Navigation systems in traffic networks
Route recommendations and performance degradation

Paolo Frasca

CNRS, GIPSA-lab, Grenoble

joint work with T. Toso and A. Kibangou

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Outline

Motivation: Navigation systems are ubiquitous

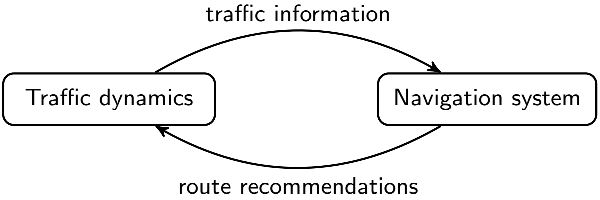
During the last decade, GPS-enabled navigation systems have spread, either embedded in vehicles or via mobile-phone apps like Waze or Google Maps

Navigation systems/apps use GPS, maps, and real-time traffic information to provide drivers with personalized route recommendations.

Route recommendations are based on the estimated shortest travel time.

Unintended feedback loop

Control systems perspective: Using real-time information implies that navigation apps are introducing a feedback loop on the traffic system.



What is the effect of navigation apps on traffic???

Literature review on traffic and navigation apps

Shortcomings of navigation apps
Recommendations benefit drivers, but navigation apps can have negative consequences at the global level

1. Increased congestion
2. Excessive traffic on small roads
3. Oscillations of traffic between roads

Previous work has mostly taken a static game-theoretic perspective (Thai et al., 2016; Cabannes, 2022), linking the drawbacks to suboptimal Wardrop equilibria

Little previous work has been devoted to modeling the dynamics:

- Bayen et al. (2019) have proposed a general dynamical framework (proved existence and uniqueness of solutions)
- Festa and Goatin (2019) have shown high pressure on secondary roads with negligible alleviation on main roads
- Bianchin and Pasqualetti (2020) have suggested the imitator dynamics to describe drivers' behavior

In this work, we address this literature gap and study a dynamical model

Outline

Objectives and features of our model

Objective of this work

Modeling the dynamics of a traffic network subject to route recommendations feedback, with the purpose of

- reconstructing the empirical phenomena,
- explaining their causes,
- highlighting which are the relevant parameters,
- identifying opportunities for interventions.

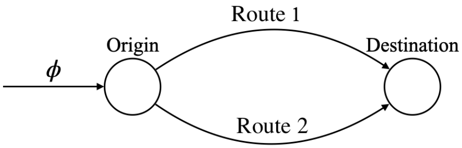
General modeling framework

Our general model will require to define three ingredients:

1. the geometry of the network of possible routes
2. the dynamics of traffic,
   aiming to include realistic congestion phenomena
3. the effect of navigation apps, which influence the choices of the drivers and ultimately the way traffic flow splits between routes.

Network geometry

We focus on the simplest network geometry:



Traffic demand $\phi > 0$ aims to travel from Origin to Destination, across two non-intersecting routes.

The demand is split between the two routes according to the routing ratios R_1, R_2, such that $0 \leq R_i \leq 1, \; i = 1, 2$ and $R_1 + R_2 = 1$

Each route i has its own features:

- length L_i (km)
- capacity F_i (veh/h)
- critical density C_i and jam density B_i (veh/km)
- free-flow speed $v_i = F_i / C_i$ (km/h)

We assume that the network is *well-dimensioned*: $\phi < F_1 + F_2$

Traffic dynamics with congestion: supply and demand functions

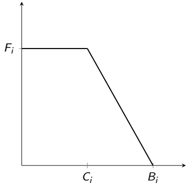
On each route $i = 1, 2$, the density x_i (veh/km) evolves according to the conservation law

$$L_i \dot{x}_i := \min\{\phi R_i(x), S_i(x_i)\} - D_i(x_i), \quad x \in \Omega := [0, B_1] \times [0, B_2], \tag{CL}$$

which accounts for congestion when $x_i > C_i$ via supply/demand functions:

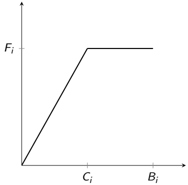
- the supply function is a Lipschitz function of the route density defined as

$$S_i(x_i) := \begin{cases} F_i & \text{if } x_i < C_i \\ \frac{F_i}{B_i - C_i}(B_i - x_i) & \text{otherwise} \end{cases}$$



- the demand function is a Lipschitz function of the route density defined as

$$D_i(x_i) := \begin{cases} v_i x_i & \text{if } x_i < C_i \\ F_i & \text{otherwise} \end{cases}$$



Travel times

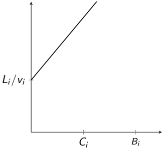
Each route has its travel time τ_i.

Travel times τ_i are increasing functions of density x_i, that is, $\frac{\partial \tau_i(x_i)}{\partial x_i} > 0$.

Examples:

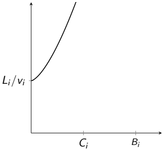
- Affine travel times:

$$\tau_i(x_i) := a_i \frac{x_i}{B_i} + \frac{L_i}{v_i}$$



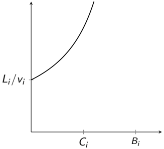
- Power-law travel times

$$\tau_i(x_i) := \frac{L_i}{v_i} + a_i \left(\frac{x_i}{B_i} \right)^m$$



- Travel times deduced from Greenshield's model:

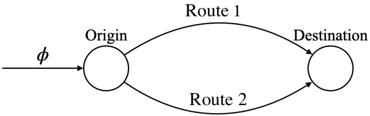
$$\tau_i(x_i) := \frac{L_i}{v_i \left(1 - \frac{x_i}{B_i}\right)}$$



From recommendations to routing ratios (through the travel times)

$$L_i \dot{x}_i := \min\{\phi R_i(x), S_i(x_i)\} - D_i(x_i),$$

The state-dependent splitting ratios $R(x)$ constitute the *coupling* between the two routes



The recommendations and the choices of drivers are reflected in the routing ratios between the two routes:

- The routing ratios R_1 and R_2 depend on the travel times τ_1 and τ_2
- Routing ratios are *monotonic* in the travel times (Como et al., 2013), that is,

$$\frac{\partial R_i}{\partial \tau_j} > 0, \quad i \neq j, \ i = 1, 2.$$

Densities $x \Rightarrow$ Travel times $\tau(x) \Rightarrow$ Routing ratios $R(\tau(x))$

Examples of monotonic ratios

We assume to have

- penetration rate α:, i.e., fraction of app-informed drivers;
- $r^0 = (r_1^0, r_2^0)$: fixed splitting of non-informed drivers;

Examples:

1. Logit routing ratios are an approximation of best response (that is, choosing the shortest travel time $\tau_i(x_i) = a_i \frac{x_i}{B_i} + \frac{L_i}{v_i}$):

$$R_1(x) := (1-\alpha) r_1^0 + \alpha \frac{1}{1 + \frac{r_2^0}{r_1^0} \exp\left(-\frac{\tau_2(x) - \tau_1(x)}{\eta}\right)} \qquad \text{(Logit)}$$

$$R_2(x) = 1 - r_1(x)$$

   where $1/\eta$ is user compliance to recommendations. Non-compliance can be due to unmodeled costs, besides travel time

2. A simpler model are linear routing ratios:

$$R_1(x) = (1-\alpha) r_1^0 + \alpha \left(\frac{1}{2} + \frac{1}{2}\left(\frac{x_2}{B_2} - \frac{x_1}{B_1}\right)\right)$$

$$R_2(x) = (1-\alpha) r_2^0 + \alpha \left(\frac{1}{2} + \frac{1}{2}\left(\frac{x_1}{B_1} - \frac{x_2}{B_2}\right)\right) \qquad \text{(Lin)}$$

   where travel times reduce to occupancy indices x_i/B_i

Measuring traffic network quality: demand satisfaction

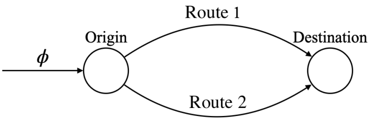
How to evaluate the (negative) effects of recommendations?

We shall check whether the network is able to satisfy the incoming demand ϕ.

Network failure

If $\min\{\phi R_i(x), S_i(x_i)\} = S_i(x_i)$, then there is unsatisfied demand $\phi R_i(x) - S_i(x_i)$

Note: Since we assumed $\phi \leq F_1 + F_2$, there cannot be unsatisfied demand on both routes simultaneously.



Unsatisfied demand in the mathematical model means that vehicles cannot enter the chosen routes, thus congestion builds up at the origin.

Outline

Theorem (Long-time behavior)

System (CL) *with monotonic ratios and increasing travel times is a monotone system and has a unique, globally asymptotically stable, equilibrium* $\overline{x}$*.*

Our model does not produce sustained oscillations, but instead convergence to an equilibrium. Then, it is important to study the properties of the equilibrium.

Proposition (No congestion at equilibrium)

$$\overline{x}_i \leq C_i \text{ for } i = 1, 2$$

Congestion (within the routes) is only a transient phenomenon for this dynamics

Linear routing: Unsatisfied demand at equilibrium

- Define for each route *virtual capacity* $E_i := v_i B_i$
- To focus on the effect of recommendations, assume that r^0 is such that $\phi r_j^0 \leq F_j$, $j = 1, 2$, i.e. the fixed splitting ratios satisfy demand.
- Recall that demand is not satisfied on route i when $\phi R_i > F_i$

Proposition (Unsatisfied demand condition)

Demand is not satisfied on route i if and only if

$$\alpha\phi^2 E_j^{-1} + \alpha\phi\left(1 - F_i E_i^{-1} - F_i E_j^{-1}\right) + (1-\alpha)\phi\, 2r_i^0 > 2F_i$$

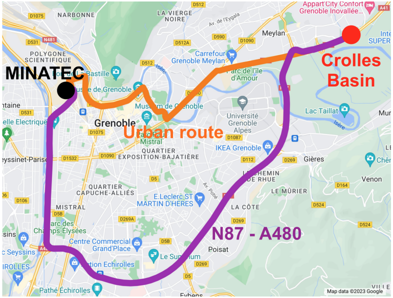
Therefore, demand is not satisfied on the "weakest" route

- if ϕ is large enough
- if ϕ is not too small and α is large enough

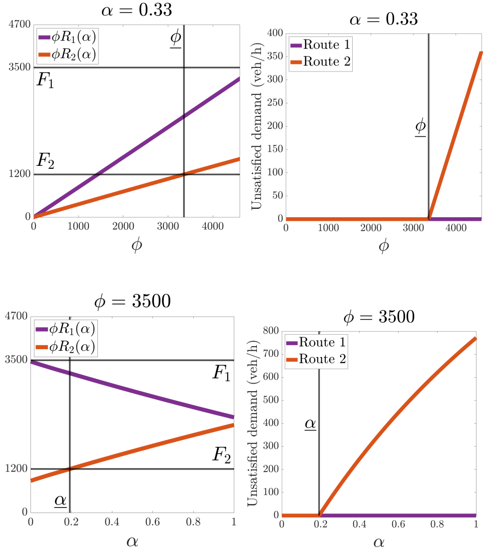
High traffic demand & high penetration rate $\Longrightarrow$ unsatisfied demand

Linear routing: numerical analysis of equilibrium

Case study: Crossing Grenoble via its South Ring or via the city center.



$F_1 = 3500$ veh/h, $v_1 = 70$ km/h, $B_1 = 250$ veh/km
$F_2 = 1200$ veh/h, $v_2 = 50$ km/h, $B_2 = 120$ veh/km



Linear routing: numerical analysis of equilibrium

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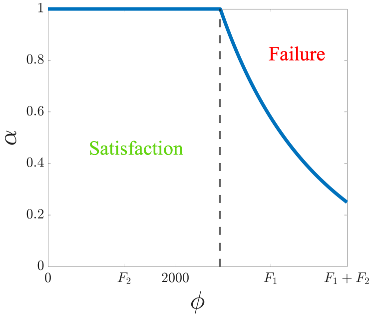

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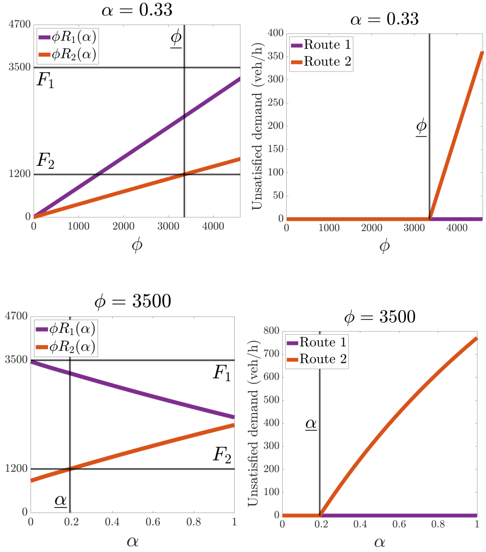
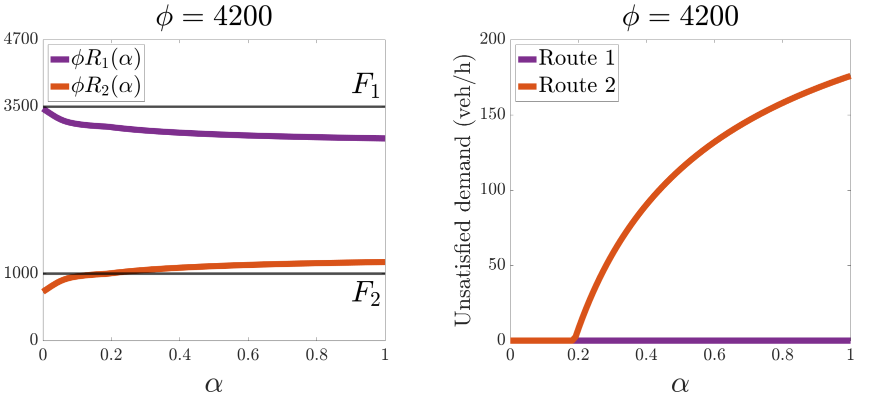


Logit routing

The nonlinearity prevents finding an explicit expression of $\overline{x}$, but numerically we can see that the system can fail to satisfy demand at equilibrium (for high ϕ and high α).



$F_1 = 3500$ veh/h, $v_1 = 70$ km/h, $B_1 = 250$ veh/km
$F_2 = 1000$ veh/h, $v_2 = 50$ km/h, $B_2 = 120$ veh/km

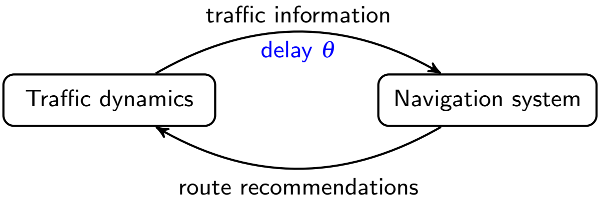
Outline

Delayed traffic information

Delay in "real-time" traffic data is unavoidable, because collection, communication, and processing are necessary before data can be used for the recommendations.

To describe this, we then introduce a delay θ in density information as it appears in the routing ratios:

$$L\dot{x}_i(t) := \min\{\phi R_i(x(t-\theta)), S_i(x_i(t))\} - D_i(x_i(t)), \quad i = 1,2$$



Reducing to scalar dynamics

Assumption (Homogeneous routes)

We assume that the two routes are homogeneous, i.e., they have the same length L and free-flow speed v.

The homogeneity assumption reduces the system to a scalar dynamics in $d(t)$:

$$\dot{d}(t) = -\frac{v}{L} d(t) + \rho(d(t-\theta)) \tag{Delay}$$

where

$$\rho(d(t-\theta)) := \frac{1}{L}\left(\frac{a_2}{B_2}\min\{F_2, \phi(1-R_1(d(t-\theta)))\} - \frac{a_1}{B_1}\min\{F_1, \phi R_1(d(t-\theta))\}\right)$$

Proposition (Equilibrium)

The dynamics (Delay) has a unique equilibrium point $\overline{d}$.

What are its stability properties?

Stability analysis: Delay-independent stability

Theorem (Delay-independent global asymptotic stability, sufficient condition)

If $\dfrac{\alpha\phi}{\eta} < \dfrac{4vB_1B_2}{a_2B_1 + a_1B_2}$*, then* $\overline{d}$ *globally asymptotically stable for any* $\theta \geq 0$.

If traffic demand, penetration rate, and compliance are high, then the system becomes sensitive to delay, in the sense that delays can destabilise it.

What happens when the system is destabilised?
Can oscillations produce unsatisfied demand??

Assumptions (to keep the problem interesting)

1. There is no unsatisfied demand when the penetration rate is zero, that is,

$$\phi r_i^0 < F_i, \quad i = 1, 2.$$

2. There is no unsatisfied demand at equilibrium, that is,

$$1 - \frac{F_2}{\phi} < R_1(\overline{d}) < \frac{F_1}{\phi}.$$

3. The user demand ϕ and the penetration rate α are high enough to allow unsatisfied demand to emerge on one of the two routes. This requirement is satisfied if

$$\phi > F_i,\ \alpha > \underline{\alpha}_i := \frac{F_i - \phi r_i^0}{\phi(1 - r_i^0)}, \quad i = 1, 2.$$

Delay-dependent stability

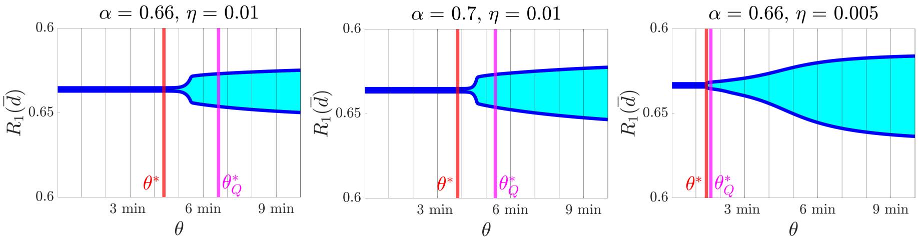
Define $\gamma_i := F_i - \phi(1-\alpha)r_i^0$ and $Q := \frac{\phi}{\eta L}\left(\frac{a_1}{B_1} + \frac{a_2}{B_2}\right)\min\{\gamma_1\left(1-\frac{\gamma_1}{\alpha}\right), \gamma_2\left(1-\frac{\gamma_2}{\alpha}\right)\}$

Theorem (Hopf bifurcation, sufficient condition)

If $v/L < Q$, then there exists a critical delay θ^* such that

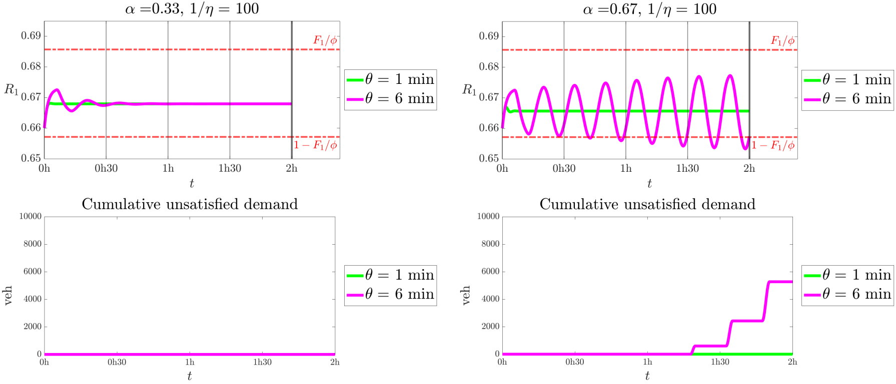
- *$\overline{d}$ is asymptotically stable for $\theta < \theta^*$ and unstable for $\theta > \theta^*$;*
- *system (Delay) undergoes a Hopf bifurcation at $d = \overline{d}$ when $\theta = \theta^*$.*

Furthermore, $\theta^* \le \theta_Q^*$, where $\theta_Q^* := \left(Q^2 - v^2/L^2\right)^{-1/2} \arccos(-v/LQ)$.



Bifurcation diagrams: θ_Q^* decreases as α and $1/\eta$ increase.

The oscillations of the solutions cause periodic failure to satisfy demand

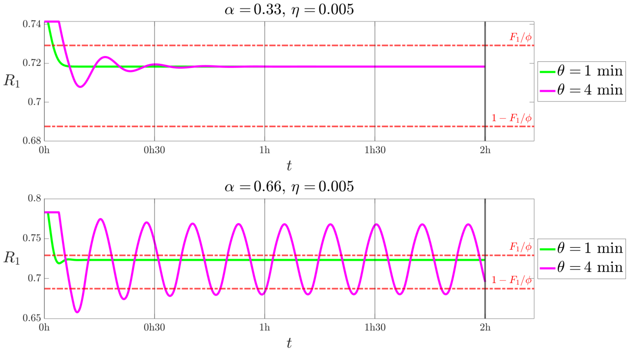


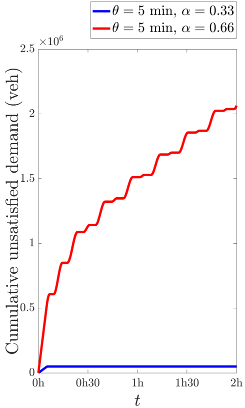
Red dashed lines delimit the states in which demand is satisfied

$F_1 = 1200$ veh/h, $C_1 = 24$ veh/km, $B_1 = 120$ veh/km,

$F_2 = 600$ veh/h, $C_2 = 12$ veh/km, $B_2 = 60$ veh/km, $a_1 = a_2 = 6$ min, $r_1^0 = 0.67$, $r_2^0 = 0.33$.

Oscillations are also present in the case of non-homogeneous routes.





Red dashed lines delimit the states in which demand is satisfied

$$F_1 = 3500 \text{ veh/h},\ B_1 = 250 \text{ veh/km},\ L_1 = 10 \text{ km},\ v_1 = 70 \text{ km/h},$$
$$F_2 = 1500 \text{ veh/h},\ B_2 = 120 \text{ veh/km},\ L_2 = 7 \text{ km},\ v_2 = 50 \text{ km/h},$$
$$\phi = 4800 \text{ veh/h},\ a_1 = a_2 = 6 \text{ min},\ r_1^0 = 0.7,\ r_2^0 = 0.3.$$

Outline

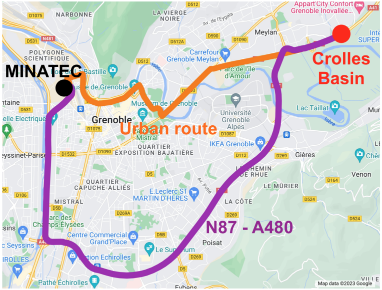
Final comments

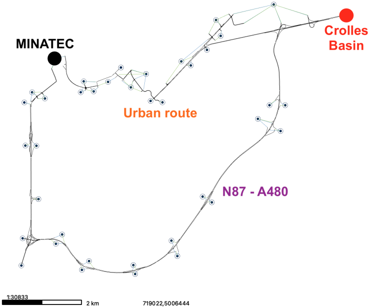
Summary

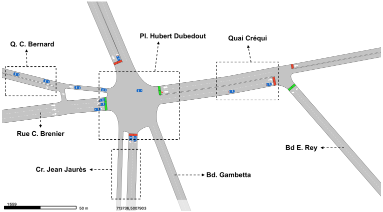
1. Routing apps provide information on road congestion and travel times, thus making drivers prioritize faster (less crowded) roads.
2. We model the effect of navigation apps on drivers by using state-dependent splitting ratios, which prioritize less crowded roads.
3. Our analytical and numerical results indicate that these state-dependent splitting ratios can lead to poor steady-state performance:
   - the network can fail to satisfy demand at equilibrium;
   - in presence of delays, the network can exhibit oscillations and therefore periodically fail to satisfy demand.

   Both drawbacks are more likely and more pronounced for high demand and high penetration rate.

Detailed Aimsun simulations







Future work

Extensions of the model

Network geometry: More complex network topologies, with multiple intersecting routes.

Traffic dynamics: Allow for saturation at route exit, and thus endogenous congestion.

Routing ratios: More complex routing models, possibly with internal dynamics, develop game-theoretic interpretation.

We would also like to further look into the case of incomplete/imperfect/delayed information (and remove assumption of route homogeneity in delayed model)

Empirical research

- In collaboration with behavioral scientists in Grenoble, seek insights on penetration rate, compliance, app usage by drivers... $\rightarrow$ better model calibration

From dynamics to control

Design mitigation strategies against the issues brought by navigation apps

- Variable speed limits, variable capacities
- Incentives, tolls
- Autonomous vehicles

Some references

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