A social power game for the concatenated opinion dynamics with stubborn agents

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Social opinion dynamics

- Individuals' opinions are influenced by their neighbors over social networks, and evolve following some cognitive patterns.

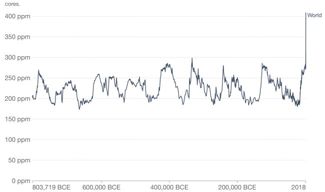
Opinion dynamics: to investigate opinion evolution by system theory

$$\left.\begin{array}{c}\text{opinions - scalars, vectors...}\\ \text{social networks - matrices}\\ \text{cognitive pattern - dynamics}\end{array}\right\}\Longrightarrow\begin{array}{l}\text{collective behaviors:}\\ \text{consensus, polarization,}\\ \text{oscillation...}\end{array}$$

Contents

UNFCCC

Atmospheric CO_2 in last 800K years



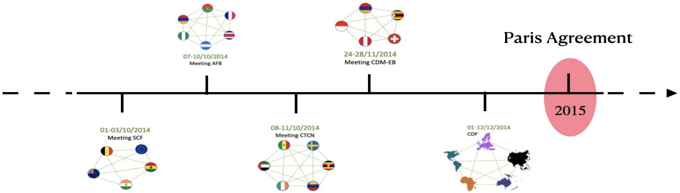
Source: EPICA Dome C CO_2 record (2015) & NOAA (2018) OurWorldInData.org/co2-and-other-greenhouse-gas-emissions • CC BY

UNFCCC

United Nations
Framework Convention on
Climate Change

- **UNFCCC**: an international environmental framework to combat ``dangerous human interference with the climate system''
- Parties in the UNFCCC: 195 countries + EU
- ``Supreme'' governing body: Conference of the parties (COP)

Negotiation process of the UNFCCC



- COP meets annually and decides on climate actions
- Many constituted bodies help the COP
- COP is plenary
- Constituted bodies have restricted participation (not plenary)
- Each constituted body meets once/twice a year

What is the Paris Agreement?

- **Comprehensive accord** for coordinating the international effort to keep the effects of global warming to below 2 ℃ relative to the pre-industrial level
- **Many aspects**: carbon emission mitigation, adaptation to the effects of climate change, climate finance, green technology transfer, climate agreement implementation, legal and procedural matters linked to climate agreements, etc.
- **Agreement**: all parties (195 countries + EU) agree on common measures $\Longrightarrow$ consensus is needed
- **Issues at stake**:
  - Future of our planet
  - Many trillions of US $\$$...

  $\Longrightarrow$ long (15 years), complex negotiation process

Mathematical model for the Paris Agreement

Task

Develop a dynamical opinion model that describes the process of ``achieving an agreement´´ like the Paris agreement

- **Ingredients**:
  1. Agents: 196 parties
  2. State variables: opinions on the agreement
  3. Interaction graph: time-varying
- **Dynamics**
  1. agents are stubborn (defend their opinions)
  2. negotiation leads to compromise
     $\Longrightarrow$ at each meeting final opinions must be closer than initial opinions
     $\Longrightarrow$ at each meeting: convergence inside the convex hull of the initial conditions
  3. over the long time horizon consensus must be achieved

Mathematical model for the Paris Agreement

Task

Develop a dynamical opinion model that describes the process of ``achieving an agreement´´ like the Paris agreement

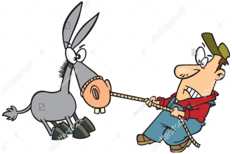
- Candidate model for each meeting: Friedkin-Johnsen (FJ) model
- Model for multiple meetings in a sequence:
  $\Longrightarrow$ concatenated FJ model

Contents

The Friedkin-Johnsen (FJ) model

- Motivation: people's stubbornness will influence their opinions
- FJ model:

$$\mathbf{y}(t+1) = (I - \Theta)W\mathbf{y}(t) + \Theta\mathbf{y}(0)$$

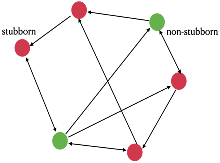


- Opinions: $\mathbf{y}(t) \in \mathbb{R}^m$; weight matrix: W
- Stubbornness (``memory'' of initial opinions):

$$\Theta = \mathrm{diag}\{\theta_1, \ldots, \theta_m\}, \theta_i \in [0, 1)$$

- Possible agents:

$$\begin{cases} \theta_i > 0 & \text{stubborn ``●''} \\ \theta_i = 0 & \text{non-stubborn ``●''} \end{cases}$$

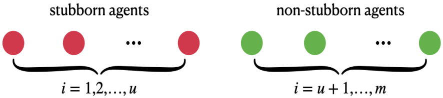


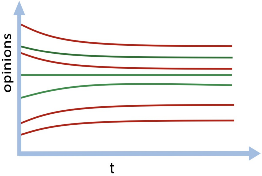
Aymptotic behavior for a single FJ model

- Solution:

$$\mathbf{y}(\infty) = \lim_{t \to +\infty} \mathbf{y}(t) = \underbrace{(I - (I - \Theta)W)^{-1}\Theta}_{V} \mathbf{y}(0)$$

- V is a stochastic matrix





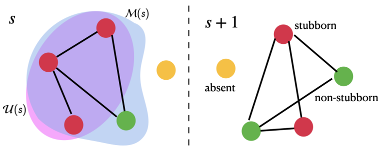
- If $\theta_i > 0, i = 1, \dots, u$, and $\theta_i = 0, i = u + 1, \dots, m$,

$$V = [\, \underbrace{R}_{u} \mid \underbrace{0}_{m-u} \,], \quad R \in \mathbb{R}^{m \times u}_{>0}$$

Concatenated FJ model

- Agent set $\mathcal{V} = \{1, \ldots, n\}$
- Opinion states $\mathbf{y}(s, t) \in \mathbb{R}^n$ (two time scales)
- Partial participation
  - ● stubborn participants $\mathcal{U}(s)$
  - ● non-stubborn participants
  - ● absent agents

$\left.\begin{matrix} \text{stubborn participants } \mathcal{U}(s) \\ \text{non-stubborn participants} \end{matrix}\right\} \mathcal{M}(s)$



- For a single discussion s, a FJ model is applied to $\mathcal{M}(s)$

$$\mathbf{y}(s, t+1)|_{\mathcal{M}(s)} = \text{FJ}(\mathbf{y}(s, t)|_{\mathcal{M}(s)})$$

- Opinions are concatenated:

$$\mathbf{y}(s, \infty) = \mathbf{y}(s+1, 0)$$

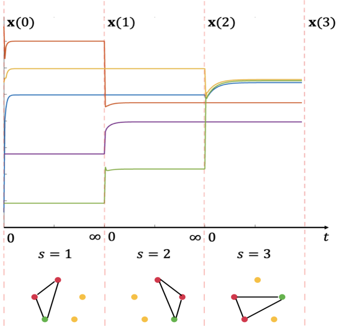
Concatenated FJ model (compact form)

- Let $\mathbf{x}(s) = \mathbf{y}(s, \infty)$
- Update rule: $\mathbf{x}(s) = P(s)\mathbf{x}(s-1)$

$$P(s) = \Pi(s)^\top \left[\begin{array}{c|c|c} R(s) & 0 & 0 \\ \hline 0 & 0 & I_{n-m(s)} \end{array}\right] \Pi(s)$$

- $P(s)$ is stochastic
- $R(s) \in \mathbb{R}^{m(s)\times u(s)}$ is positive
- **Concatenated FJ model**:

$$\mathbf{x}(s) = P(s)P(s-1)\dots P(1)\mathbf{x}(0)$$



Convergence of the CFJ model

- Consensus: $\lim_{s\to\infty} \mathbf{x}(s) = c\mathbf{1} \Leftrightarrow \lim_{s\to\infty} P(s)\ldots P(1) = \mathbf{1}\mathbf{c}^\top$

Consensus condition (existing result)

Given stochastic matrices $Q(s), s \geq 1$

1. $\exists \epsilon > 0$ s.t. $[Q(s)]_{ij} > \epsilon$ if $[Q(s)]_{ij} > 0, \forall i, j, s$
2. $\exists s_1 < s_2 < \ldots$ s.t. $Q(s_k)$ has a positive column

$$\Longrightarrow \qquad \lim_{s\to\infty} Q(s)Q(s-1)\ldots Q(1) = \mathbf{1}\mathbf{c}^\top$$

- By exploiting the existing result, conditions for the CFJ model to achieve consensus can be given[1]

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[1] L. Wang, et., al. IEEE Trans. on Automatic Control (2022)

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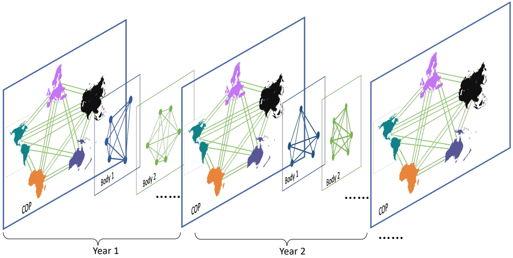
Back to the UNFCCC



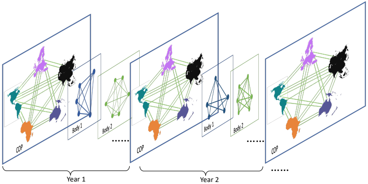
Body	meetings
COP	15
AC	8
AFB	26
CTCN	6
CC-E	27
CC-F	17
CGE	24
CDM EB	86
JISC	37
LEG	28
SCF	11
TEC	10
total	295

Data collected for 295 meetings (2001-2015)

1. Meeting participants $\Longrightarrow \mathcal{M}(s)$
2. Speakers ($\Longleftrightarrow$ stubborn agents) $\Longrightarrow \mathcal{U}(s)$
3. N. of speeches ($\Longleftrightarrow$ stubbornness level) $\Longrightarrow \theta_i(s)$

From the CFJ model to the Paris Agreement

- Each year of UNFCCC:
  1. COP (plenary)
  2. many meetings of 11 constituted bodies



- Split the overall 2001— 2015 product of stochastic matrices into yearly intervals with yearly matrices $Q(k)$

$$Q(k) = \underbrace{P^{\mathrm{COP}}(k)}_{\text{COP}} \underbrace{P^{11}(k)P^{10}(k)\dots P^{1}(k)}_{\text{constituted bodies}}, \quad k = 1, \dots, 15$$

- ``Yearly'' opinion dynamics:

$$x(k) = Q(k)x(k-1), \quad k = 1, \dots, 15$$

- COP is plenary $\Longrightarrow$ $Q(k)$ has positive columns

  $\Longrightarrow$ ``practical convergence'' is predicted

  $\Longrightarrow$ Paris Agreement

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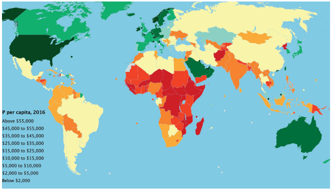
Strategic interactions in the UNFCCC



- The participating parties are rational, with many issues bargaining on table
- In the CFJ model, agents' opinions are only passively evolving

Task

Develop the concatenated FJ model to reflect the rationality of the parties for the UNFCCC

Revisit of the concatenated FJ model

- **Observation 1**: parties can choose to speak or not

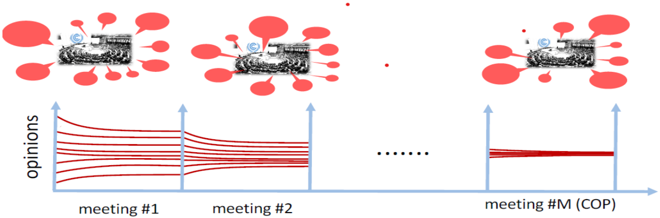
$\Rightarrow$ "speaking" is linked with stubbornness of the model

$\Rightarrow$ stubbornness can be decided as an action!

- **Observation 2**: $\mathbf{x}(s) = P(s)\mathbf{x}(s-1) = \underbrace{P(s)\dots P(1)}_{Q(s)}\mathbf{x}(0)$

$$\Rightarrow \quad \lim_{s\to\infty} Q(s) = \underbrace{\mathbf{1c}^\top}_{\text{rank-1}}, \quad \lim_{s\to\infty} \mathbf{x}(s) = \underbrace{c\mathbf{1}}_{\text{consensus}}$$

$\Rightarrow$ $Q(s)$ encodes the eigenvector centrality of each agent!



Social power for the concatenated FJ model

- (Cumulated) social power = overall influence accumulated by agent i over all agents in the sequence of discussions $1, \dots, M$

$$\mathbf{x}(M) = Q(M)\mathbf{x}(0) = P(M) \dots P(1)\mathbf{x}(0)$$

$$\mathbf{sp}(M)^\top = \frac{1}{n}\mathbf{1}^\top Q(M) = \frac{1}{n}\mathbf{1}^\top \begin{bmatrix} \cdots & Q_{1i}(M) & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & Q_{ni}(M) & \cdots \end{bmatrix}$$

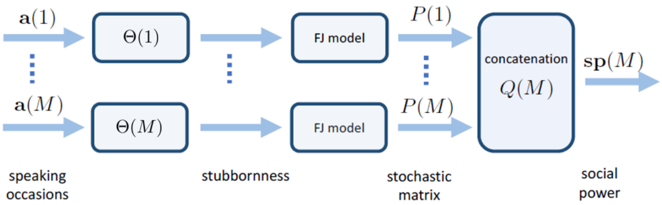
i-th agent

- $\mathbf{sp}(M) \sim$ eigenvector centrality: $\lim_{M \to \infty} \mathbf{sp}(M) = \mathbf{c}$
- $\mathbf{sp}(M)$ = nonlinear function of the stubbornness parameters $\Theta(1), \dots, \Theta(M)$

$$P(s) = (I - (I - \Theta(s))W(s))^{-1}\Theta(s)$$

Maximizing social power

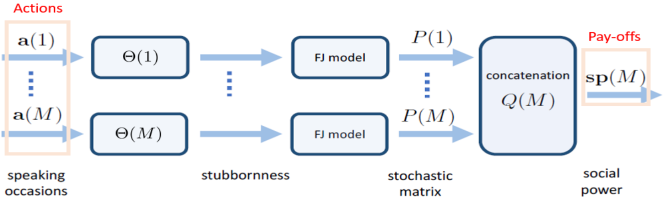
- $\mathbf{sp}(M)$ is determined by the speaking occasions $\mathbf{a}(1), \ldots, \mathbf{a}(M)$ through the concatenated FJ model



Question

How should an agent take speaking opportunities to maximize its social power?

Social power game



Social power game

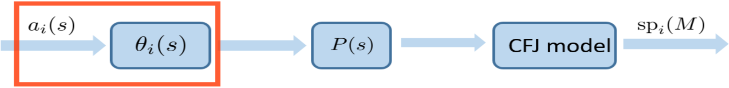
- Players: agents $\mathcal{V} = \{1, \ldots, n\}$
- Actions: allocation of speaking occasions

$$\mathbf{a}_i = (a_i(1), \ldots, a_i(M)) \quad \Leftrightarrow \quad \boldsymbol{\theta}_i = (\theta_i(1), \ldots, \theta_i(M))$$

- Pay-off function: social power

$$u_i(\mathbf{a}_i, \mathbf{a}_{-i}) = \mathrm{sp}_i(M)$$

Social power game: constraints



1. More speaking, more stubborn

$$\theta_i(s) = \theta a_i(s)$$

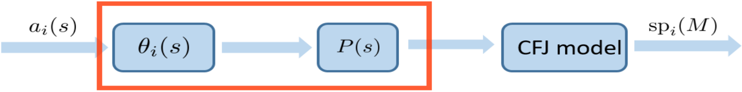
2. Limited budget of overall speaking opportunities: γ, K

$$a_i(s) \leq \gamma, \quad a_i(1) + \cdots + a_i(M) \leq K$$

3. Limited capacity of speaking occasions per meeting: C

$$\sum_{i \in \mathcal{V}} a_i(s) \leq C$$

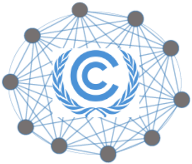
Social power game: network topology



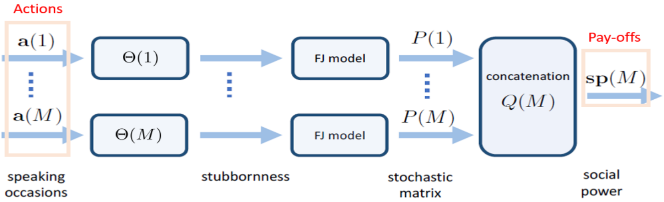
- The network is a complete graph

$$W(s) = W = \frac{1}{n}\mathbf{1}\mathbf{1}^\top, \quad s = 1, \ldots, M$$

- Meaning: meetings are all plenary



Problems of interest



- **P1**: given the actions of two agents, who will obtain a higher social power (social power comparison)?
- **P2**: what is the (generalized) NE of the social power game (Nash equilibrium)?
- **P3**: for a given agent, if the actions of the other agents are fixed, what is the best strategy for her (best strategy)?

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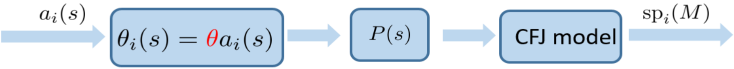
Problem P1: social power comparison



- strategies of agents *i* and *j*

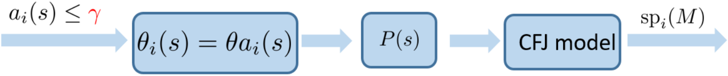
$$\mathbf{a}_i = (a_i(1), \dots, a_i(M)) \quad \mathbf{a}_j = (a_j(1), \dots, a_j(M))$$

Theorem (Comparison of social powers)

For small enough θ,
$$\left.\begin{array}{c} a_i(s) = a_j(s), \ \forall s < s' \\ a_i(s') < a_j(s') \end{array}\right\} \implies \mathbf{sp}_i(M) < \mathbf{sp}_j(M).$$

- Meaning: speaking more at early meetings gives higher social power
- $\implies$ early mover earns more

Problem P1: binary stubbornness



- Assume $\gamma = 1$, i.e., agents can choose to speak or be silent

Theorem (Comparison of social powers)

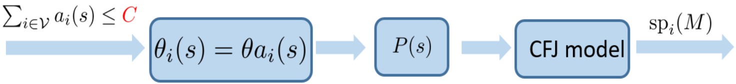
Let $\tau_i = \arg\min_s \{a_i(s) = 0\}$.

$$\tau_i < \tau_j \qquad \Longrightarrow \qquad u_i < u_j$$

- No constraint is made on θ
- Example

$$\begin{aligned} \mathbf{a}_1 &= (1,1,1,0,0,1) \\ \mathbf{a}_2 &= (1,0,1,1,1,0) \end{aligned} \quad \Longrightarrow \quad \text{agent 1 wins!}$$

Problem P2: (generalized) Nash Equilibrium



- Nash equilibrium: $\mathbf{a}_i^* = \arg\max_{\mathbf{a}_i} u_i(\mathbf{a}_i, \mathbf{a}_{-i}^*)$

Theorem (Generalized Nash equilibrium)

For θ small enough, if $\gamma|C$, any $\mathbf{a}^*$ taking the following form is a GNE

- *For* $i = 1, \dots, \frac{C}{\gamma}$: $\quad \mathbf{a}_i^* = (\underbrace{\gamma, \dots, \gamma}_{\lceil\frac{K}{\gamma}\rceil \text{ meetings}}, K - \gamma\lceil\frac{K}{\gamma}\rceil, 0, \dots, 0)$
- *For* $i > \frac{C}{\gamma}$, $\mathbf{a}_i^*$ *can be arbitrarily chosen such that*

$$a_i^*(1) = \dots = a_i^*(\lceil\frac{K}{\gamma}\rceil) = 0, \quad \sum_{j\in\mathcal{V}} a_j^*(\lceil\frac{K}{\gamma}\rceil + 1) = C$$

Problem P2: Nash equilibrium (cont′d)

- Multiple GNEs
- On the equlibrium agents tend to speak more in early meetings
- $\Longrightarrow$ early mover strategies consist the GNE

Theorem (Nash equlibrium: binary stubbornness)

Assume $\gamma = 1$ and $C = |\mathcal{V}|$. For small enough θ, the unique NE is

$$\mathbf{a}_i^* = (\underbrace{1, \ldots, 1}_{K \text{ meetings}}, 0, \ldots, 0)$$

- $\Longrightarrow$ everyone takes the early mover strategy!

Problem 3: best strategy

- Early mover strategy

$$\tilde{\mathbf{a}}_i = (\underbrace{\gamma, \ldots, \gamma}_{\lceil \frac{K}{\gamma} \rceil \text{ meetings}}, K - \gamma \lceil \frac{K}{\gamma} \rceil, 0, \ldots, 0)$$

Theorem (Best strategy)

For θ small enough, it holds

$$\tilde{\mathbf{a}}_i = \arg\max_{\mathbf{a}_i} u_i(\mathbf{a}_i, \mathbf{a}_{-i}), \quad \forall \mathbf{a}_{-i}.$$

- Meaning: the early mover strategy is a dominant strategy
- $\Longrightarrow$ early mover advantage

Problem 3: best strategy (cont'd)

- Early mover strategy

$$\tilde{\mathbf{a}}_i = (\underbrace{\gamma, \ldots, \gamma}_{\lceil \frac{K}{\gamma} \rceil \text{ meetings}}, K - \gamma \lceil \frac{K}{\gamma} \rceil, 0, \ldots, 0)$$

- The early mover strategy might not be optimal for larger θ

Example. $\gamma = 1, K = 6$ and $\theta = 0.6$

$$\begin{aligned} \mathbf{a}_1' &= (1, 1, 1, 1, 1, 0, 1, 0, 0, 0) \\ \mathbf{a}_2 &= (0, 1, 1, 1, 0, 1, 0, 1, 1, 0) \\ \mathbf{a}_3 &= (1, 1, 1, 1, 0, 1, 0, 0, 1, 0) \\ \mathbf{a}_4 &= (1, 1, 0, 1, 1, 1, 1, 0, 0, 0) \end{aligned} \implies u_1(\tilde{\mathbf{a}}_1, \mathbf{a}_{-1}) < u_1(\mathbf{a}_1', \mathbf{a}_{-1})$$

Early mover advantage for general stubbornness

- Early mover strategy

$$\tilde{\mathbf{a}}_i = (\underbrace{\gamma, \ldots, \gamma}_{\lceil \frac{K}{\gamma} \rceil \text{ meetings}}, K - \gamma \lceil \frac{K}{\gamma} \rceil, 0, \ldots, 0)$$

Theorem (General stubbornness)

For any $\mathbf{a}_{-i}$ *it must be*

$$u_i(\tilde{\mathbf{a}}_i, \mathbf{a}_{-i}) \geq \max_{\mathbf{a}_i \in \mathcal{A}_i(\mathbf{a}_{-i})} u_i(\mathbf{a}_i, \mathbf{a}_{-i}) - 2(1 - \frac{1}{n}) \sum_{s=\lfloor \frac{K}{\gamma} \rfloor}^{M-1} (\underbrace{\gamma\theta}_{\text{less than 1}})^s.$$

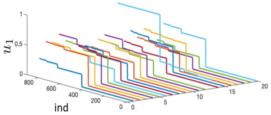
- Meaning: the early mover strategy is at least suboptimal
- $\Longrightarrow$ early mover advantage holds for general stubbornness

Beyond complete graph: simulation results

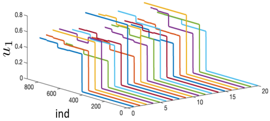
Graphs

Parameters: $M = 10, K = 6, C = 24, \theta = 0.05$

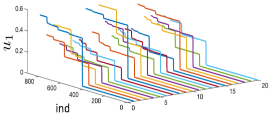
Social power of agent 1 w.r.t $\mathbf{a}_1$: ind $=$ lexicographical order



cycle



star (center)



star (leaf)

- Social power roughly increases along the lexicographical order
- $\Longrightarrow$ early mover advantage still holds!

Why early mover advantage?

- Concatenated FJ model has contracting dynamics
- Closer to consensus, harder to impact the final outcome
- $\Longrightarrow$ early discussions are more important
- $\Longrightarrow$ diminishing return law

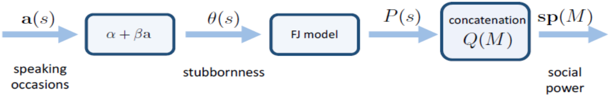
Theorem (Diminishing returns)

Let $\boldsymbol{\Theta} = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_n)$ *be the strategy profile. It holds for* $\forall i$

$$\max_{\boldsymbol{\Theta}} \{\mathrm{sp}_i(s_1+1) - \mathrm{sp}_i(s_1)\} = (1 - \frac{1}{n})\Pi_{s=1}^{s_1} \max_{j \in \mathrm{V}} \theta_j(s)$$

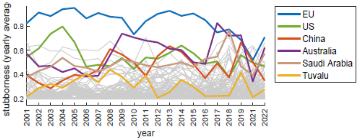
- The diminishing return law does not depend on how $\mathbf{a}_i$ is associated with $\boldsymbol{\theta}_i$

Back to UNFCCC: social power

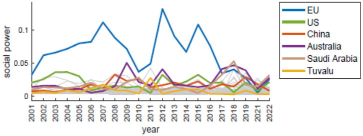


$\mathbf{a}(s)$ speaking occasions → $\alpha + \beta\mathbf{a}$ → $\theta(s)$ stubbornness → FJ model → $P(s)$ → concatenation $Q(M)$ → $\mathbf{sp}(M)$ social power

Stubbornness
(average over the meetings of each year)

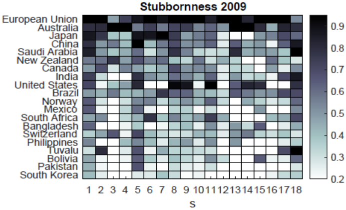


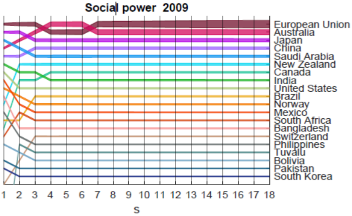
Social power

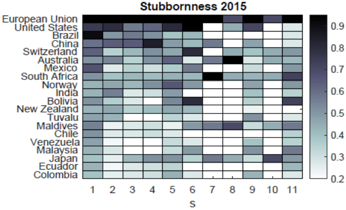


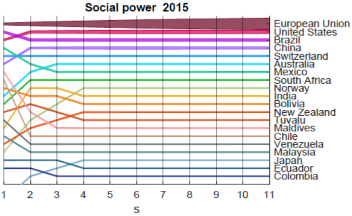
- The EU has the highest social power for most of the years
- Is the EU using an early mover strategy?

UNFCCC Negotiations: a few years


Stubbornness 2009


Social power 2009

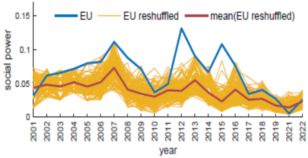

Stubbornness 2015


Social power 2015

UNFCCC Negotiations: early mover strategy

Is EU taking early mover advantage?

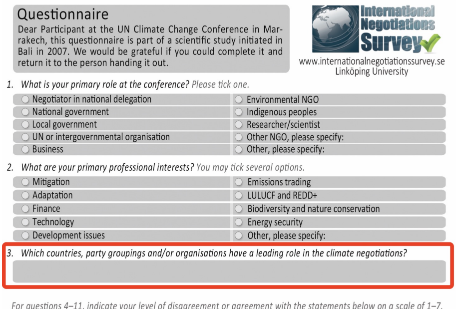
- null model: reshuffle order in the action $\mathbf{a}_{\mathrm{EU}} \to \mathrm{perm}(\mathbf{a}_{\mathrm{EU}})$ recompute the social powers



$\mathrm{mean}(\mathbf{sp}_{\mathrm{EU,\ reshuffled}}) < \mathbf{sp}_{\mathrm{EU}} \implies$ the EU is taking an early mover advantage!

Validation: UNFCCC leadership

- To assess leadership in climate negotiations: use survey data from International Negotiations Survey
- $\Longrightarrow$ perceived leadership
- data collected in years 2008-2022
- total of 5530 responses



Questionnaire

Dear Participant at the UN Climate Change Conference in Marrakech, this questionnaire is part of a scientific study initiated in Bali in 2007. We would be grateful if you could complete it and return it to the person handing it out.

International Negotiations Survey
www.internationalnegotiationssurvey.se
Linköping University

1. *What is your primary role at the conference?* Please tick one.

○ Negotiator in national delegation
○ National government
○ Local government
○ UN or intergovernmental organisation
○ Business
○ Environmental NGO
○ Indigenous peoples
○ Researcher/scientist
○ Other NGO, please specify:
○ Other, please specify:

2. *What are your primary professional interests?* You may tick several options.

○ Mitigation
○ Adaptation
○ Finance
○ Technology
○ Development issues
○ Emissions trading
○ LULUCF and REDD+
○ Biodiversity and nature conservation
○ Energy security
○ Other, please specify:

3. *Which countries, party groupings and/or organisations have a leading role in the climate negotiations?*

For questions 4–11, indicate your level of disagreement or agreement with the statements below on a scale of 1–7.

Validation: UNFCCC leadership

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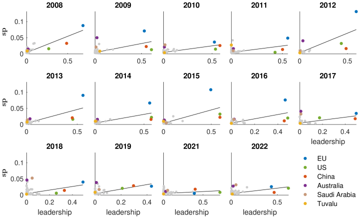

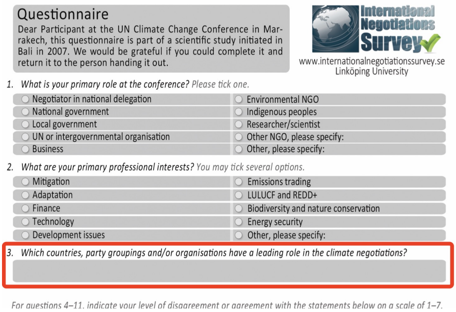

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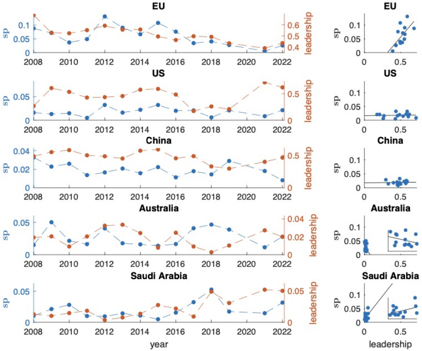
$\Rightarrow$ mean(corr(leadership, sp))=0.6

Validation: UNFCCC leadership

- Temporal trend for the EU is captured very well
- Less precise for other countries like China and US



- Summary: the model-based social powers seem rather close to the perceived leadership!

Summary

- Concatenated FJ model
  - a two time scale model representing consecutive FJ discussion events
  - opinions are contracting for each discussion
- Social power game
  - strategic game for the concatenated FJ model
  - allocate speaking opportunities to maximize social power
- Results
  - Early mover advantage: speaking more in early discussions makes an advantage
  - Diminishing return law: later discussions have lower influence on the social power
- Application: UNFCCC, Paris Agreement

References

(1) L. Wang, C. Bernardo, Y. Hong, F. Vasca, G. Shi, C. Altafini, ``Achieving consensus in spite of stubbornness: time-varying concatenated Friedkin-Johnsen models'', CDC, 2021.

(2) C. Bernardo, L. Wang, F. Vasca, Y. Hong, G. Shi, C. Altafini, ``Achieving consensus in multilateral international negotiations: The case study of the 2015 Paris Agreement on climate change'', Science Advances, 2021.

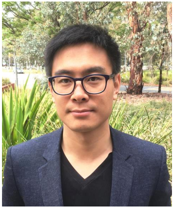
(3) L. Wang, C. Bernardo, Y. Hong, F. Vasca, G. Shi, C. Altafini, ``Consensus in concatenated opinion dynamics with stubborn agents'', TAC, 2022.

(4) L. Wang, G. Chen, Y. Hong, G. Shi, C. Altafini, ``A social power game for the concatenated Friedkin-Johnsen model'', CDC, 2022.

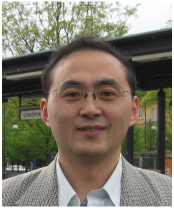
Collaborators

Claudio Altafini

Carmela Bernado



Guodong Shi



Yiguang Hong

Thank You!