How bacteria generate beneficial phenotypic heterogeneity through mixed positive/negative feedback loops



# Biological systems are noisy



Noise is caused by e.g. randomness in (diffusion based) reactions.

Cellular noise is phenotypically important



**Bacterial Chemotaxis** 



Several decisions are based on noise-induced randomness.

Bulk measurements obscure single-cell dynamics



Single-cell measurements may reveal population heterogeneity.

Bacteria activate  $\sigma$ -factors in response to stress



These then activate the stress response genes. We will use them as a system to study cellular noise.

## Bacteria often have several different $\sigma$ -factors

A stress is sensed - A  $\sigma\text{-}factor$  is activated - Stress response genes are activated



Each activates in response to a specific stress.

Most  $\sigma$ -factors circuits have a similar structure



The  $\sigma$ -factor activates both its own production, and that of an anti- $\sigma$ -factor. This creates a mixed positive/negative feedback loop.

We measure  $\sigma$ -factors activity through fluorescent markers



Cellular fluorescence corresponds to  $\sigma$ -factors activity.

We measure  $\sigma$ -factors activity using single-cell fluorescent microscopy

σ (Au) 80 Time (Hours)

Using image analysis, time trajectories are created.

 $\sigma\text{-factor B}~(\sigma^B)$  responds to environmental stress



It displays a stochastic pulsing behaviour (in the presence e.g. ethanol).

Park et al. (2018) Cell Systems

This creates population heterogeneity, allowing adaption to an uncertain future



If stress wipes out the non-expressers, the expressers can repopulate the colony.

 $\sigma\text{-factor}~V~(\sigma^V)$  regulates the lysozyme stress response



It activates genes for cell wall protection and repair.

 $\sigma^{V}$  responds through a heterogeneous activation behaviour



The time to activation is heterogeneous across an isogenic population.

We can model the  $\sigma^V$  circuit



Our *chemical reaction network* model is based on the circuit's reaction events.

The model recreates the heterogeneous response



Stochastic reaction network interpretation (Gillespie's algorithm) is used to implement noise.

Our model predicts a memory of previous stresses



The reactivation (after a stress holiday) is homogenous, not heterogeneous.

We validate the prediction experimentally



Each validation step increase our confidence in the model.

Both the  $\sigma^B$  and  $\sigma^V$  circuits' contain a mixed positive/negative feedback loop



Can this motif reproduce the two distinct response behaviours?

We model a general  $\sigma$ -factors circuit



The negative feedback is subject to a *time delay*.

System noise is accounted for by making simulations *stochastic*.

The model depends on only three parameters:

- *S*: The strength of the self-activation loop.
- *D*: The strength of the self-deactivation loop.
- *τ*: The length of the self-deactivation delay.



The system's behaviour is determined by these three properties.

(The model is a one-variable stochastic delay differential equation)

For every parameter set, we get a specific response behaviour



Here we can recreate the behaviours of both the  $\sigma^B$  and  $\sigma^V$  systems.

## We have found all the possible response behaviours of the system



(An automated algorithm is used to classify parameter sets)





( $\tau$  = length of self-deactivation delay)



(*D* = strength of self-deactivation,  $\tau$  = length of self-deactivation delay)

















Real systems can be located on our map



This makes predictions on their system-properties.

(Here,  $\sigma^{B}$  should have a higher *D* = strength of self-deactivation than  $\sigma^{V}$ )

The model predicts a behavioural transition as the parameter *S* is varied



This transition should be observable in the  $\sigma^{B}$  system (which contains stochastic pulsing).

We can modulate this parameter in the real circuit



This confirms that  $\sigma^{B}$  undergoes the predicted transition.

An increase in IPTG corresponds to an increase in *S*. For each level of IPTG, three repeats are shown.

We can classify model and experiment behaviours as **S** is varied



A similar transition occurs in both systems.

# Goal

- Build system as a tunable synthetic regulator.
- Can be used in synthetic organisms.

## Summary

- Biological systems are noisy.
- Cellular noise can generate population heterogeneity.
- Single-cell measurements and models are required to detect this.

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Our model is a two-variable Stochastic Differential Equation

$$\begin{cases} \frac{d\sigma}{dt} = \mathbf{v_0} + \frac{(S \cdot \sigma)^n}{(S \cdot \sigma)^n + (\mathbf{D} \cdot A)^n + 1} - \sigma & + \eta \cdot noise_1(\bar{x}, \bar{p}) \\ \frac{dA}{dt} = \frac{1}{\tau} (\sigma - A) & + \eta \cdot noise_2(\bar{x}, \bar{p}) \end{cases}$$

It depends on only 6 parameters:

- *S*: The degree of system *self-activation*
- *D*: The degree of system *self-deactivation n*: The degree of *system cooperativity*
- *τ*: The length of the *time delay*

- $v_0$ : The *base production* of the  $\sigma$ -factor
- $\eta$ : The noise amplitude

(The final terms are functions determining the degree of noise) (The variable *A* models the time delay)

By simulating the model we can observe its behaviour

Here it exhibits a *Stochastic Pulsing* behaviour.

(A single simulation displayed in *phase space* and over time) (Nullclines are drawn in red and blue)