6-DOF Radar Absolute Pose Estimation

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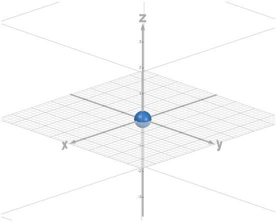
- Radar at origin, spinning in the xy-plane gets a hit on some point $\boldsymbol{x}$. Where can $\boldsymbol{x}$ be?



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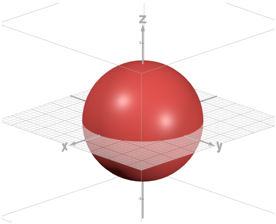

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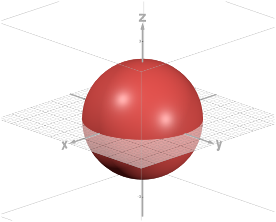

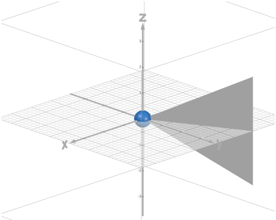

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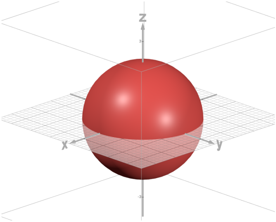

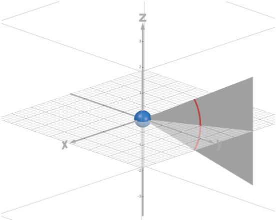

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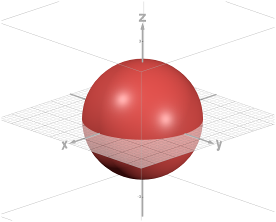

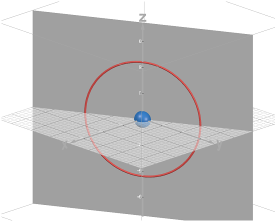

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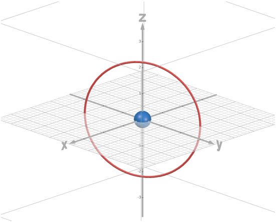
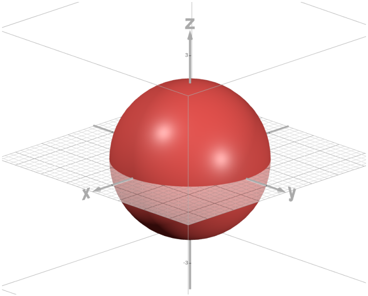


Radar model equations

- Sphere with radius r:

$$\|\boldsymbol{x}\|^2 - r^2 \approx 0 \tag{1}$$



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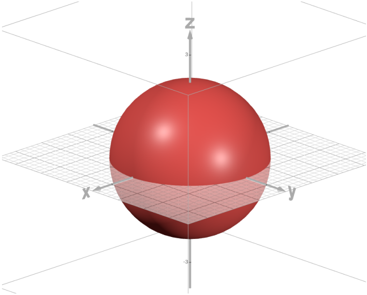

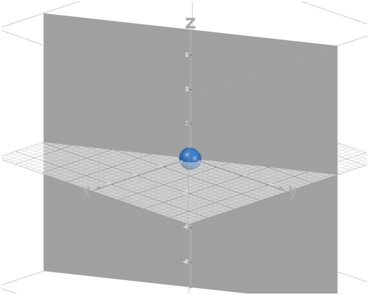

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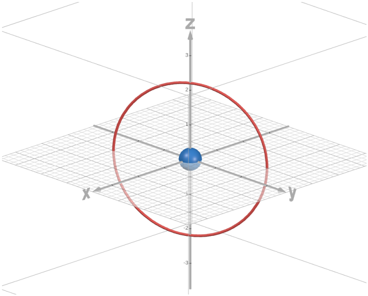
- Together they give the circle:



Problem formulation

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N known 3D points $\boldsymbol{x}_i$ with corresponding radii r_i and azimuth θ_i

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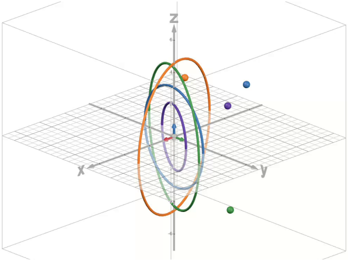
Goal

Estimate $\boldsymbol{c}, \boldsymbol{R}$ to align the circles with their corresponding points



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$$L(\boldsymbol{R}, \boldsymbol{c}) = \sum_{i=1}^{N} \omega_i (\underbrace{\|\boldsymbol{c} - \boldsymbol{x}_i\|_2^2 - r_i^2}_{\text{Distance error}})^2 + \gamma_i (\underbrace{\boldsymbol{n}_i^T \boldsymbol{R}(\boldsymbol{x}_i - \boldsymbol{c})}_{\text{Azimuth error}})^2 \tag{4}$$

Optimization problem

- Loss reminder:

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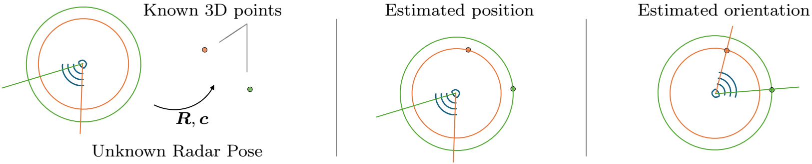

Solving for translation

- Define the loss using only $\boldsymbol{c}$:

$$L_c(\boldsymbol{c}) = \sum_{i=1}^{N} \omega_i (\|\boldsymbol{c} - \boldsymbol{x}_i\|_2^2 - r_i^2)^2 \tag{7}$$

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- Approximation of optimal trilateration, solved by Martin Larsson et al.

Rotation optimization problem

- Define the loss using only $\boldsymbol{R}$ (note $\boldsymbol{c}$ is now fixed):

$$L_R(\boldsymbol{R}) = \sum_{i=1}^{N} \gamma_i (\underbrace{\boldsymbol{n}_i^T \boldsymbol{R}}_{\text{Some 0s}} (\boldsymbol{x}_i - \boldsymbol{c}))^2 \tag{8}$$

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- Quadratic in $\hat{\boldsymbol{R}}$, rewrite it as a quadratic form:

$$L_R(\boldsymbol{R}) = \frac{1}{2} \boldsymbol{r}^T \boldsymbol{M} \boldsymbol{r}, \text{ with } \boldsymbol{r} = \text{vec}(\hat{\boldsymbol{R}}) \tag{10}$$

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- Only constraint is $\hat{\boldsymbol{R}}\hat{\boldsymbol{R}}^T = \boldsymbol{I}_{2\times 2}$. Determinant is irrelevant since bottom row can be constructed afterwards.

Solving for rotation

- Optimization problem:

$$\min_{\hat{\boldsymbol{R}}\hat{\boldsymbol{R}}^T = \boldsymbol{I}_{2\times 2}} \frac{1}{2}\boldsymbol{r}^T \boldsymbol{M} \boldsymbol{r} \tag{11}$$

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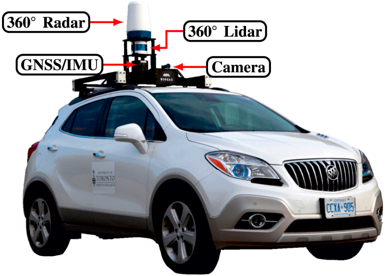
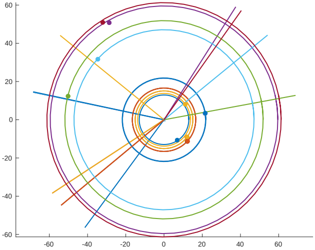
- This is a special case of a problem solved by Gaku Nakano

Dataset

- Boreas dataset, car with sensors in Toronto
- Same route in different seasons



Qualitative results



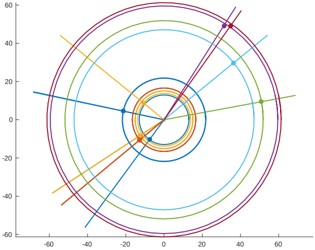


Figure 1: Left: After translation estimation. Right: Rotation estimation

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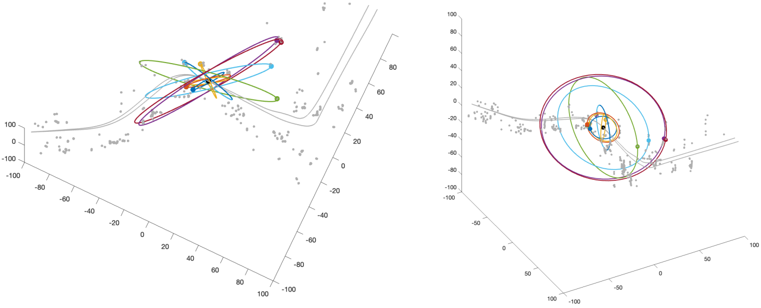

Figure 2: Two different views in 3D

Results

- Median of results on 27 sequences of the same route during different parts of the year

	Acc@ (0.25m, 5°)	Acc@ (0.5m, 10°)	Acc@ (5m, 15°)	Median $\epsilon_{\boldsymbol{c}}$ (m)	Median $\epsilon_{\boldsymbol{R}}$ (°)
Median	0.44	0.73	0.89	0.17	4.09

References

1. Larsson, M., Larsson, V., Åström, K., Oskarsson, M.: Single-source localization as an eigenvalue problem. IEEE Transactions on Signal Processing (2025)
2. Nakano, G.: Globally Optimal DLS Method for PnP Problem with Cayley parameterization. In: BMVC. vol. 78, pp. 1–78 (2015)