Scalable Adaptive Control

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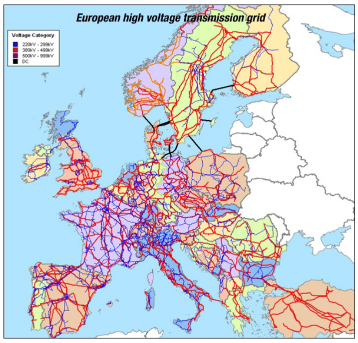
ELLIIT focus workshop, Linköping 2022:
Hybrid AI – Where data-driven and model-based methods meet

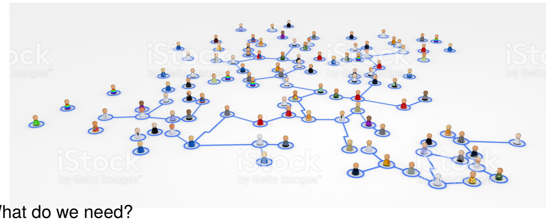
The Challenge of Modern Control: Scalability

[Source: geospatial.blogs.com]



Control challenges:
More producers. Variable capacity. Limited storage. Flexible components.

Towards a Scalable Theory of Control



What do we need?

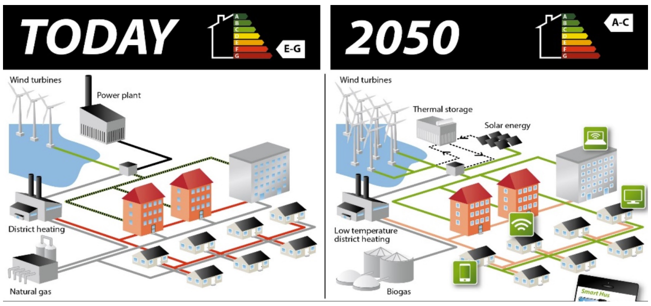
- Scalable Synthesis
- Scalable Verification
- Scalable Modeling
- Scalable Objectives

Outline

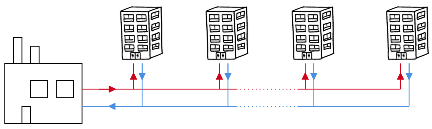
- Introduction
- **Scalable optimal control**
- Minimax adaptive control
- Scalable adaptive control

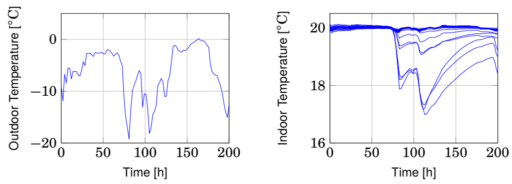
Thermal Networks in a Renewable Energy System

From [Mathiesen, Drysdale, Lund, Paardekooper, Ridjan, Connolly, Thellufsen, Jensen, Aalborg University 2016]:



Fairness Problem in District Heating





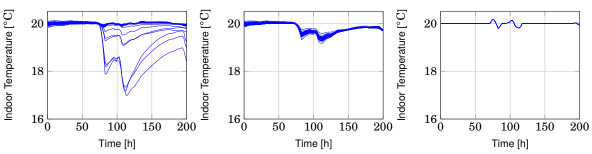
(a) Outdoor Temperature (b) Indoor Temperatures

[Problem mentioned in recent annual report by Stockholm Exergi]

Load Coordination

minimize$_\delta$	$\max_i \lvert\gamma_i \delta_i\rvert$	Min. max deviation
subject to	$\delta = \widehat{\mathbf{q}} - \mathbf{q}$	Actual flows $\mathbf{q}$ differ from demands $\widehat{\mathbf{q}}$
	$\mathbf{q} \in \mathcal{Q}$	Actual flows should be feasible

Use building models to choose γ_i for fair temperature deviations



(a) Traditional (b) Load Coordination (c) Optimal

[Agner/Kergus/Pates/Rantzer, Smart Energy, March 2022]

Wind Farms Need Control

Picture from http://www.hochtief.com/hochtief_en/9164.jhtml

Most wind farms today are paid to maximize power production. Future farms will have to curtial power at contracted levels.

New control objective:
Minimize fatigue loads subject to fixed total production.

Minimizing Fatigue Loads

Single turbine control:
Minimize tower pressure variance subject to linearized dynamics with measurements of pitch angle and rotor speed.

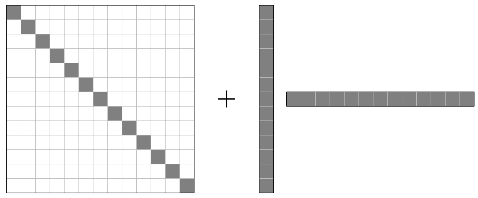
Optimal controller: $u_i^{\text{loc}}(t)$

Wind farm control:
Minimize sum of all tower pressure variances subject to *fixed total production of the farm:* $\sum_{i=1}^m u_i = 0$

Optimal controller: $u_i(t) = u_i^{\text{loc}}(t) - \frac{1}{m}\sum_{j=1}^m u_j^{\text{loc}}(t)$.

[D Madjidian, L Mirkin, A Rantzer, IEEE Trans. on Automatic Control 2016]

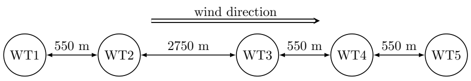
Controller Structure

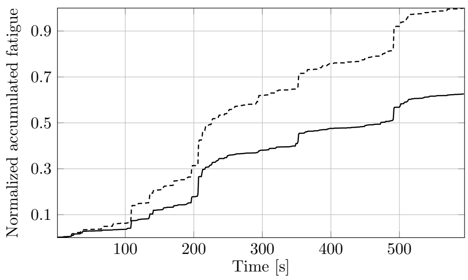


Linear quadratic control of m identical systems and a constraint $\sum_{i=1}^m u_i = 0$ gives an optimal feedback matrix with two parts:

- One is localized (diagonal).
- The other has rank one (control of the average state).

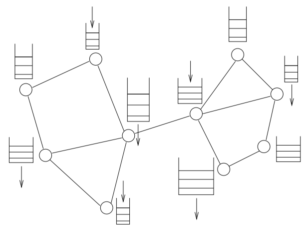
Simulations with Real Wind Data





[Data from Vestas wind farm collected within EU project AEOLUS]

Dynamic Buffer Networks



- Producers, consumers and storages
- Examples: water, power, traffic, data
- Discrete/continuous, stochastic/deterministic
- Multiple commodities, human interaction

H_∞ Optimal Static Control on Networks

Problem:
Given a graph $(\mathcal{V}, \mathcal{E})$ *and*

$$\dot{x}_i = a_i x_i + \sum_{(i,j)\in\mathcal{E}} (u_{ij} - u_{ji}) + w_i \qquad i \in \mathcal{V}$$

find control law $u = Kx$ *that minimizes the* H_∞ *norm of the map from* w *to* (x, u).

Solution:
An optimal control law when $a_i < 0$ is given by

$$u_{ij} = x_i/a_i - x_j/a_j \qquad (i,j) \in \mathcal{E}.$$

[Lidström/Rantzer, ACC2016]

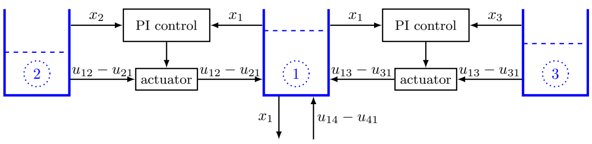
Structure Preserving Static Feedback

Problem
Consider the system $\dot{x} = Ax + Bu + w$ with A symmetric and Hurwitz. Find a state feedback controller $u = Kx$ that minimizes the H_∞ norm of the map from w to (x, u) in the closed loop system $\dot{x} = (A + BK)x + w$.

Theorem
A solution is given by $u = K_* x$ where $K_* = B^T A^{-1}$. The minimal value of the norm is $\sqrt{\|(A^2 + BB^T)^{-1}\|}$.

Proof idea
$K_* = B^T A^{-1}$ minimizes the static gain. Other frequencies are better off.

Optimal Network Control with Edge Integrators



Given a graph $(\mathcal{V}, \mathcal{E})$, let $P(s)$ be the transfer matrix from u to x given by $\dot{x}_i = a_i x_i + \sum_{(i,j)\in\mathcal{E}}(u_{ij} - u_{ji})$, $i \in \mathcal{V}$ with $a_i < 0$. Then $\widehat{K}(s)$ is a separate PI controller for each graph edge:

$$\begin{cases} \dot{z}_{ij} = k(x_i/a_i - x_j/a_j) \\ u_{ij} = z_{ij} - x_i/a_i^2 + x_j/a_j^2 \end{cases}$$

(Works if the graph is a tree!)

Outline

- Introduction
- Scalable optimal control
- Minimax adaptive control
  - **Problem formulation**
  - Solution using Dynamic Programming
- Scalable adaptive control

Status of Learning Based Control of Linear Systems

Adaptive control literature since the 1950s

- Searching Google Scholar for “Adaptive Control” gives 4 890 000 hits
- Theory focus on stability and optimal asymptotic performance
- Valuable counterexamples

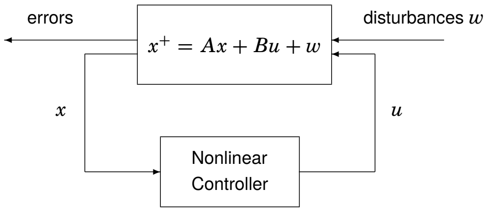
Recent applications of learning theory for control

- Focus is shifted to transient performance and regret bounds
- Usually assumes stabilizing linear controller known
- No robustness bounds for unmodelled dynamics

Minimax adaptive control

- Early work by [Didinsky/Basar 1994, Vinnicombe, Megretski 2004]
- Scalar case [Rantzer, IFAC 2020]
- Multivariable case [Rantzer, L4DC 2021]

Problem Formulation



Find a controller that simultaneously keeps the l_2-gain from disturbances to errors below a given bound for all $(A, B) \in \{(A_1, B_1), \dots, (A_N, B_N)\}$.

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Min-max Dynamic Games

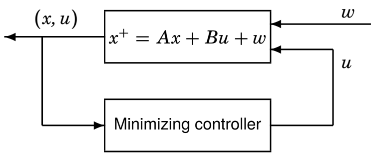
Given a cost function, find a control law μ that attains the infimum

$$\inf_{\mu} \sup_{w} \left(\sum_{t=0}^{\infty} g(x_t, u_t, w_t) \right)$$

when x and u are generated from w according to the dynamical system $x_{t+1} = f(x_t, u_t, w_t)$ and $u_t = \mu(x_t)$.

Isaacs Bellman Pontryagin Zachrisson

Game Formulation of H_∞ Control



Find a control law μ that attains the minimum

$$\min_{\mu} \max_{w} \sum_{t=0}^{\infty} \left(|x_t|_Q^2 + |u_t|_R^2 - \gamma^2 |w_t|^2 \right)$$

(with notation $|x|_Q^2 = x^\top Q x$), when x_t, u_t are generated according to

$$\begin{cases} x_{t+1} = Ax_t + Bu_t + w_t \\ u_t = \mu_t(x_0, \dots, x_t, u_0, \dots, u_{t-1}). \end{cases}$$

Optimal control $u = -Kx$ defined by the minimizing u in the Riccati equation

$$|x|_P^2 = \min_u \max_w \left\{ |x|_Q^2 + |u|_R^2 - \gamma^2 |w|^2 + |Ax + Bu + w|_P^2 \right\}$$

Minimax Adaptive Control

Suppose $Q, R \succ 0$ and let $\mathcal{M}$ be a set of possible (A, B)-pairs. Given a number $\gamma > 0$, find a control law μ that attains the infimum

$$\inf_{\mu} \sup_{w} \max_{(A,B) \in \mathcal{M}} \sum_{t=0}^{\infty} \left(|x_t|_Q^2 + |u_t|_R^2 - \gamma^2 |w_t|^2 \right)$$

when supremum is taken over all solutions to

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t + w_t \qquad\qquad t \geq 0 \\ u_t &= \mu_t(x_0, \dots, x_t, u_0, \dots, u_{t-1}). \end{aligned}$$

- γ quantifies robustness to unmodeled dynamics.
- In general, nonlinear feedback with memory is needed.
- Early work by [Didinsky/Basar, CDC 1994]

Theorem 1: Equivalent Dynamic Game

Optimal controller has the form

$$u_t = \eta(x_t, Z_t)$$

where

$$Z_T = \sum_{t=0}^{T-1} \begin{bmatrix} -x_{t+1} \\ x_t \\ u_t \end{bmatrix} \begin{bmatrix} -x_{t+1} \\ x_t \\ u_t \end{bmatrix}^\top .$$

(Compare to statistical sample covariance)

[Rantzer, L4DC 2021]

Theorem 2: Gain Bound from Riccati Inequalities

Given $Q \succ 0$, $R \succ 0$ and $\mathcal{M} := \{(A_1, B_1), \dots, (A_N, B_N))\}$, suppose there exist $K_1, \dots, K_N$ and P_{ij} with $0 \prec P_{ij} \prec \gamma^2 I$ and

$$\begin{aligned} |x|_{P_{ik}}^2 \geq |x|_Q^2 + |K_k x|_R^2 + &\left| (A_i - B_i K_k + A_j - B_j K_k) x/2 \right|^2_{(P_{ij}^{-1} - \gamma^2 I)^{-1}} \\ &- \gamma^2 |(A_i - B_i K_k - A_j + B_j K_k) x/2|^2 \end{aligned}$$

for $x \in \mathbb{R}^n$ and $i, j, k \in \{1, \dots, N\}$. Then $\max_{i,j} |x|_{P_{ij}}^2$ bounds the game value for the **certainty equivalence control law**

$$u_t = -K_{i_t} x_t, \text{ where } i_t = \arg\min_i \sum_{\tau=0}^{t-1} |A_i x_\tau + B_i u_\tau - x_{\tau+1}|^2.$$

[Rantzer, L4DC 2021], [Cederberg/Hansson/Rantzer, CDC 2022]

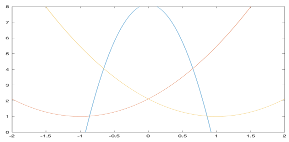
Theorem 2: Excitation/Exploitation

Given $Q \succ 0$, $R \succ 0$ and $\mathcal{M} := \{(A_1, B_1), \dots, (A_N, B_N)\}$, suppose there exist $K_1, \dots, K_N$ and P_{ij} with $0 \prec P_{ij} \prec \gamma^2 I$ and

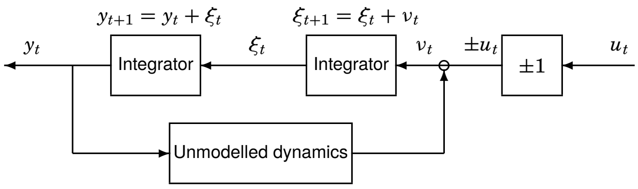
$$|x|^2_{P_{ik}} \geq |x|^2_Q + |K_k x|^2_R + \left|(A_i - B_i K_k + A_j - B_j K_k)x/2\right|^2_{(P_{ij}^{-1} - \gamma^2 I)^{-1}}$$
$$- \gamma^2 |(A_i - B_i K_k - A_j + B_j K_k)x/2|^2$$

$x \in \mathbb{R}^n$, $i, j, k \in \{1, \dots, N\}$. Then $\max_{i,j} |x|^2_{P_{ij}}$ bounds game value.

Last term encourages control activity when model uncertainty is large.



Example: Double Integrator with Uncertain Sign



No linear controller can stabilize a double integrator with unknown sign.

Minimax adaptive controller first estimates sign, then plays H_∞ game.

Example: Double Integrator with Uncertain Sign

The double integrator can be written as

$$x_{t+1} = \underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A} x_t \pm \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u_t + w_t$$

with the state $x_t = \begin{bmatrix} y_t & y_{t-1} & u_{t-1} \end{bmatrix}^\top$. Theorem 2 can be applied with $\mathcal{M} = \{(A, \pm B)\}$. By first solving the Riccati equation for $P_{11} = P_{22}$ and $K_1 = -K_2$, then solving the matrix inequalities for P_{12}, we get

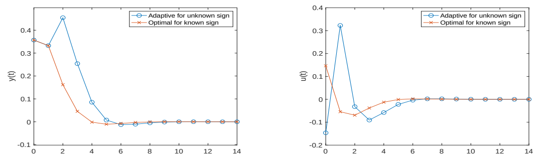
$$P_{11} = P_{22} = \begin{bmatrix} 20.61 & -11.09 & 11.09 \\ -11.09 & 7.83 & -6.83 \\ 11.09 & -6.83 & 7.83 \end{bmatrix} \qquad P_{12} = \begin{bmatrix} 155.0 & -84.4 & 84.4 \\ -84.4 & 89.0 & -87.5 \\ 84.4 & -87.5 & 89.0 \end{bmatrix}$$

$$K_1 = -K_2 = \begin{bmatrix} 1.786 & -1.288 & 1.288 \end{bmatrix} \qquad \gamma = 19$$

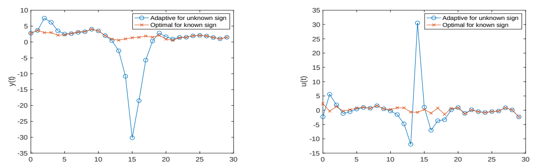
The adaptive controller has gain between $\sqrt{\|P_{12}\|} = 16.8$ and $\gamma = 19$.

Example: Double Integrator with Uncertain Sign

Without disturbances:



With white noise disturbances and sudden input gain sign change:



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Solution:
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$$u_{ij} = x_i/a_i - x_j/a_j \qquad (i, j) \in \mathcal{E}.$$

What if the dynamic parameters a_i are not known?

Is the Minimax Adaptive Controller Scalable?

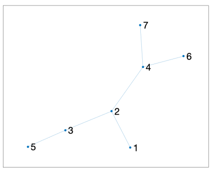
Important large-scale systems have optimal controllers where local computational complexity is independent of network size.

But the minimax adaptive state Z_t grows quadratically with network size.

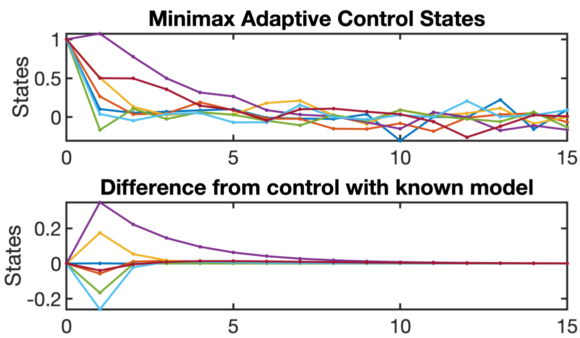
Instead, nodes update only parts relevant for estimation of local dynamics!

- Node 7 updates sample covariances of states, inputs in nodes 7 & 4.
- Node 2 updates sample covariances of states, inputs in nodes 1-4.

Then, complexity remains independent of network size!



Example: Adaptive Control of Water Network



[Renganathan/Kjellqvist/Rantzer 2022]

Summary

- Important large-scale systems have **sparse optimal controllers**
- Minimax optimal adaptive controllers have finite state involving **sample covariance matrix**
- Riccati type inequalities verify **rigorous gain bound**
- Optimal adaptive controller **scales linearly** with network size
- Many natural generalizations

Thanks

Pauline Kergus, Felix Agner, Fethi Bencherki, Martin Heyden, Richard Pates, Taouba Jouini

Emil Vladu, Olle Kjellqvist, Carolina Bergeling, Johan Grönqvist, Zhiyong Sun, Venkatraman Renganathan