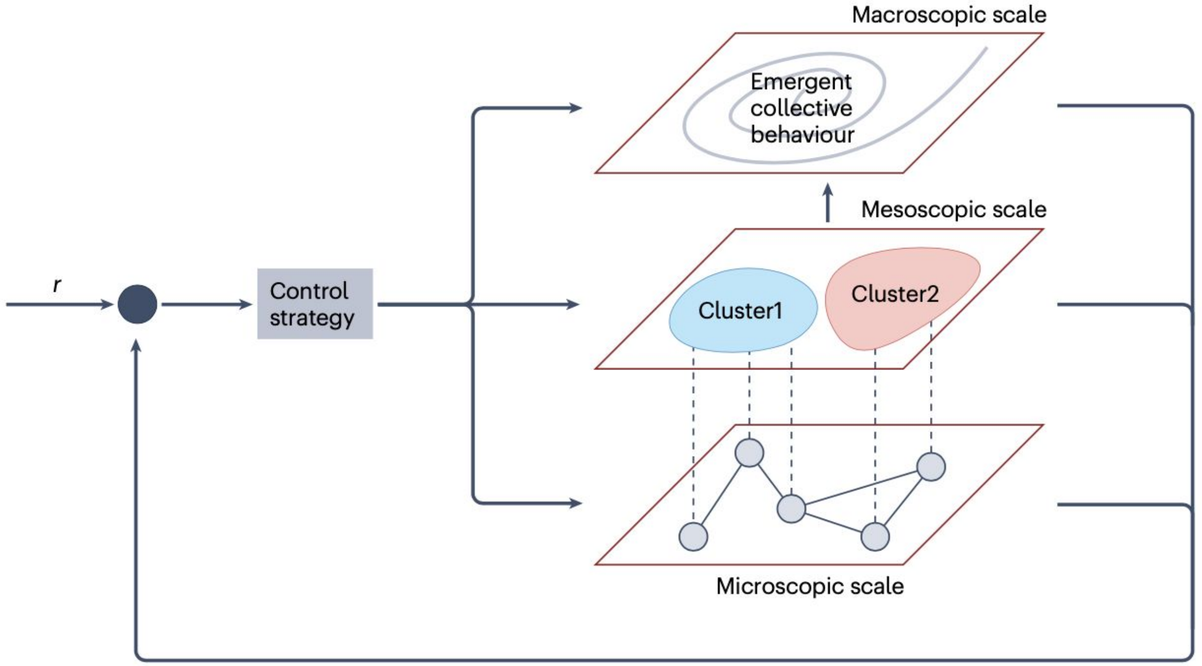
Perspective

Check for updates

Controlling complex networks with complex nodes

Published online: 24 March 2023

Raissa M. D'Souza[1,2,3], Mario di Bernardo[4,5] & Yang-Yu Liu[6,7]



Conclusions

Modern systems are made of interconnected complex systems:
Socio-technical, cyber-physical, eco-social.

Can't necessarily reduce to a set of interconnected differential equations.

- **Emergent interactions:**
  - Decoupled states – nodal dynamics & network structure
  - BTW meets Kuramoto – sandpile cascades on oscillator networks
  - SOC and Dragon kings – self-amplifying cascades

- **Controlling complex networks:**
  - Structural control (linear nodal dynamics)
  - Non-linear dynamics and basin structure
  - Data driven model prediction
  - Control of phase transitions and SOC

Guram Mikaberidze
Anastasiya Salova
Jeff Emenheiser
Matt Matheny
Micheal Roukes
Mario di Bernardo
Yang-Yu Liu
SCIENCE TO SHAPE THE FUTURE OF THE ARMY
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