Phase Transitions and Universality: A Case Study in Democratic Instablity

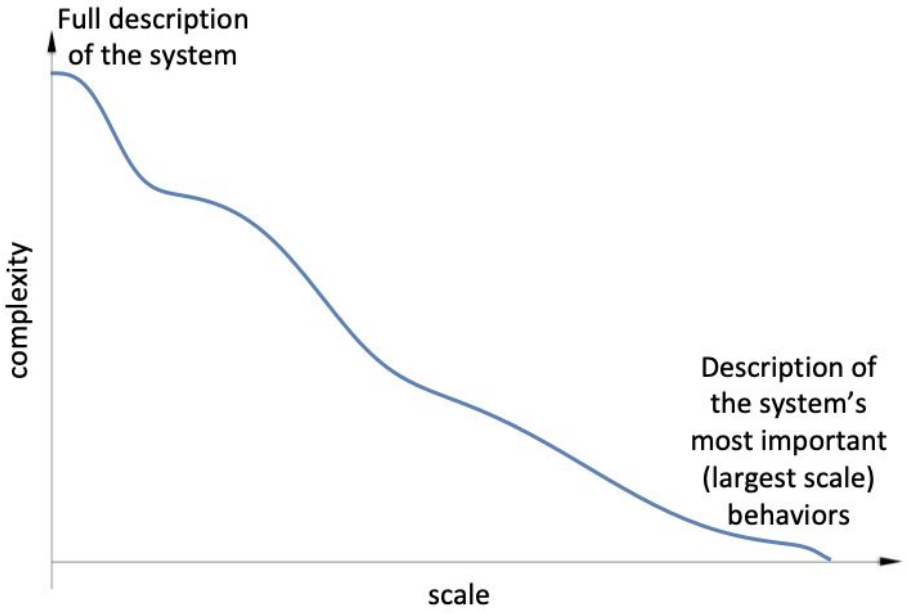
Alexander F. Siegenfeld

ELLIIT Focus Period
2 October 2023

What does physics have to do with social systems?

Universality and the renormalization group

- Impossible to capture all details
- Can’t even model individual person
- Model must fall into the same universality class as system
- Phase transitions between universality classes



Outline

- General approach
  - Need for theory
    - Example: why the West got masks wrong
  - The space of possibilities (choosing the right variables/description)
    - Example: travel restrictions in pandemic response
  - Multiscale analysis
    - Example: complexity profiles
- Negative representation and instability in democratic elections
  - Extension to federal democracies (if time)

The need for theory: avoiding naive empiricism

Without explicit theory, default assumptions of independence and homogeneity tend to be implicitly smuggled in

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While more than 100 people have died in the #coronavirus outbreak, seasonal flu kills between 250,000-650,000 people annually.

For most people, "you're probably more likely to be catching flu than you are to be getting coronavirus," says @devisridhar.

These 2 questions will determine if the coronavirus becomes a deadly pandemic
We still don't have firm answers to the most important questions about this virus.
vox.com

9:58 AM · Feb 1, 2020 · SocialFlow

Vox
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A travel ban isn't likely to prevent the spread of #coronavirus. Here's why.

The evidence on travel bans for diseases like coronavirus is clear: They don't work
They're political theater, not good public health policy.
vox.com

6:00 PM · Feb 3, 2020 · SocialFlow

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The coronavirus exposes the history of racism and "cleanliness"

The coronavirus exposes the history of racism and "cleanliness"
While the epidemic may be new, xenophobia has been intertwined with public health discourse for a very long time.
vox.com

7:40 AM · Feb 7, 2020 · Vox Media

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@voxdotcom
10:20 AM · Mar 2, 2020 · Twitter Web App

2/ Oh, and face masks? You can pass on them.

Masks are only useful if you have a respiratory infection already and want to limit the risk of spreading, or if you're working in a hospital in direct contact with people who have respiratory illnesses. bit.ly/2PzvrSl

Vox
@voxdotcom · 2/1/20

What is this #coronavirus?
It's part of a family of viruses that attack the respiratory system.

Should I travel during the outbreak?
The CDC and the State Department advise avoiding travel to China for now.

Is this going to be a deadly pandemic?
No.

9 questions about the Covid-19 coronavirus pandemic, answered
vox.com

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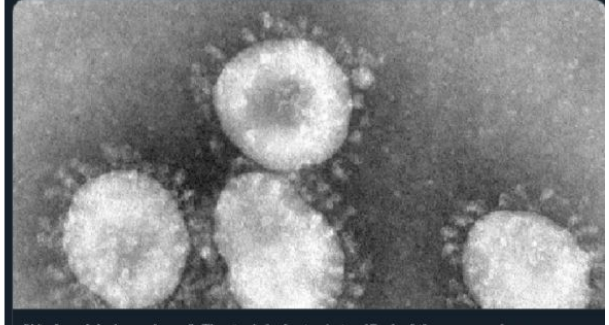
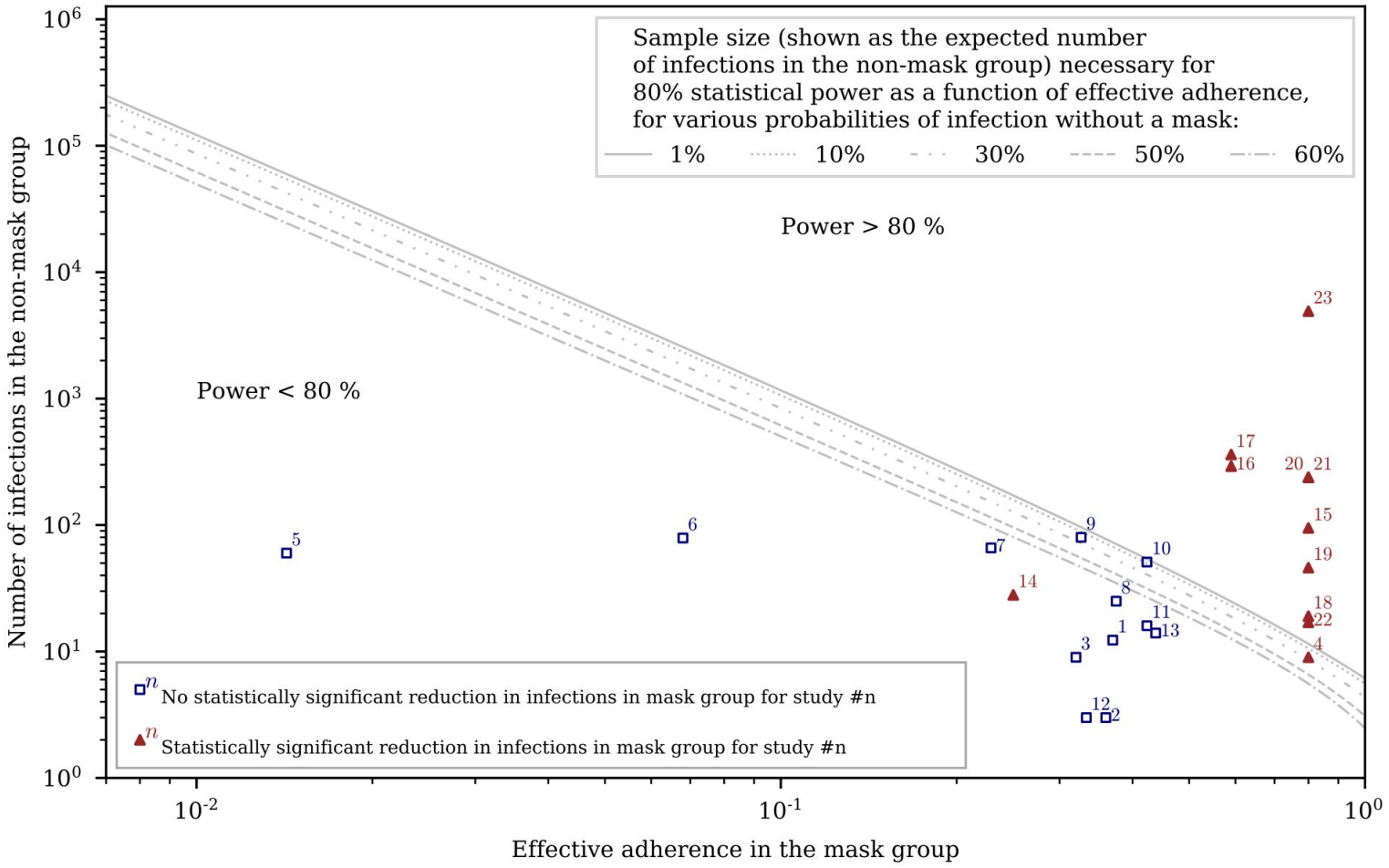
"No handshakes, please": The tech industry is terrified of the coronavirus



"No handshakes, please": The tech industry is terrified of the coronavirus
Although public officials in the area say the virus is contained for now, some in the tech industry fear the virus will spread out of control.
vox.com

7:00 AM · Feb 13, 2020 · Vox Media

The need for theory: avoiding naive empiricism



Pratyush K. Kollepara, Alexander F. Siegenfeld, Nassim Nicholas Taleb, Yaneer Bar-Yam. Unmasking the mask studies: why the effectiveness of surgical masks in preventing respiratory infections has been underestimated. *Journal of Travel Medicine* **28**, taab144 (2021)

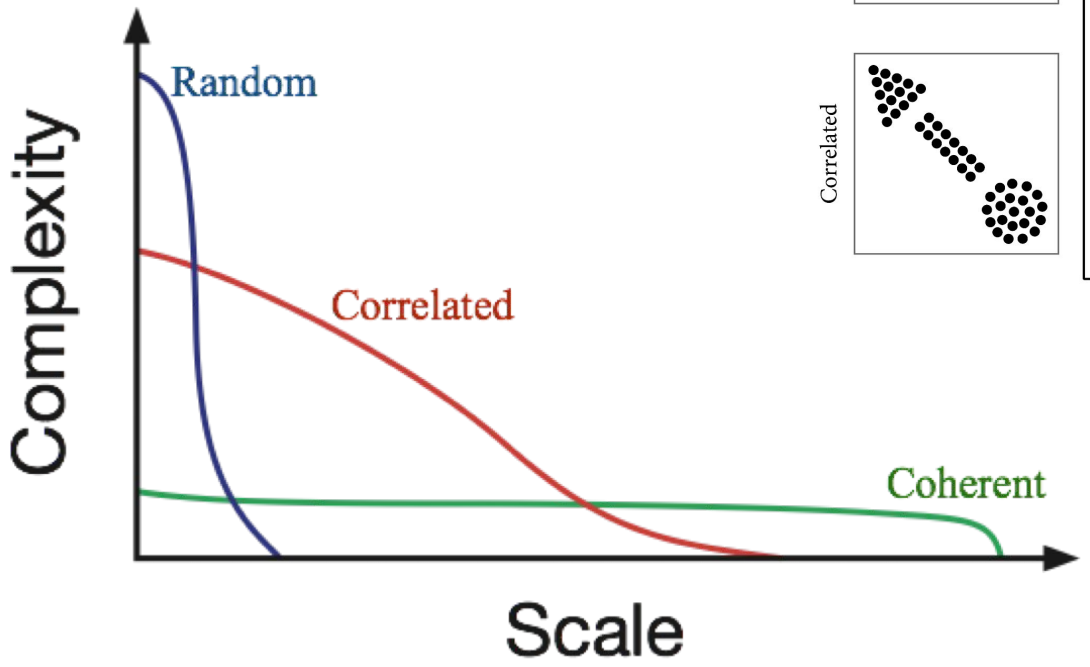
The space of possibilities: choosing the right variables

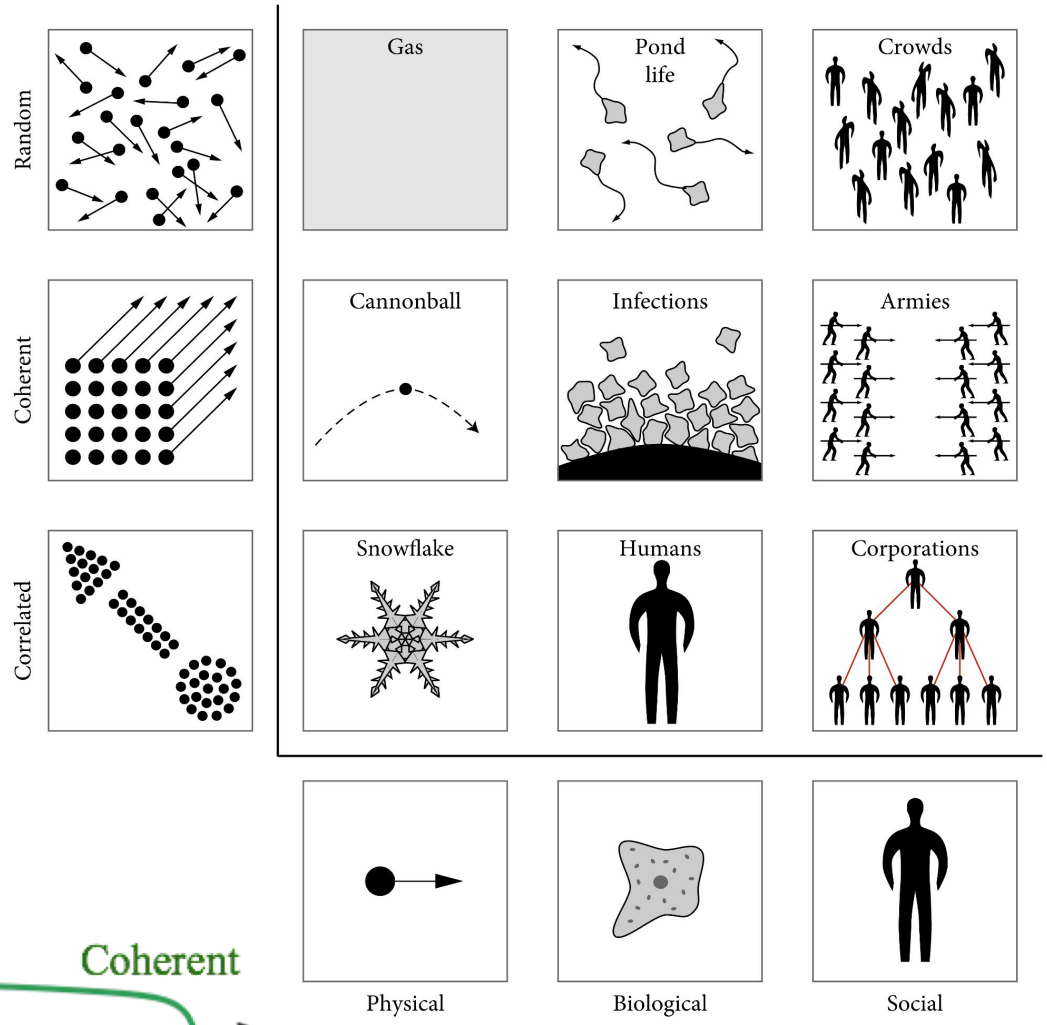
- Hidden assumption in the variables chosen
  - Describing disease by average number of cases per population
    - Rules out travel restrictions from the outset
  - Social contact rate constant in time
    - Rules out non-pharmaceutical interventions

Alexander F. Siegenfeld, Nassim Nicholas Taleb, Yaneer Bar-Yam. What models can and cannot tell us about COVID-19. *PNAS* **117**, 16092-16095 (2020).

Alexander F. Siegenfeld and Yaneer Bar-Yam. The impact of travel and timing in eliminating COVID-19. *Communications Physics* **3**, 204 (2020).

Multiscale analysis: complexity profiles

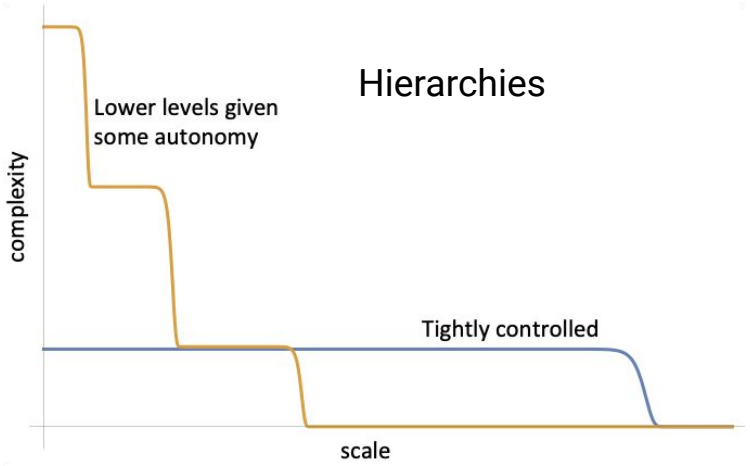




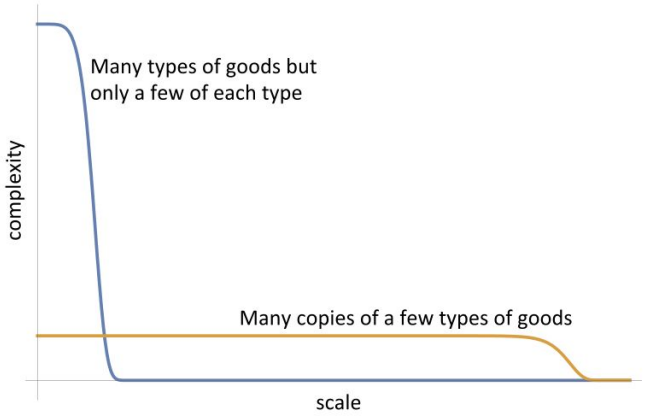
Alexander F. Siegenfeld and Yaneer Bar-Yam. An introduction to complex systems science and its applications. *Complexity* 6105872 (2020).

Examples

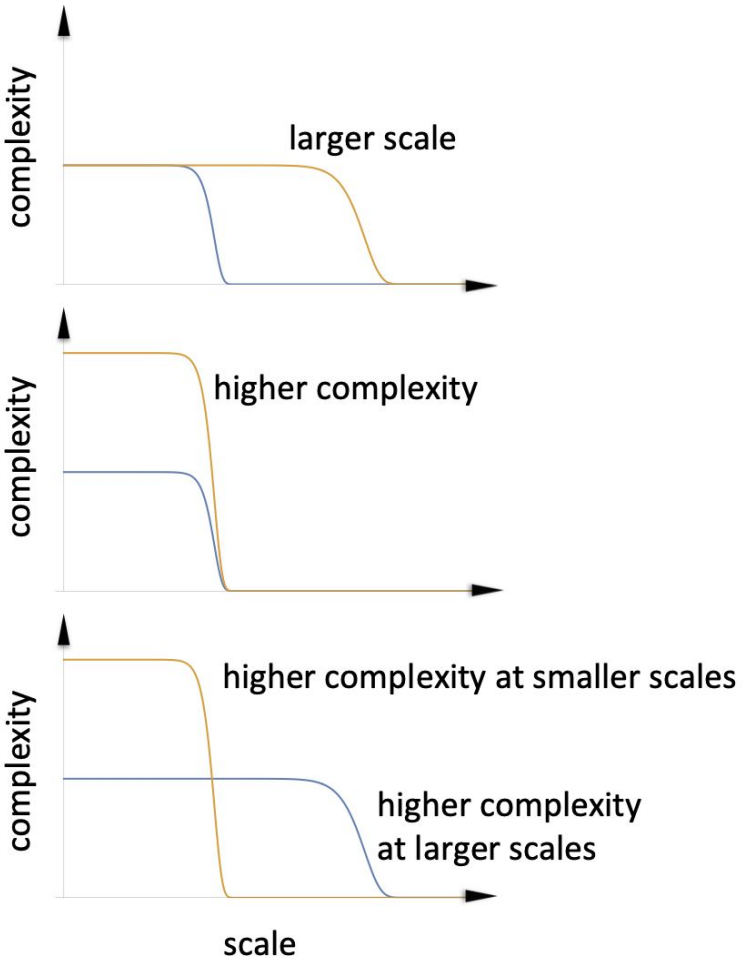
Hierarchies



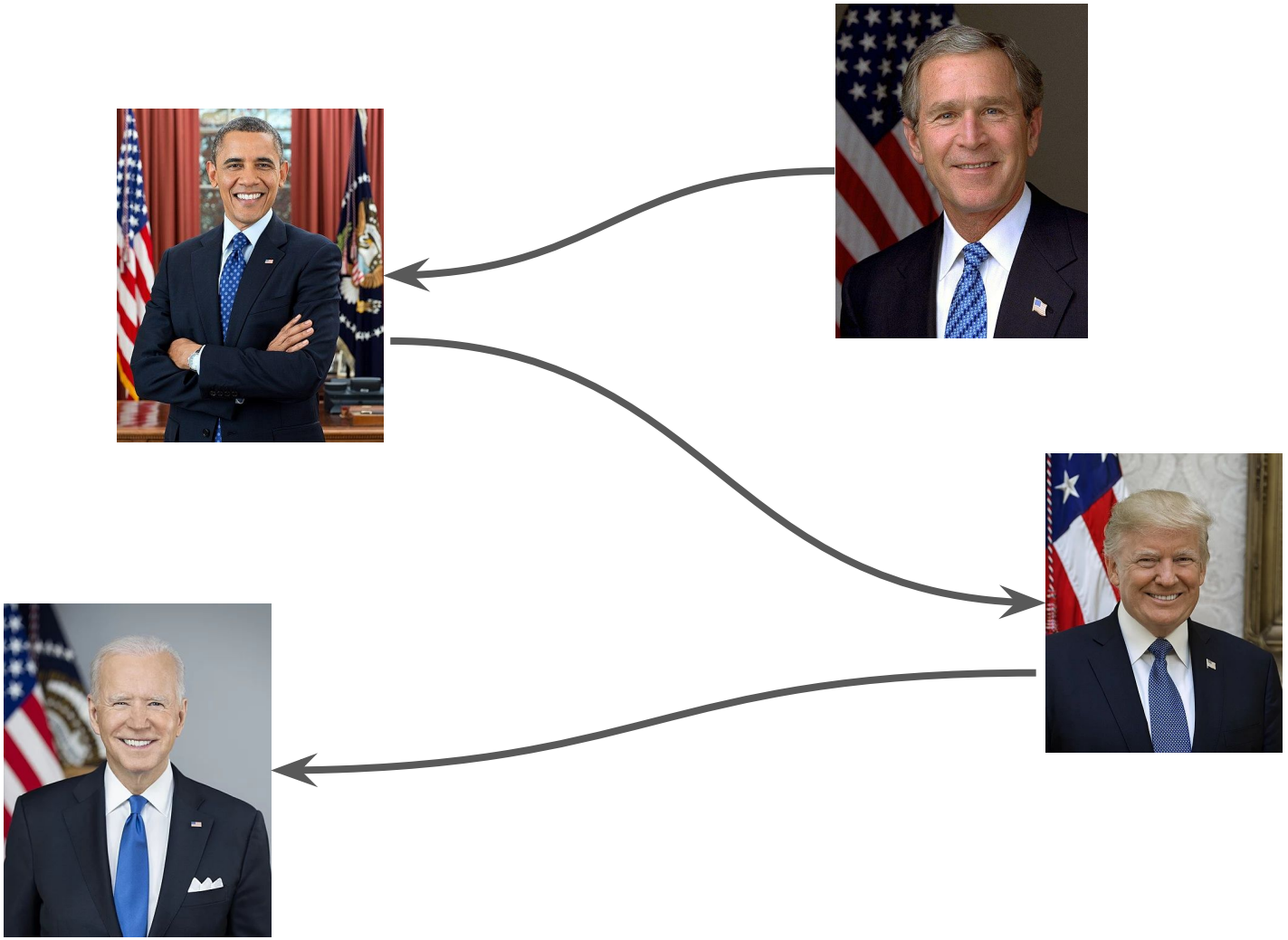
Factory (efficiency vs adaptability)



Military conflict



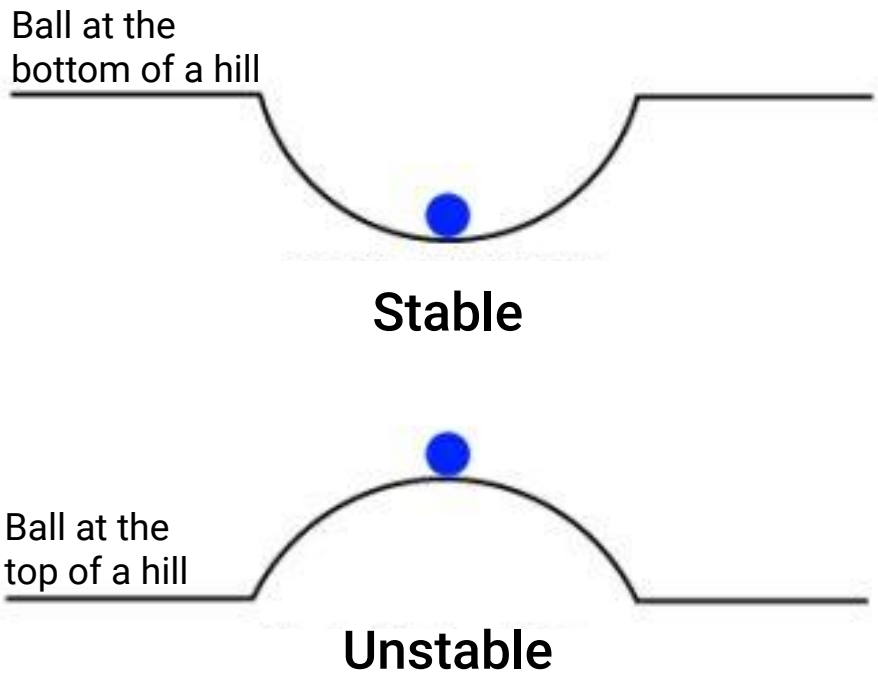
Negative representation and instability in democratic elections



Alexander F. Siegenfeld and Yaneer Bar-Yam. Negative representation and instability in democratic elections. *Nature Physics* **16**, 186-190 (2020)

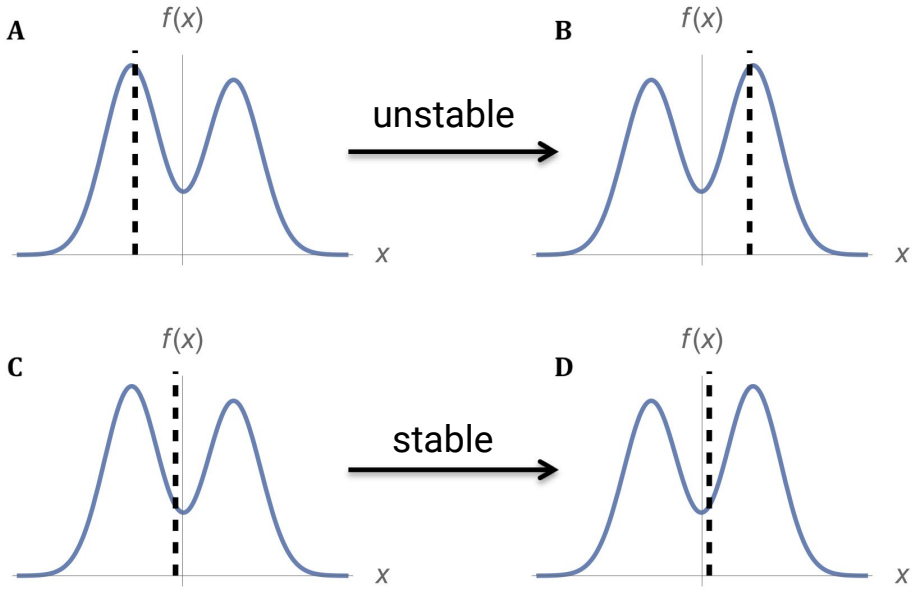
Conceptual overview

- Goal: characterize and understand electoral instability



Conceptual overview

- Goal: characterize and understand electoral instability
- Election
  - Input: electorate opinions
  - Output: opinion of elected official

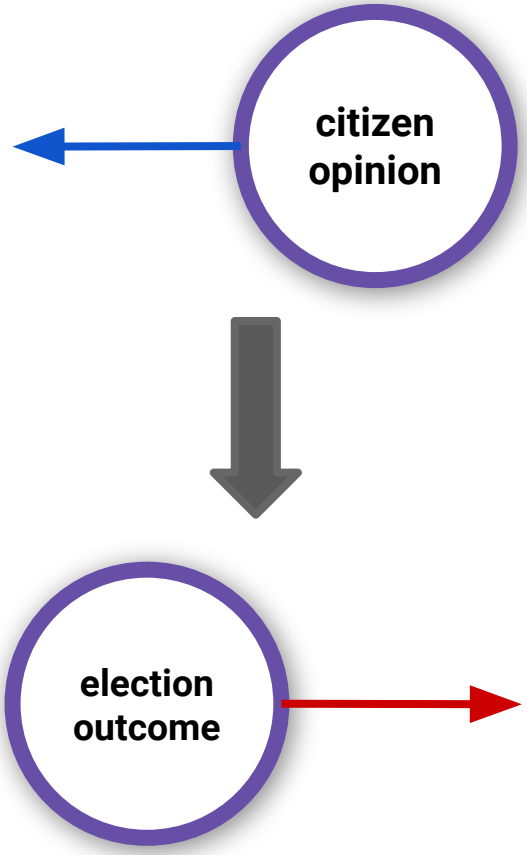


Conceptual overview

- Goal: characterize and understand electoral instability

- Election
  - Input: electorate opinions
  - Output: opinion of elected official

- Key Results
  - All unstable elections contain *negatively represented* opinions
  - Phase transition between stable and unstable regime



Negative representation

General model

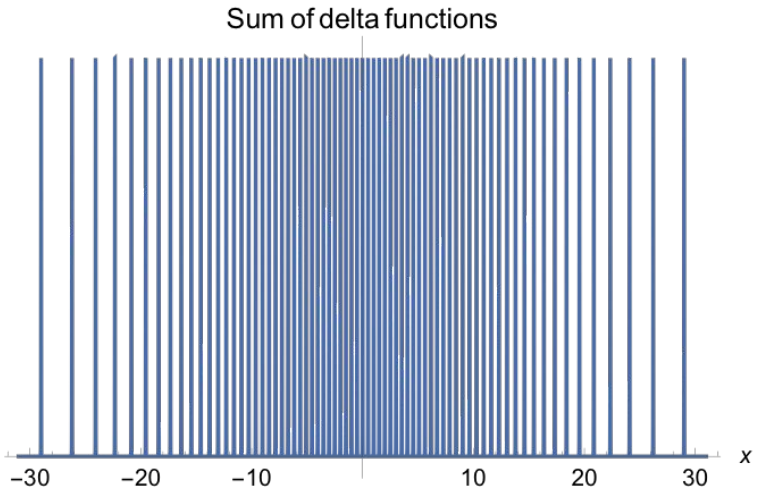
- Opinion $x \in \mathbb{R}^d$ describes *entire set* of political beliefs
- Electorate opinions: $\{x_1, ..., x_N\}$
- Election outcome: $y \in \mathbb{R}^d$
- Election: a mapping from $(\mathbb{R}^d)^N \to \mathbb{R}^d$
- Representation (in 1D): $r_i^c \equiv \frac{\delta y}{c}$ for $x_i \to x_i + c$
  - If the derivative exists: $r_i \equiv \frac{\partial y}{\partial x_i}$
- Instability: election discontinuous
- Translational invariance: $y(x_1 + c, ..., x_N + c) = y(x_1, ..., x_N) + c$

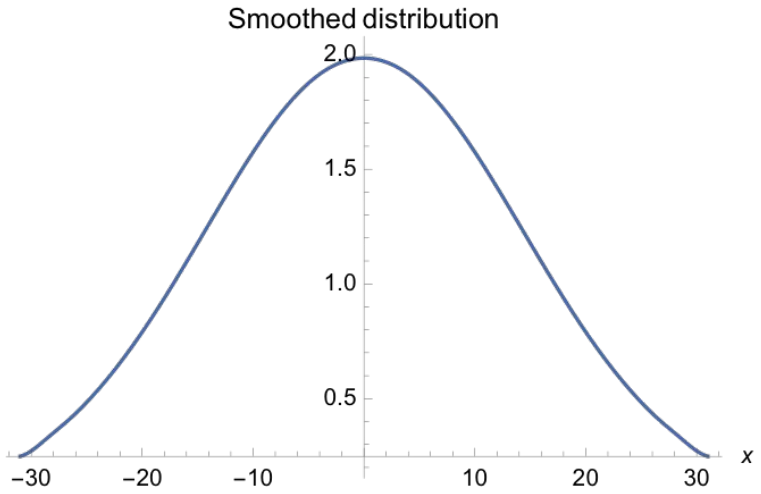
All unstable elections contain negatively represented opinions

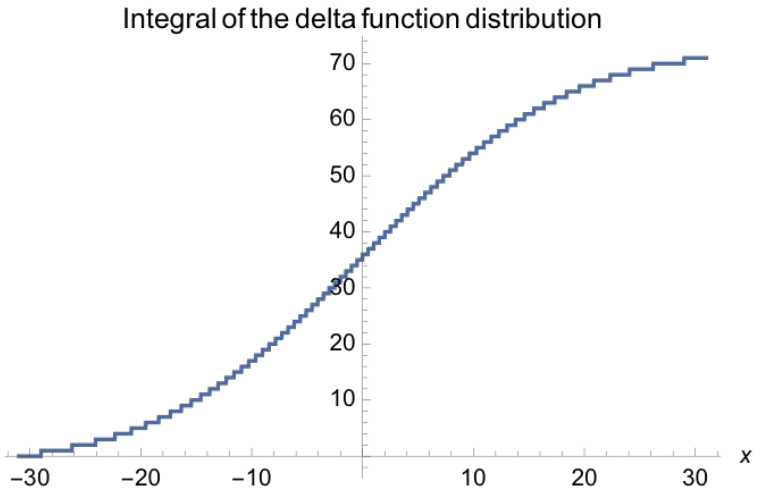
- Instability implies $\delta y_1 \equiv y(x_1, ..., x_i + \epsilon, ..., x_N) - y(x_1, ..., x_N) \equiv C$ with $|C| > |\epsilon|$
- Trans. symmetry: $\delta y_2 \equiv y(x_1 - \epsilon, ..., x_i, ..., x_N - \epsilon) - y(x_1, ..., x_N) = C - \epsilon$
- $\delta y_1 \delta y_2 > 0 \rightarrow$ exists negatively represented opinion

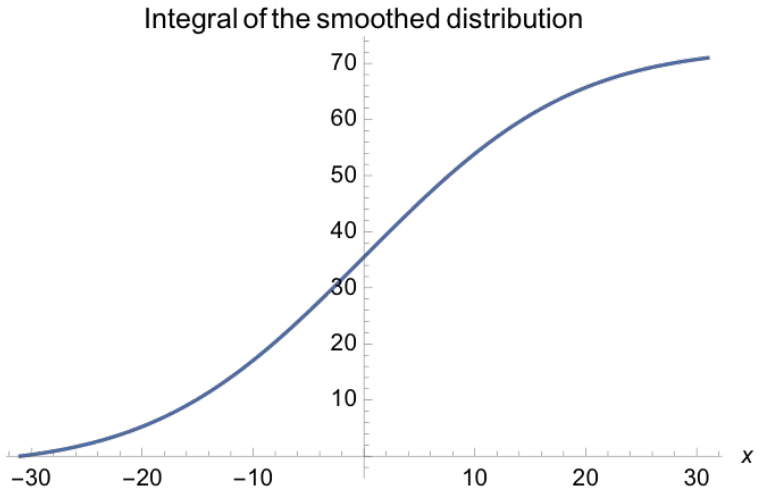
Distributions of opinions

- Consider distribution of opinions $f(x) \approx \sum_{i=1}^{N} \delta(x - x_i)$







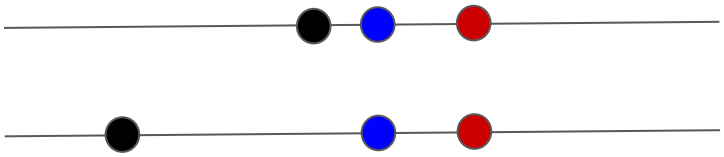


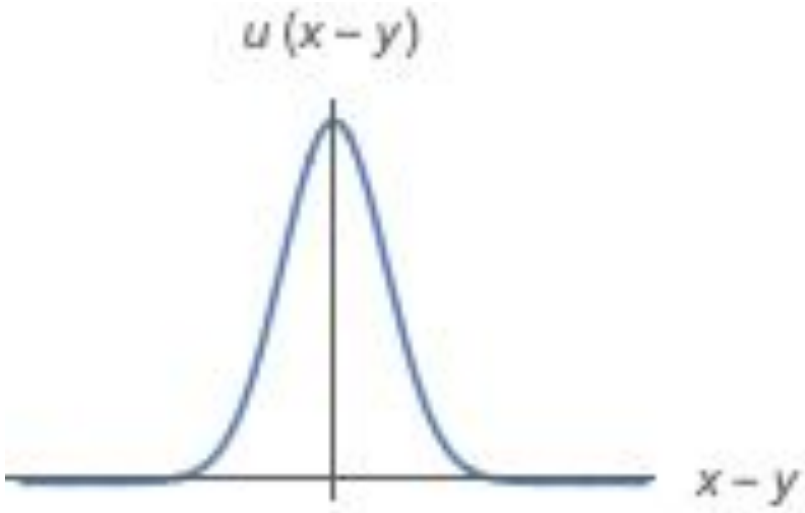
Distributions of opinions

- Consider distribution of opinions $f(x) \approx \sum_{i=1}^{N} \delta(x - x_i)$
- Election: $f \rightarrow y[f]$
- In the limit where any one voter has a small effect on the outcome:
  - Representation (in 1D): $r(f, x) = \frac{d}{dx} \frac{\delta y}{\delta f(x)}$
  - $\int_{-\infty}^{\infty} dx f(x) r(f, x) = 1$

Application to utility difference models

- Preferences of citizen with opinion x: $p_{y_1} - p_{y_2} = u_x(y_1) - u_x(y_2)$
- Outcome: $\text{argmax}_{y \in \mathbb{R}} \int_{-\infty}^{\infty} dx f(x) u_x(y)$
- Can reproduce outcome of median voter theorem: $u_x(y) = -|y - x|$
- Mean voting: $u_x(y) = -(y - x)^2$
- Alienation: $u_x(y) = e^{\frac{-(y-x)^2}{2a^2}}$
  - Reduces to mean voting as $a \to \infty$

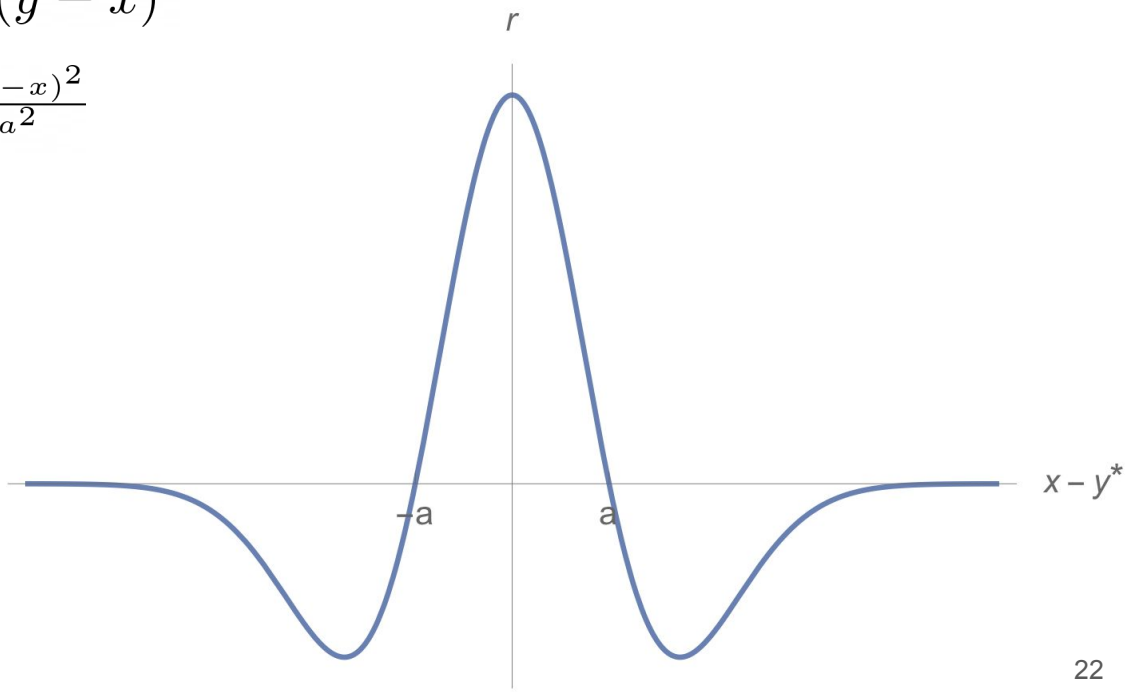




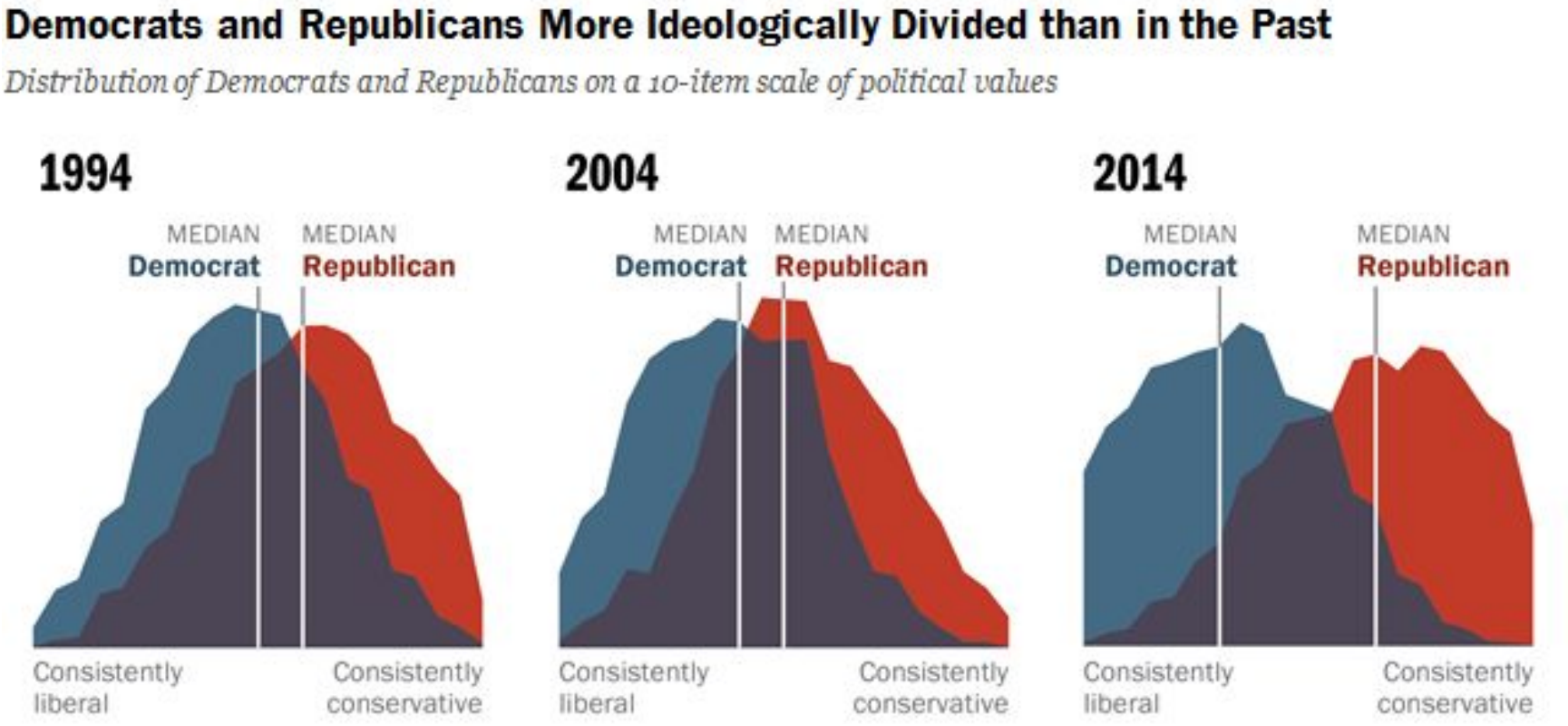
Representation

- $r(f,x) = \frac{d}{dx}\frac{\delta y}{\delta f(x)} \propto -\frac{1}{N}u''(y-x)$
- Negative representation unless concave utility
- Median voting: $u_x(y) = -|y-x|$
- Mean voting: $u_x(y) = -(y-x)^2$
- Alienation: $u_x(y) = e^{\frac{-(y-x)^2}{2a^2}}$

Note:
instability → negative representation
(but not vice versa)

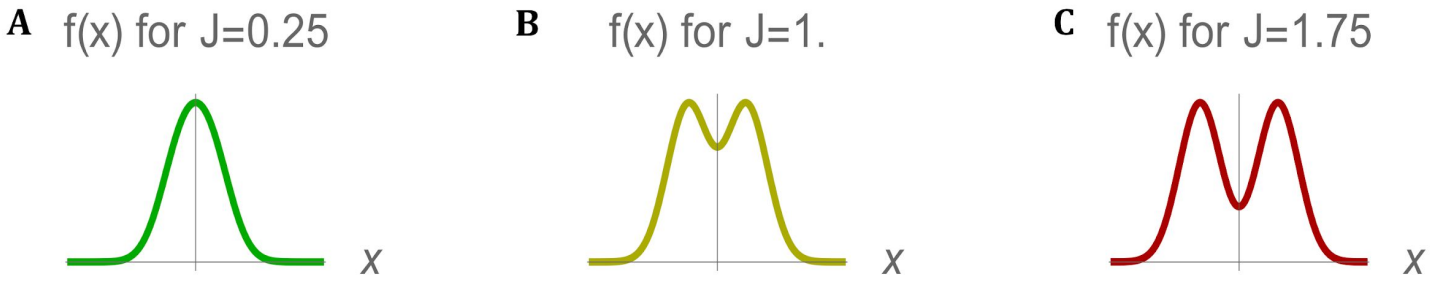


Polarizing electorate



Phase transition

Polarizing electorate:



$$f(x) = w_1 e^{\frac{-(x+\Delta)^2}{2\sigma^2}} + w_2 e^{\frac{-(x-\Delta)^2}{2\sigma^2}}$$

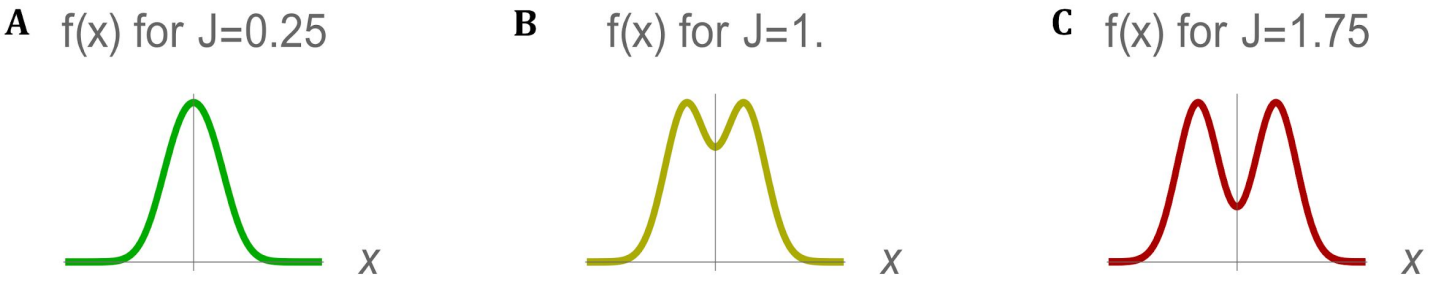
Dimensionless measure of polarization $J = \frac{\Delta^2}{a^2 + \sigma^2}$

$$h = \frac{1}{2} \ln \frac{w_2}{w_1}$$

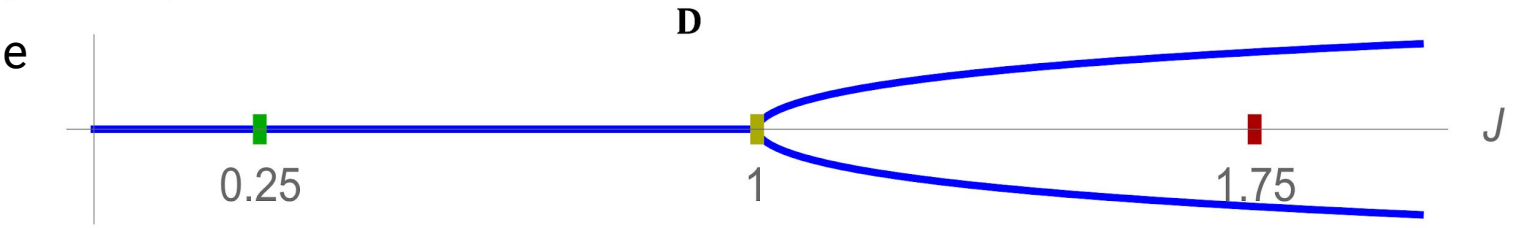
Recall: $u_x(y) = e^{\frac{-(y-x)^2}{2a^2}}$

Phase transition

Polarizing electorate:



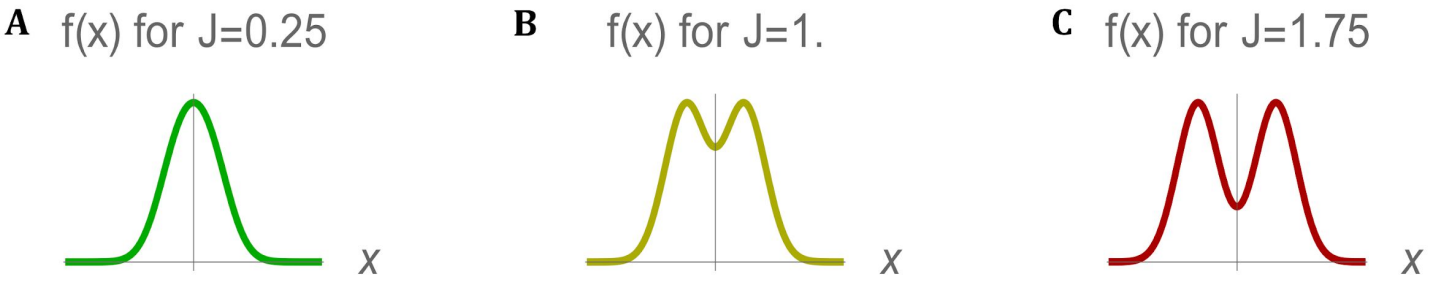
Election outcome for symmetric electorate:



$$y/\Delta = \tanh(Jy/\Delta + h)$$

Phase transition

Polarizing electorate:



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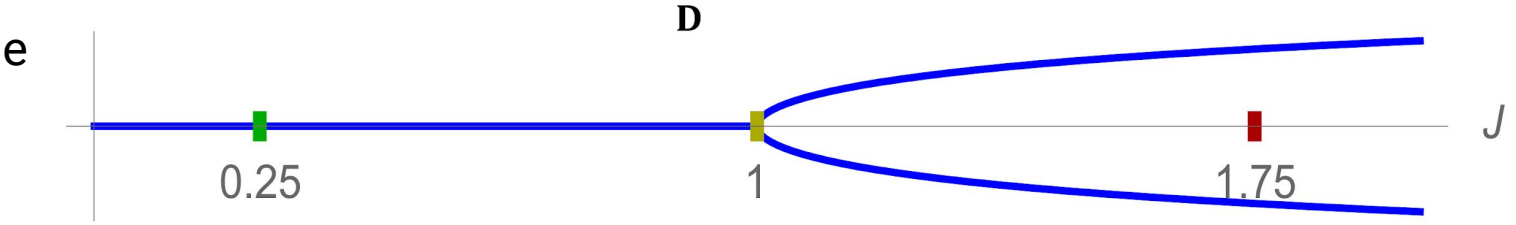
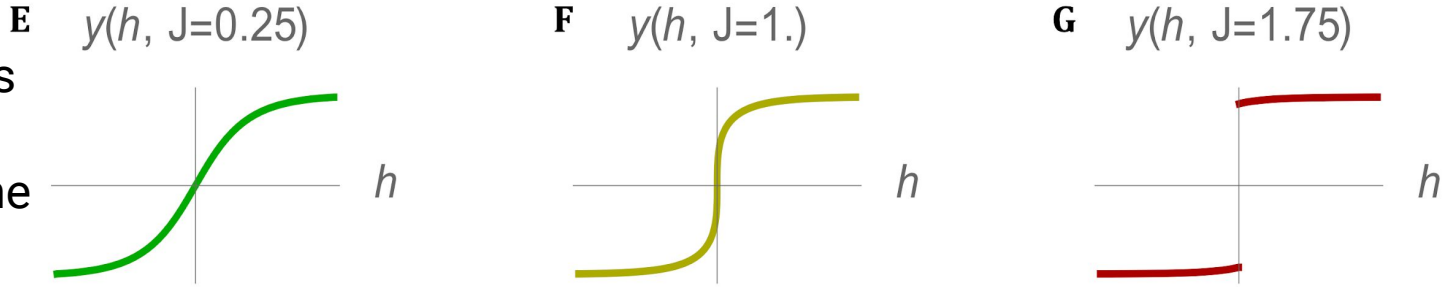


Election outcomes as function of the relative sizes of the two peaks:

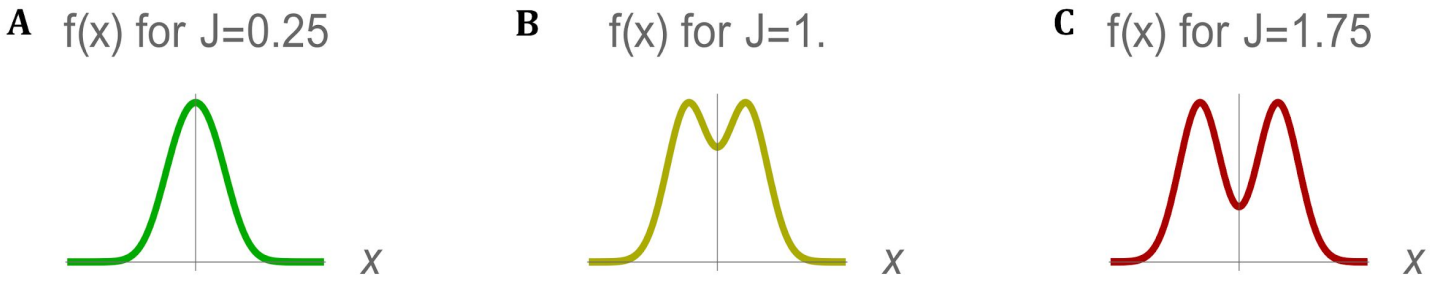


$$h = \frac{1}{2} \ln \frac{w_2}{w_1}$$

$$y/\Delta = \tanh(Jy/\Delta + h)$$

Phase transition

Polarizing electorate:



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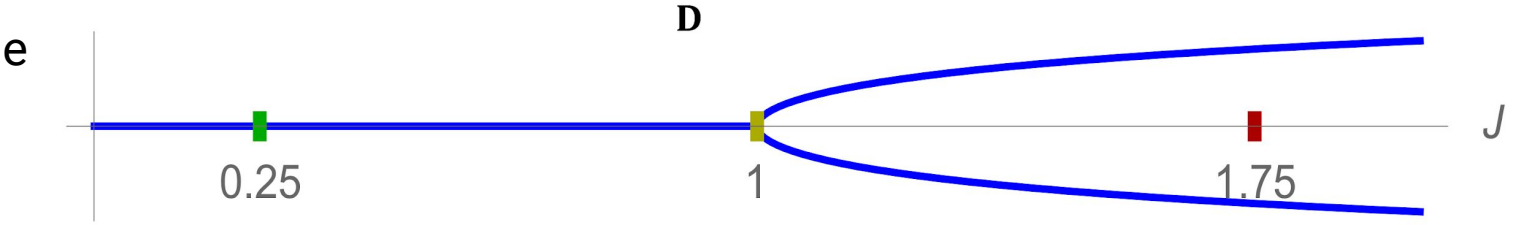

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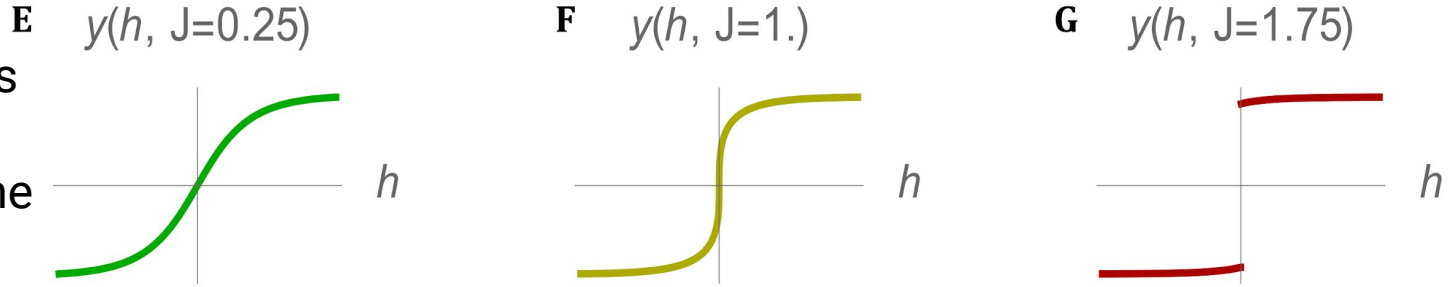
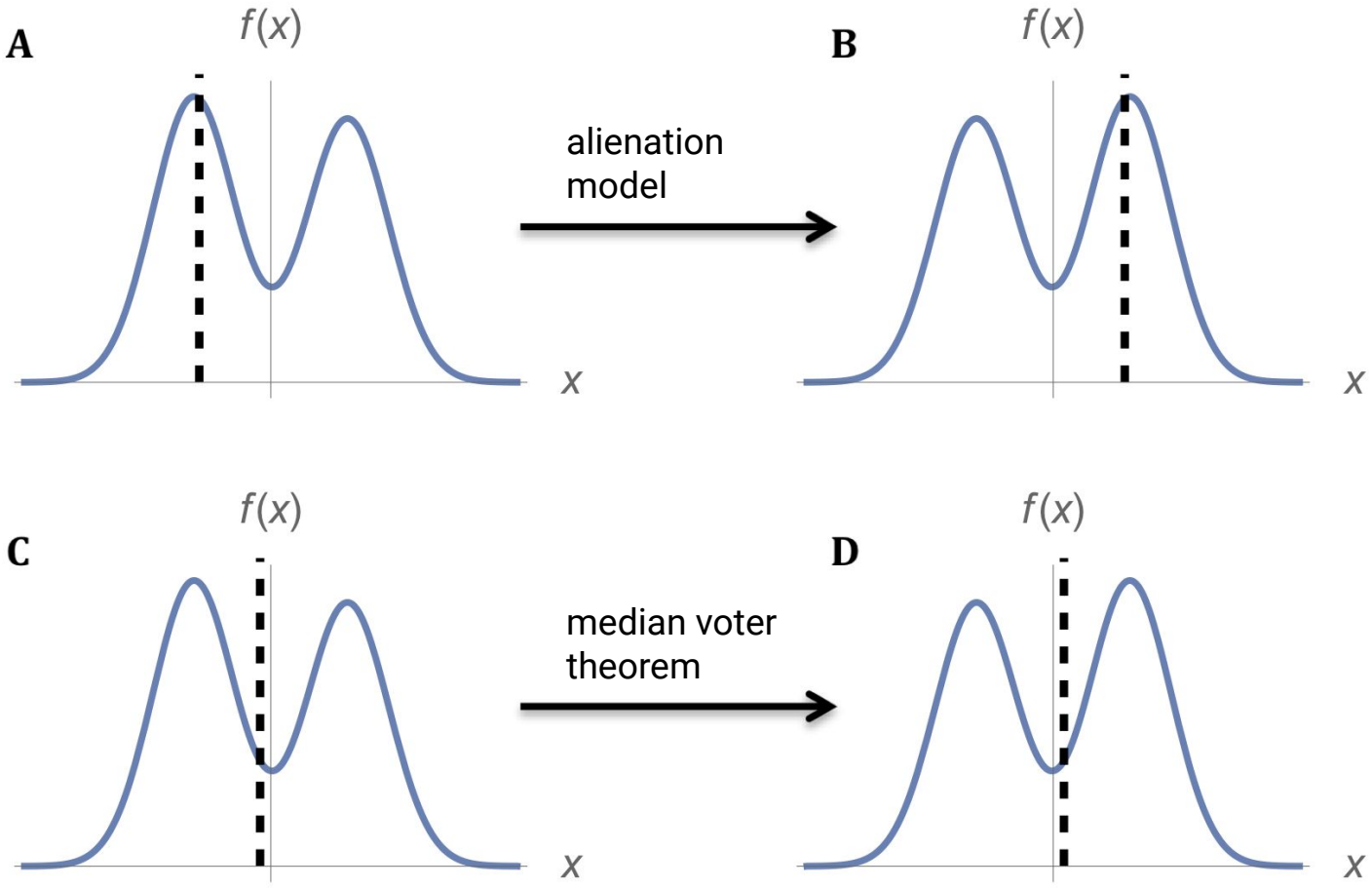
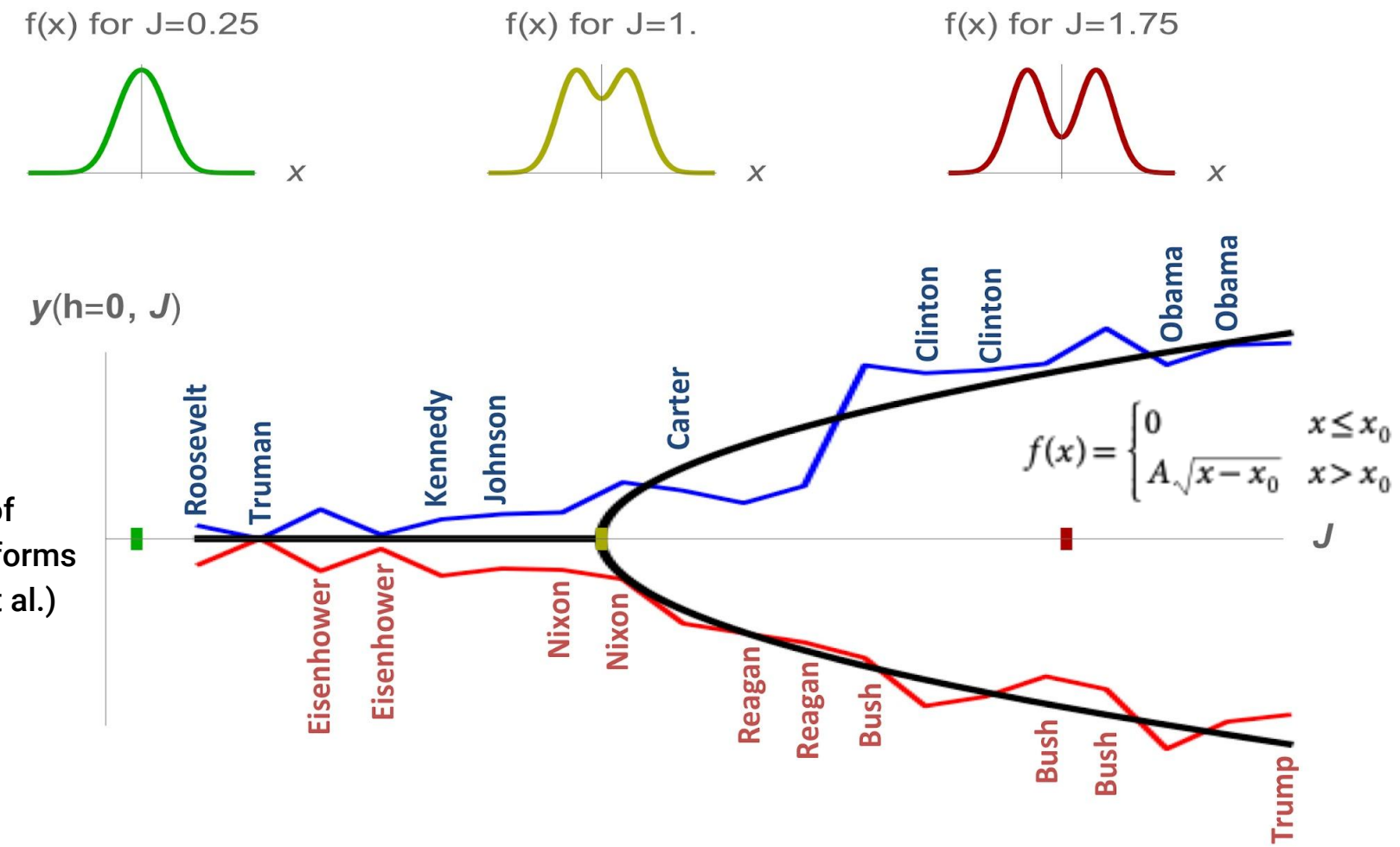


In unstable regime, majority of losing subpopulation is negatively represented.

Median voter theorem vs. alienation



Universality



Overlaid: ideology of party platforms (Jordan et al.)

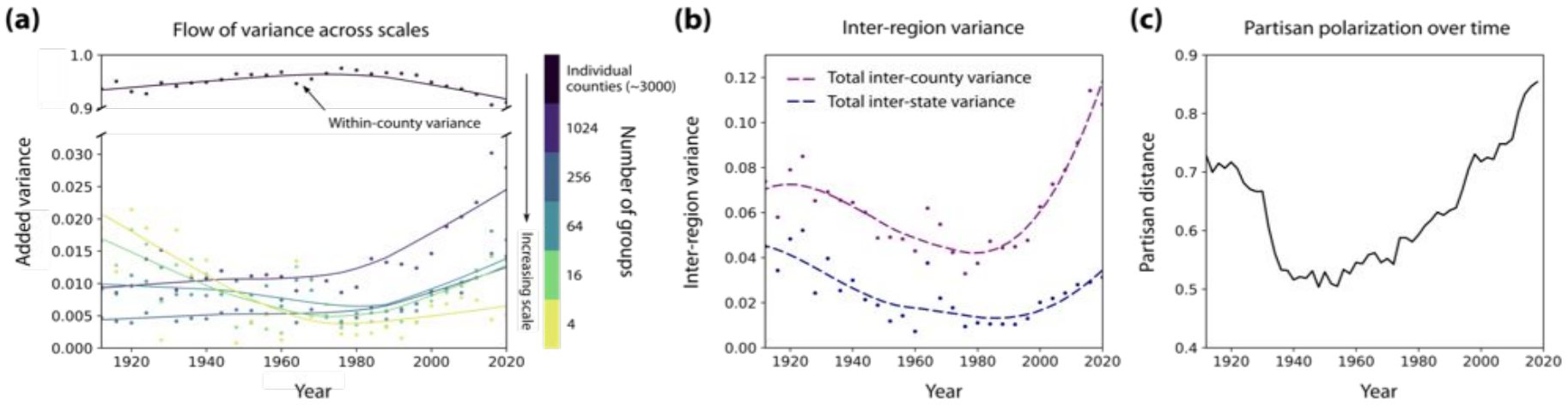
Conclusions

- Two regimes for elections: stable and unstable
  - Unstable regime contains negatively represented opinions

- Causes of instability
  - Polarization
  - Low voter turnout
  - Party primary system

$$\mathrm{Var}(z) = \mathbb{E}\left(\mathrm{Var}\left(z \mid W_1\right)\right) + \mathbb{E}\left(\mathrm{Var}\left(\mathbb{E}\left(z \mid W_1\right) \mid W_2\right)\right) + \ldots + \mathbb{E}\left(\mathrm{Var}\left(\mathbb{E}\left(z \mid W_{N-1}\right) \mid W_N\right)\right) + \mathrm{Var}\left(\mathbb{E}\left(z \mid W_N\right)\right).$$

Epilogue: federal democracies



- Three pillars (institutions, policy environment, electorate)
- Social ties
- Election axes and salience

Sihao Huang, Alexander F. Siegenfeld, and Andrew Gelman. How democracies polarize: A multilevel perspective. *arxiv:2211.01249*

2016 presidential two-party vote shares by precinct