From Statistical Relational AI to Neural Symbolic Computation

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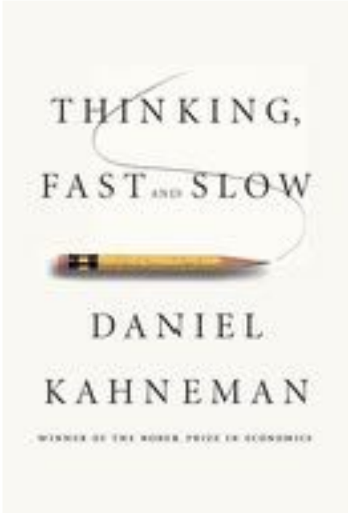
joint work with Robin Manhaeve, Angelika Kimmig, Giuseppe Marra, Sebastijan Dumancic, Thomas De Meester, Thomas Winters

Learning and Reasoning both needed

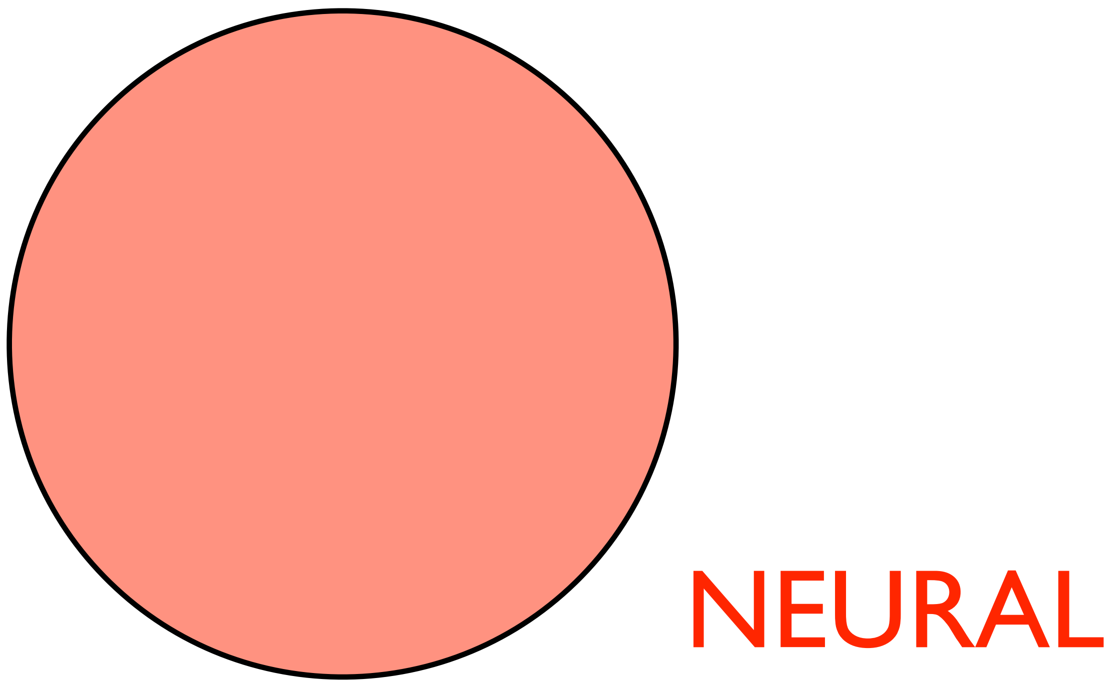


- System 1 - thinking fast - can do things like 2+2 = ? and recognise objects in image
- System 2 - thinking slow - can reason about solving complex problems - planning a complex task
- alternative terms — data-driven vs knowledge-driven, symbolic vs subsymbolic, solvers and learners, neuro-symbolic…
- **A lot of work on integrating learning and reasoning, neural symbolic computation to integrate logic / symbols reasoning with neural networks**

see also arguments
by Marcus, Darwiche, Levesque, Tenenbaum, Geffner, Bengio, Le Cun, Kautz, …

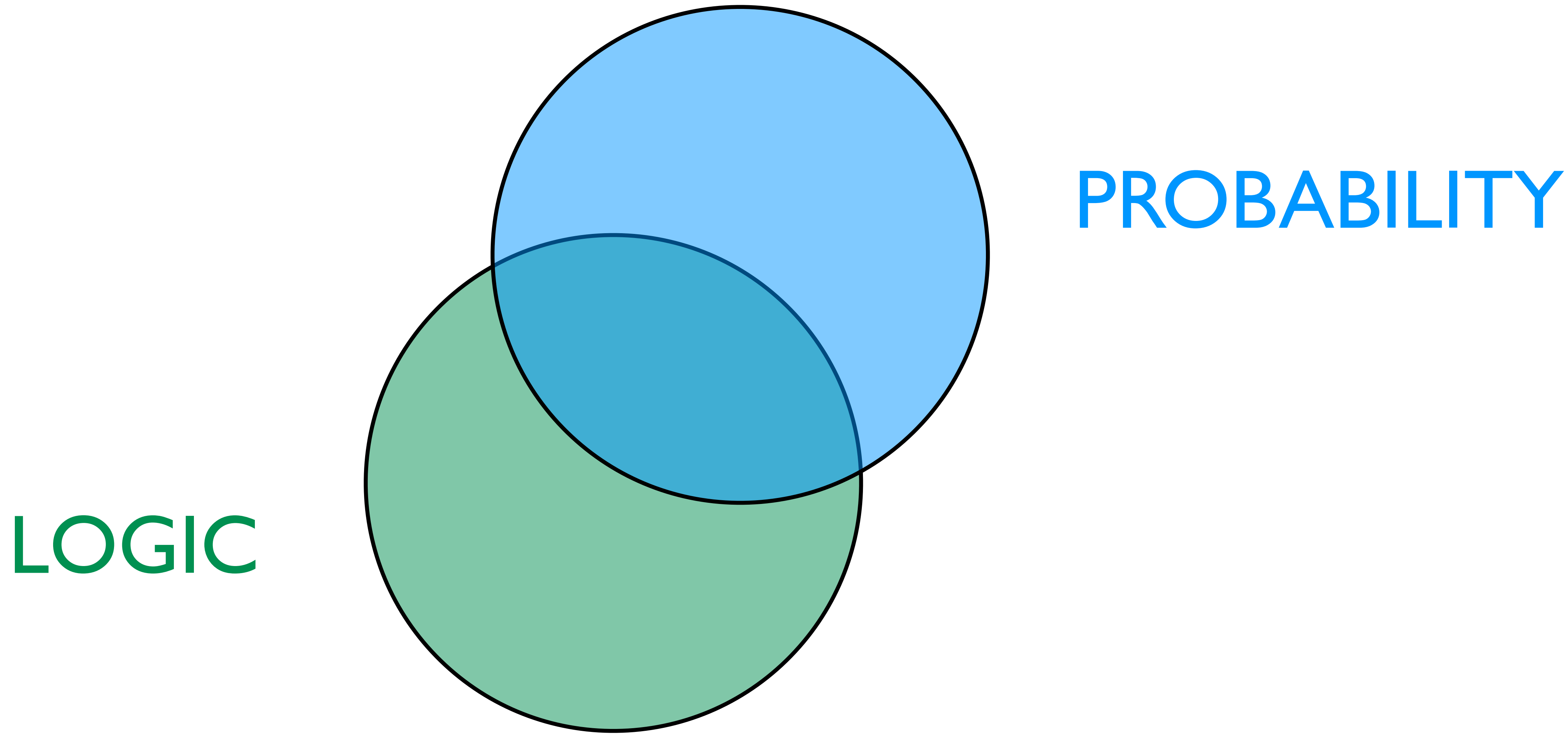
Thinking fast

MAIN PARADIGM in AI
Focus on Learning



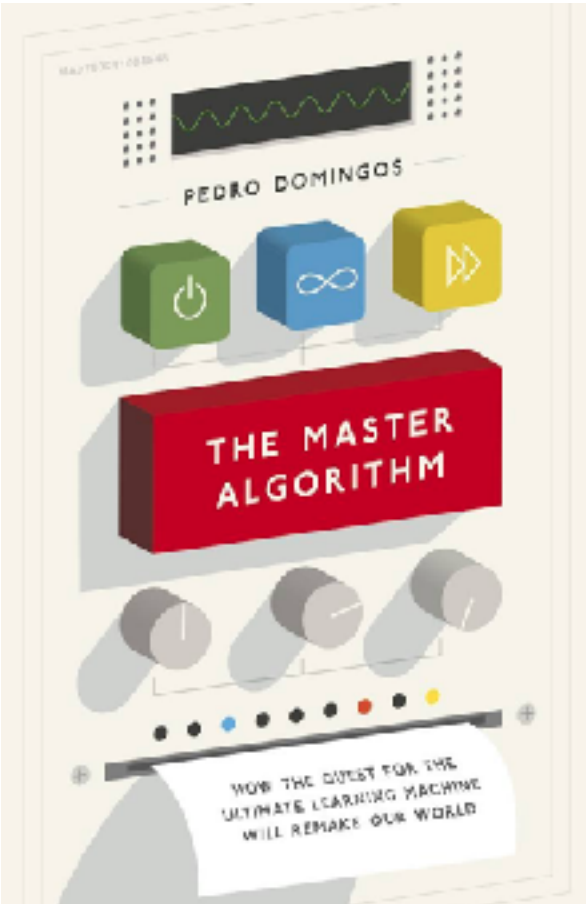
Thinking slow = reasoning

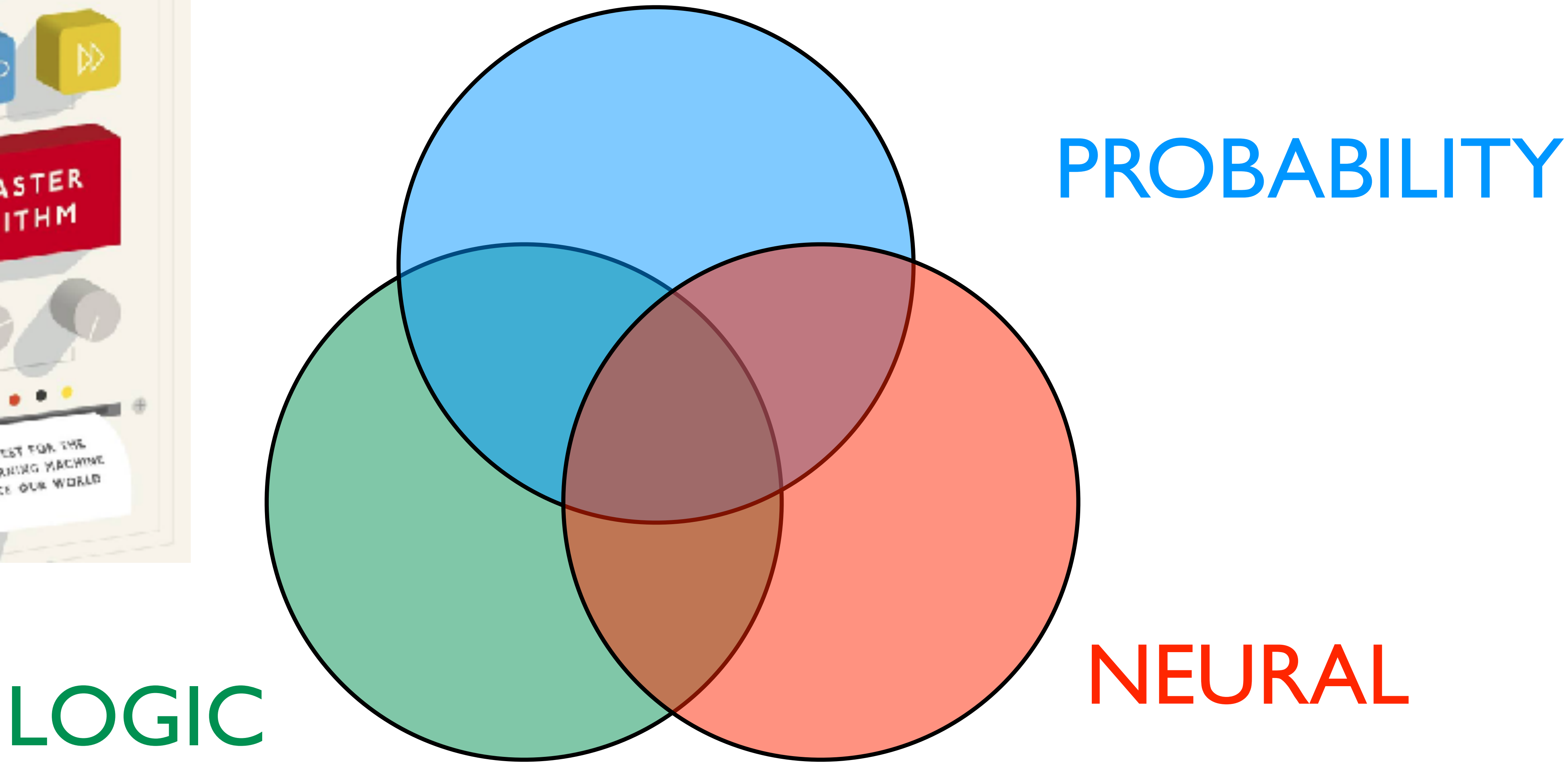
TWO MAIN PARADIGMS in AI



Their integration has been well studied in
Probabilistic (Logic) Programming and Statistical Relational AI (StarAI)

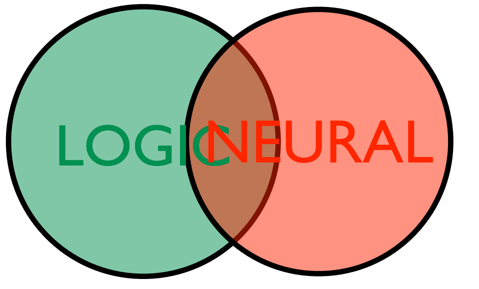
Integrating learning and reasoning



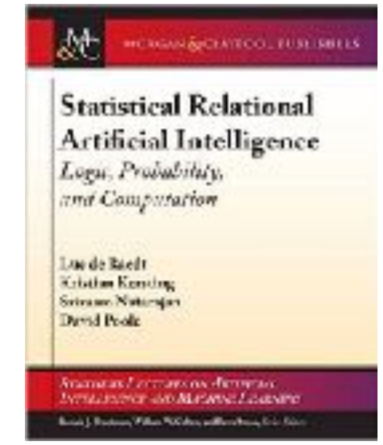


How to integrate these three paradigms in AI ?

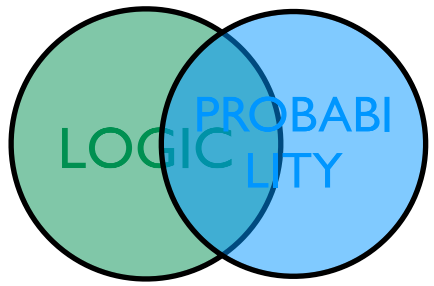
Neural Symbolic Computation:

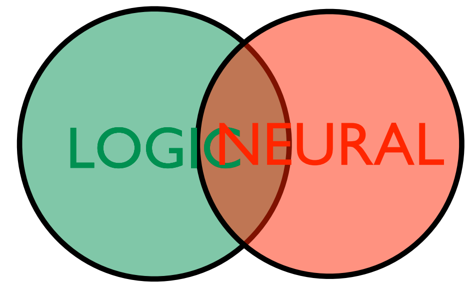


- **Neural symbolic computation** is the area combining logic / symbolic reasoning and neural networks



Key Message 1

FROM 

TO 

StarAI and NeSy share similar problems and thus similar solutions apply

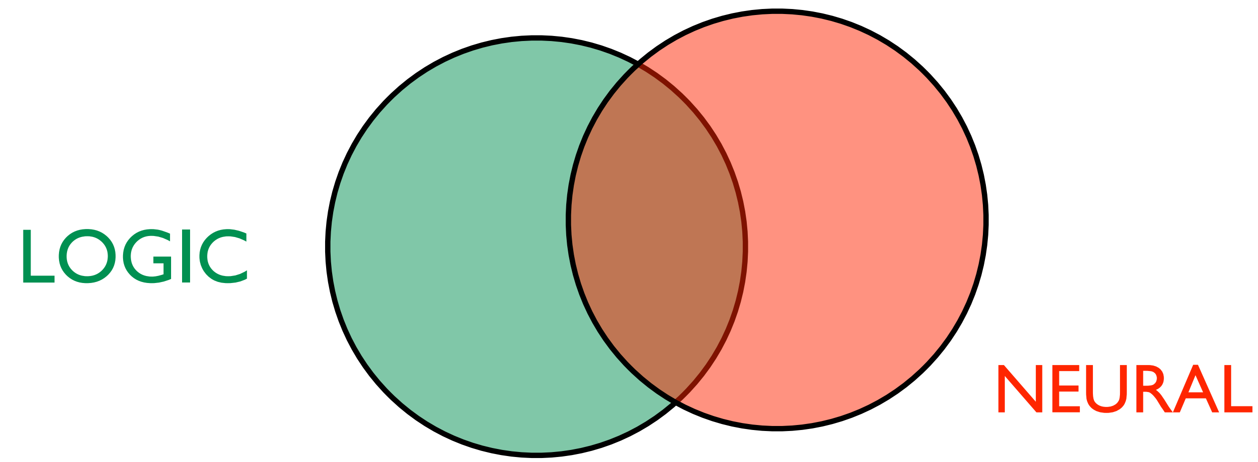
WARNING

TALK MAY NOT COVER ALL of NESY

PART 1 of the talk

See also [De Raedt et al., IJCAI 20]

Neural Symbolic Computation: state-of-the-art



- Neural symbolic computation is the area combining logic / symbolic reasoning and neural networks

- **Most NeSy approaches :** inject the logic/knowledge into neural networks, and let the neural network do the rest

- **Downside :** relies only on neural networks -> the power of reasoning, explanation and trust is (at least partly) lost

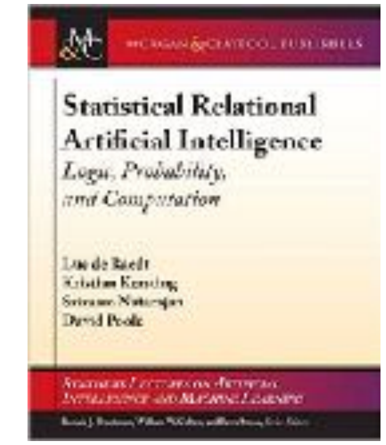
Key Message 2

A different approach

A true integration T of X and Y should allow to reconstruct X and Y as special cases of T

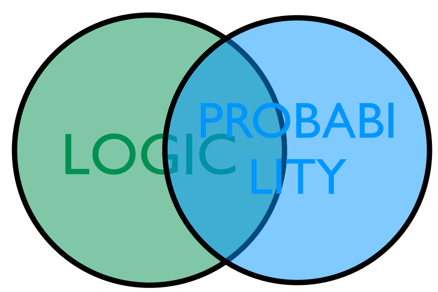
Thus, Neural Symbolic approaches should have both logic and neural networks as special cases

PART 2 of the talk — illustration with DeepProbLog [NeurIPS 2018] and DeepStochLog [AAAI 2022]

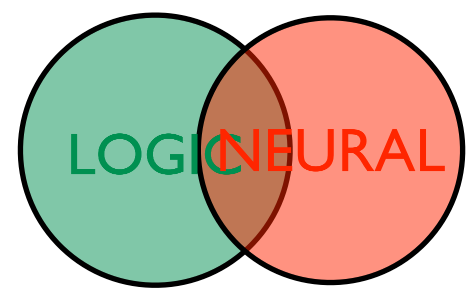


PART 1

FROM



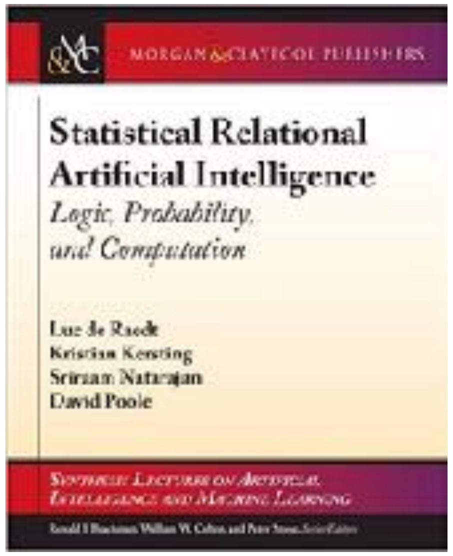
TO



Key Message 1

StarAI and NeSy share similar problems and thus similar solutions apply

There are two basic types of
(uses of) logic,
graphical models, and
neural symbolic models



Logic Programs

as in the programming language Prolog

Propositional logic program

```
burglary.
hears_alarm_mary.

earthquake.
hears_alarm_john.

alarm :– earthquake.
alarm :– burglary.

calls_mary :– alarm, hears_alarm_mary.
calls_john :– alarm, hears_alarm_john.
```

facts :
burglary = true

LOGIC

Logic Programs

as in the programming language Prolog

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```

rule:

calls_mary =true IF alarm = true AND hears_alarm_mary = true

LOGIC

erc

Logic Programs

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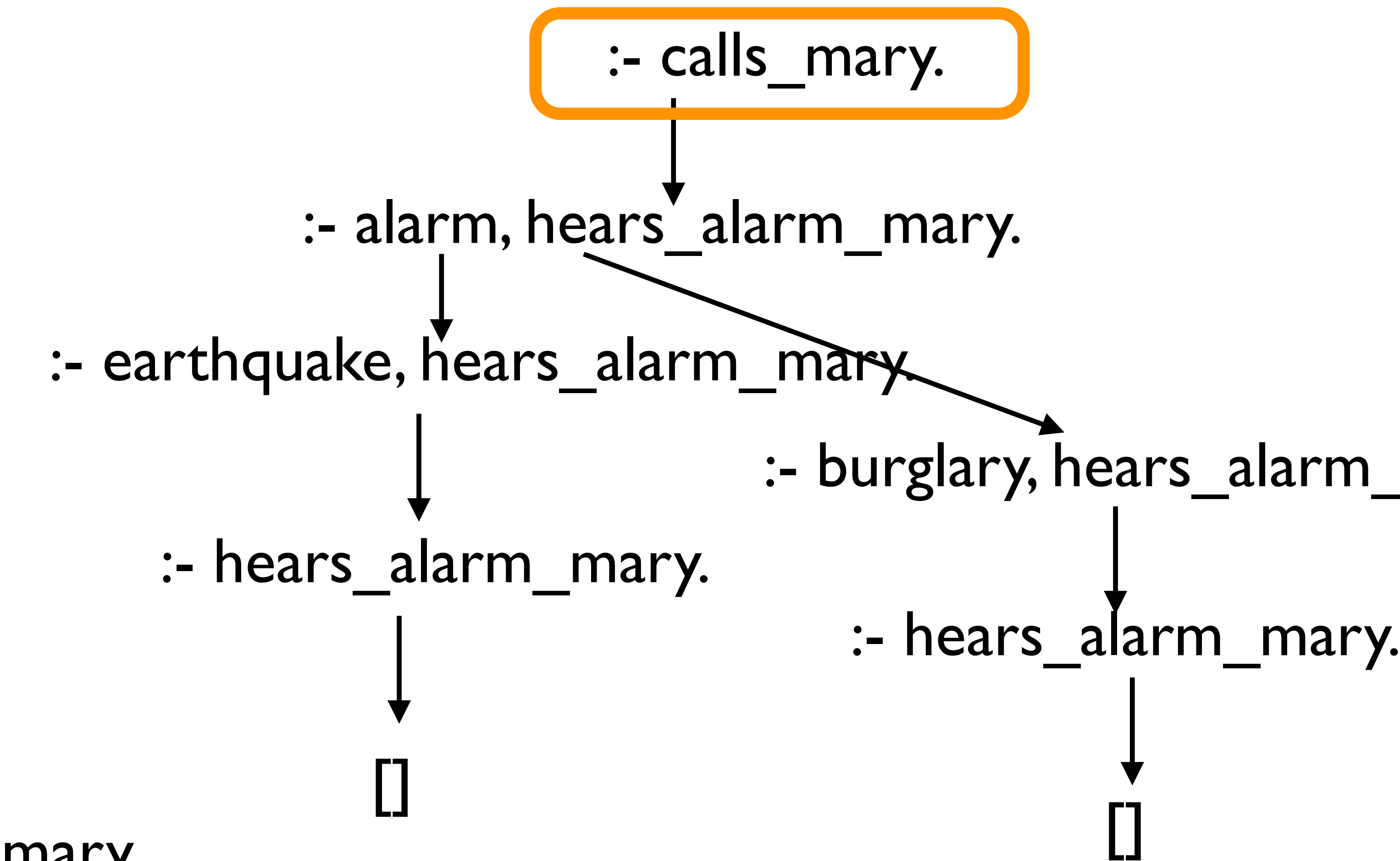
Two proofs (by refutation)



A proof-theoretic view

LOGIC

Logic as constraints

as in SAT solvers

Propositional logic

IFF / AND

calls(mary) ↔ hears_alarm(mary) ∧ alarm

calls(john) ↔ hears_alarm(john) ∧ alarm

OR

alarm ↔ earthquake v burglary

Model / Possible World

{ burglary,

hears_alarm(john),

alarm,

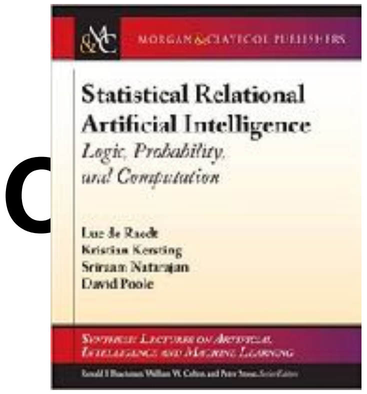
calls(john)}

the facts that are true
in this model / possible world

A model-theoretic view

LOGIC

Two types of probabilistic graphical models and StarAI systems



0.1 :: burglary.

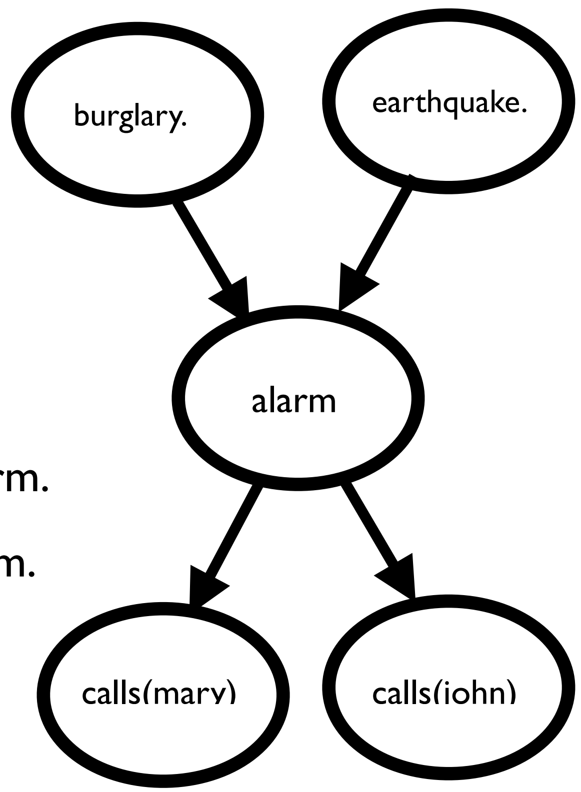
0.05 :: earthquake.

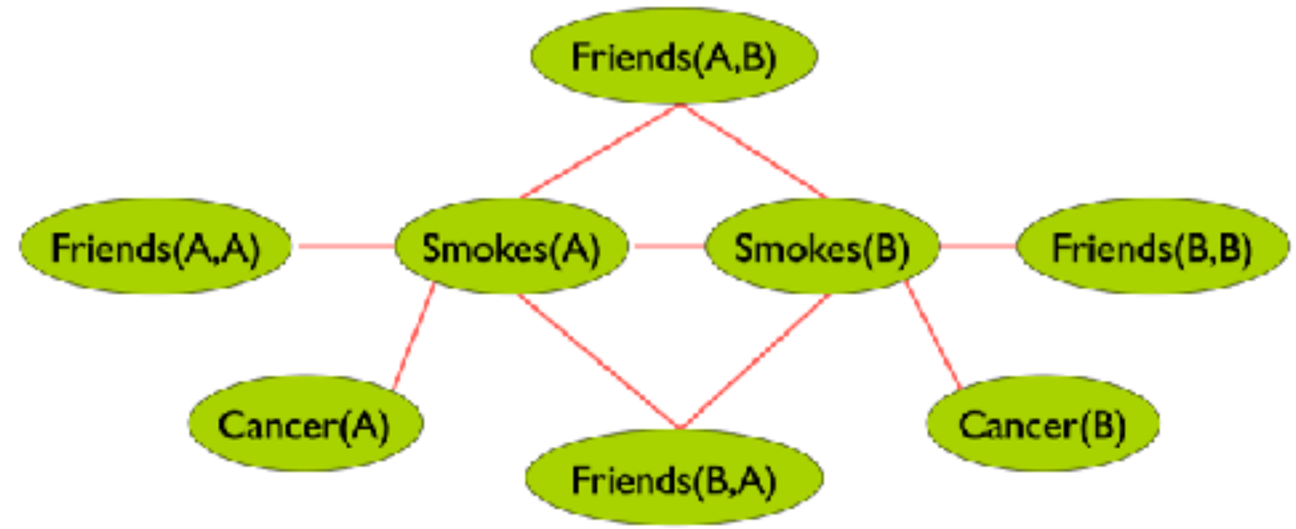
alarm :– earthquake.

alarm :– burglary.

0.7::calls(mary) :– alarm.

0.6::calls(john) :– alarm.





$$1.5 \quad \forall x \; Smokes(x) \Rightarrow Cancer(x)$$

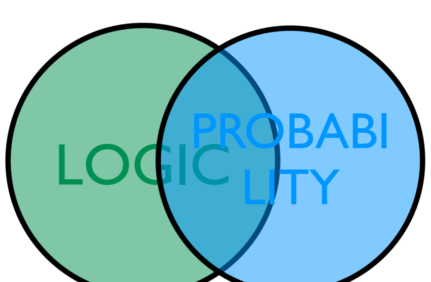
$$1.1 \quad \forall x, y \; Friends(x, y) \Rightarrow \big(Smokes(x) \Leftrightarrow Smokes(y)\big)$$

Probabilistic Logic Programs
ProbLog

directed
Bayesian Net

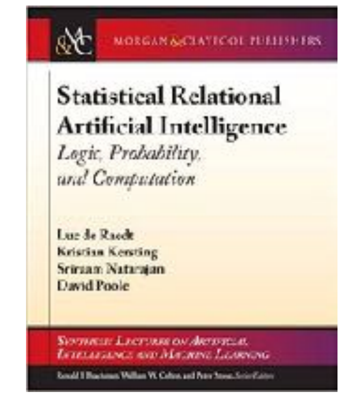
Markov Logic

undirected
Markov Net
model theoretic



key representatives

Two types of Neural Symbolic Systems



Just like in StarAI

Logic as a kind of ***neural program***

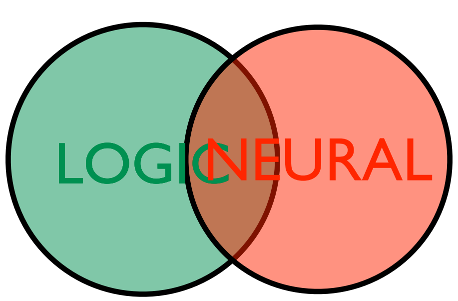
directed StarAI approach and logic programs

Logic as the ***regularizer***
(reminiscent of Markov Logic Networks)

undirected StarAI approach and (soft) constraints

Also, many NeSy systems are doing
knowledge based model construction KBMC
where logic is used as a template

Just like in StarAI



Logic as a neural program

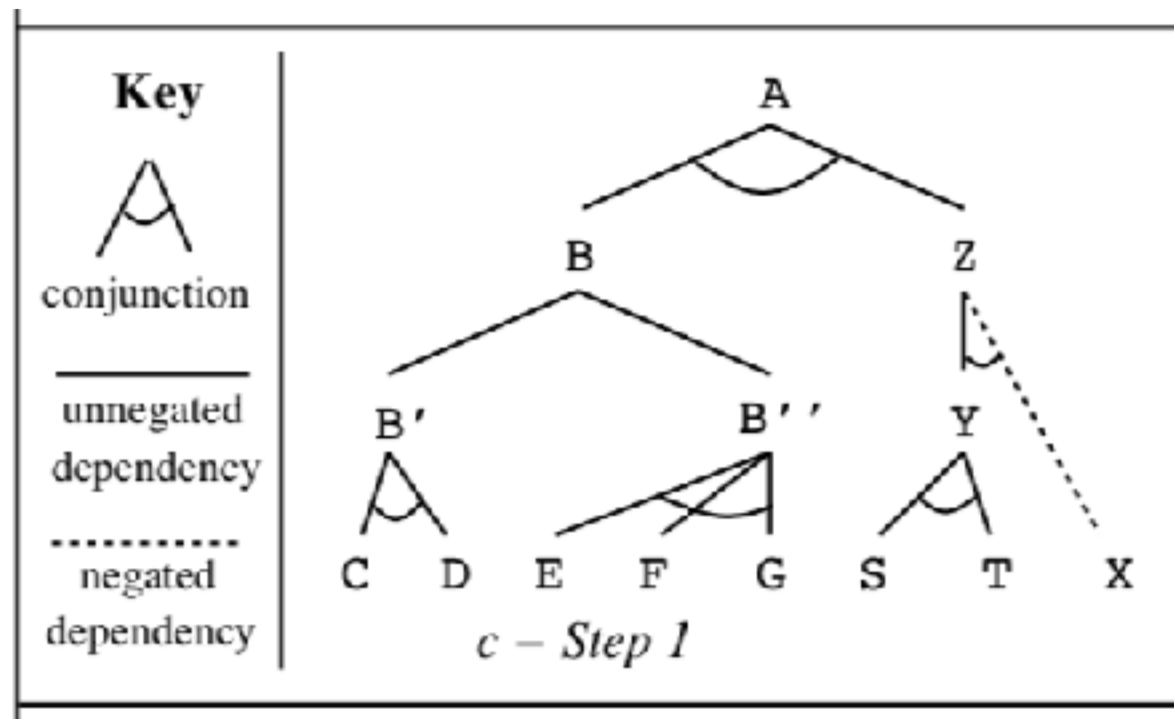
directed StarAI approach and logic programs

- KBANN (Towell and Shavlik AIJ 94)
- Turn a (propositional) Prolog program into a neural network and learn

```
A :- B, Z.
B :- C, D.
B :- E, F, G.
Z :- Y, not X.
Y :- S, T.
```

REWRITE

```
A    :- B, Z.
B    :- B'.
B    :- B''.
B'   :- C, D.
B''  :- E, F, G.
Z    :- Y, not X.
Y    :- S, T.
```

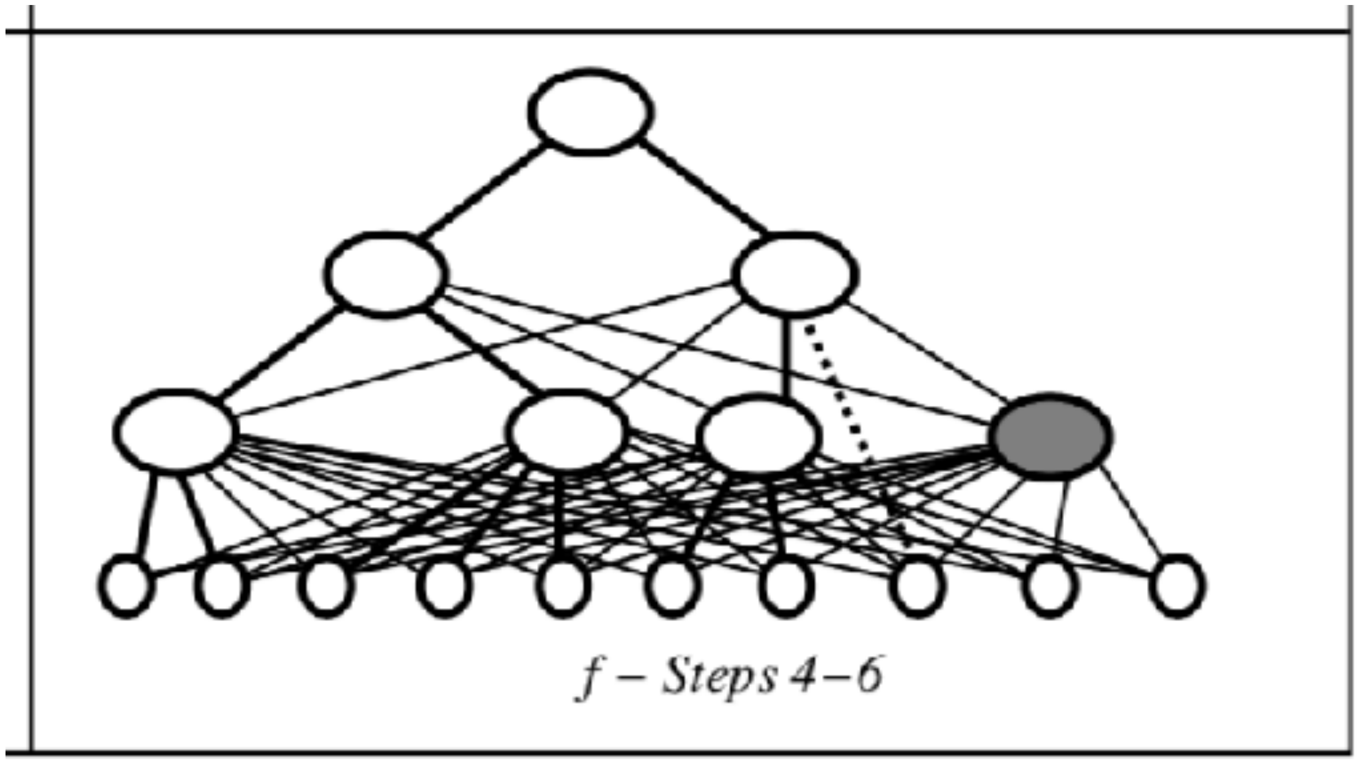


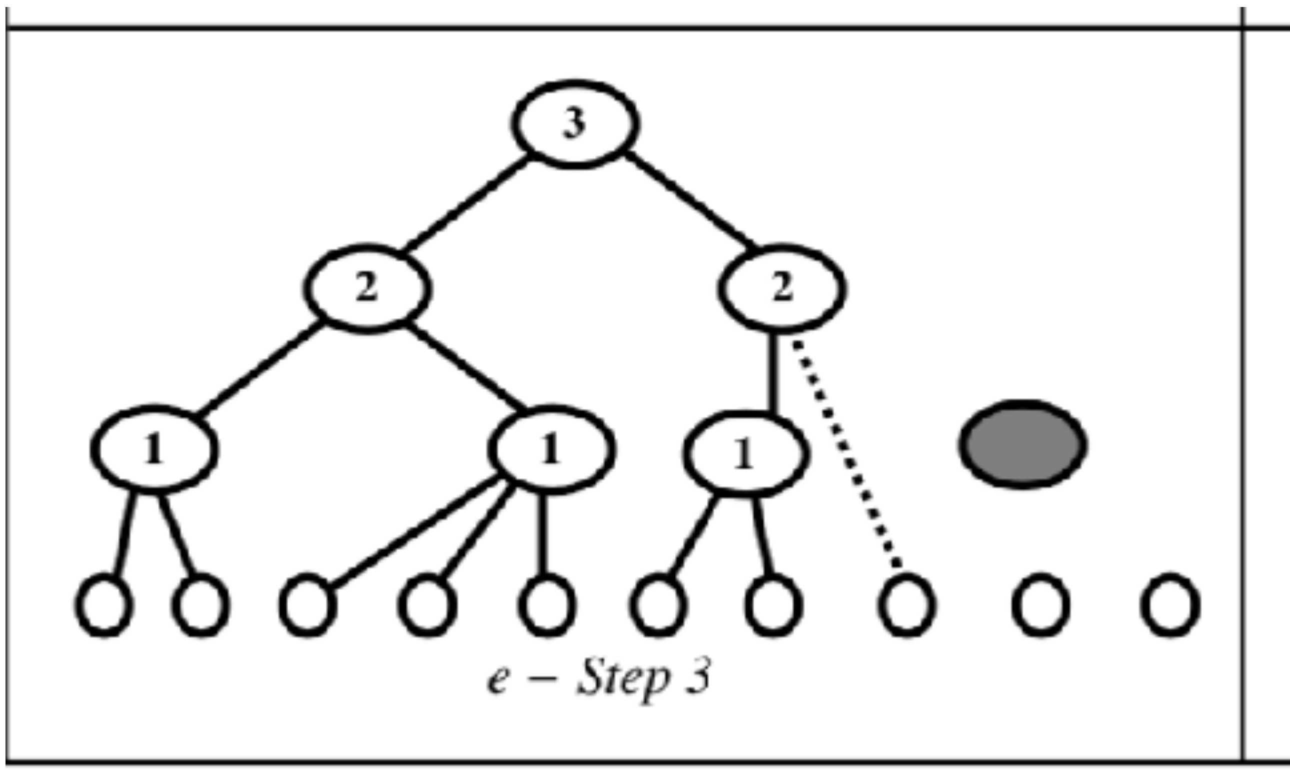
LOGIC NEURAL

erc

Logic as a neural program

directed StarAI approach and logic programs





ADD LINKS — ALSO SPURIOUS ONES

HIDDEN UNIT

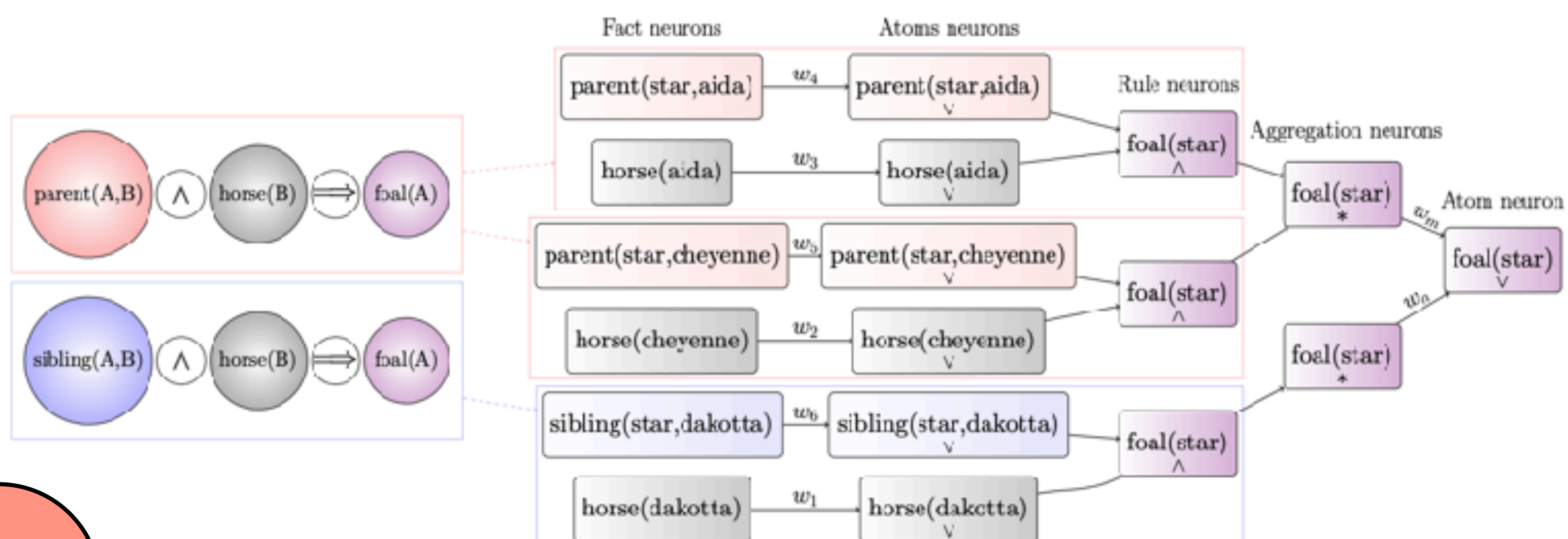
and then learn

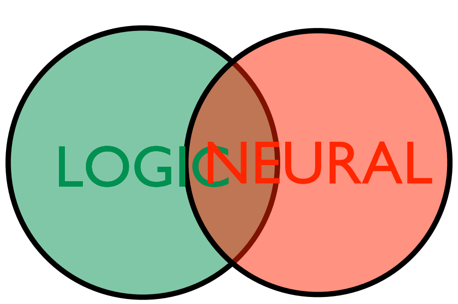
(Details of activation & loss functions not mentioned)

Lifted Relational Neural Networks

directed StarAI approach and logic programs

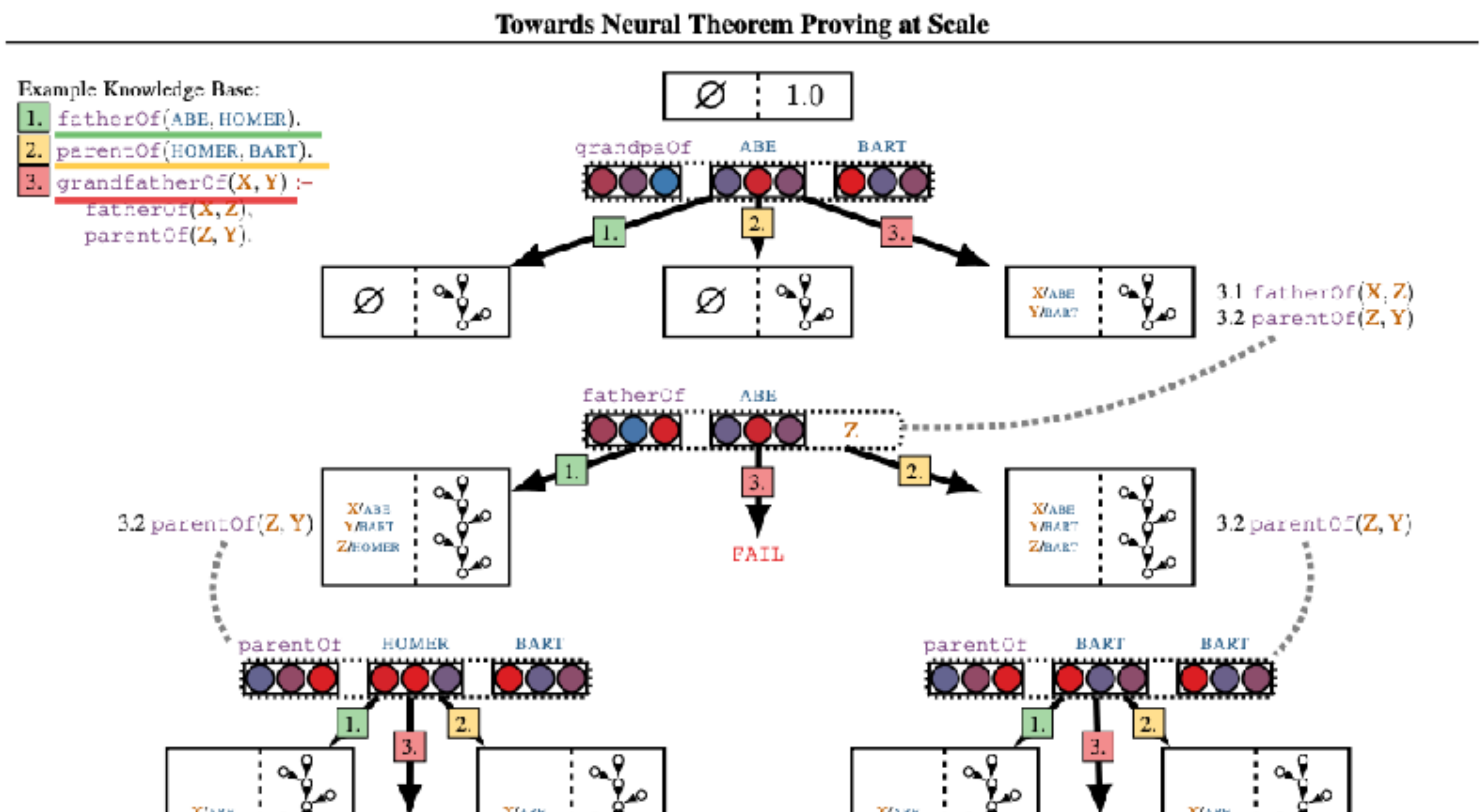
- Directed (fuzzy) NeSy
- similar in spirit to the Bayesian Logic Programs and Probabilistic Relational Models
- Of course, other kind of (fuzzy) operations for AND, OR and Aggregation (cf. later)





[Sourek, Kuzelka, et al JAIR]

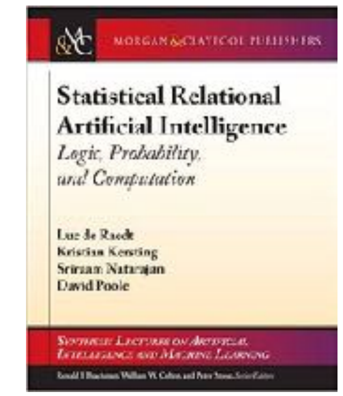
Neural Theorem Prover



the logic is encoded in the network
how to reason logically ?

[Rocktäschel Riedel, NeurIPS 17; Minervini et al.]

Two types of Neural Symbolic Systems



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Logic as a kind of ***neural program***

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Logic as the ***regularizer***
(reminiscent of Markov Logic Networks)

undirected StarAI approach and (soft) constraints

Also, many NeSy systems are doing
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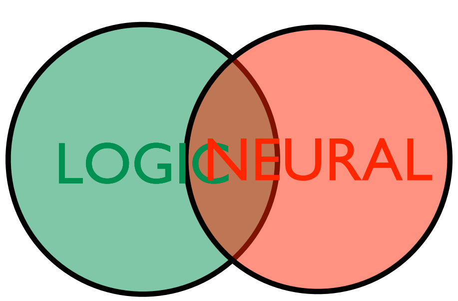
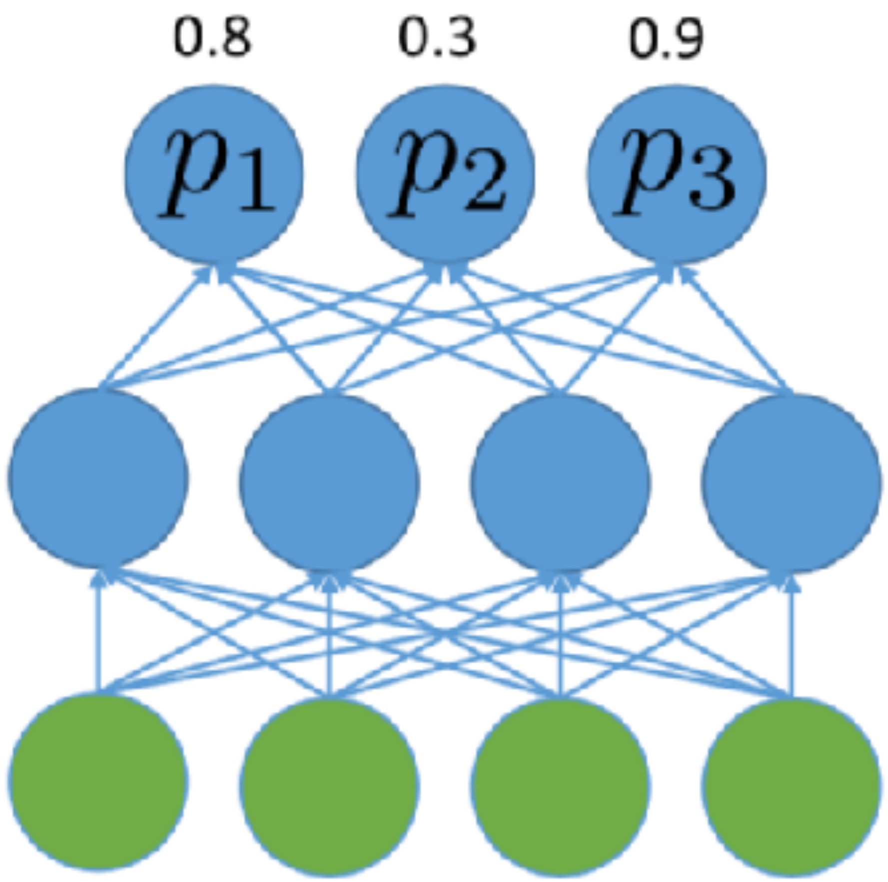


Logic as constraints

undirected StarAI approach and (soft) constraints

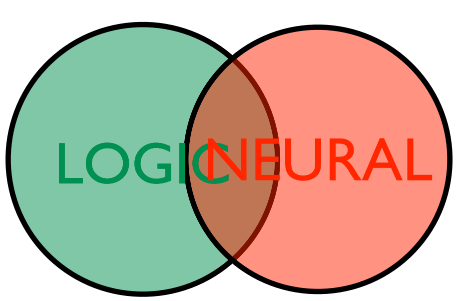
multi-class classification



This constraint should be satisfied

$$(\neg x_1 \wedge \neg x_2 \wedge x_3) \vee$$
$$(\neg x_1 \wedge x_2 \wedge \neg x_3) \vee$$
$$(x_1 \wedge \neg x_2 \wedge \neg x_3)$$

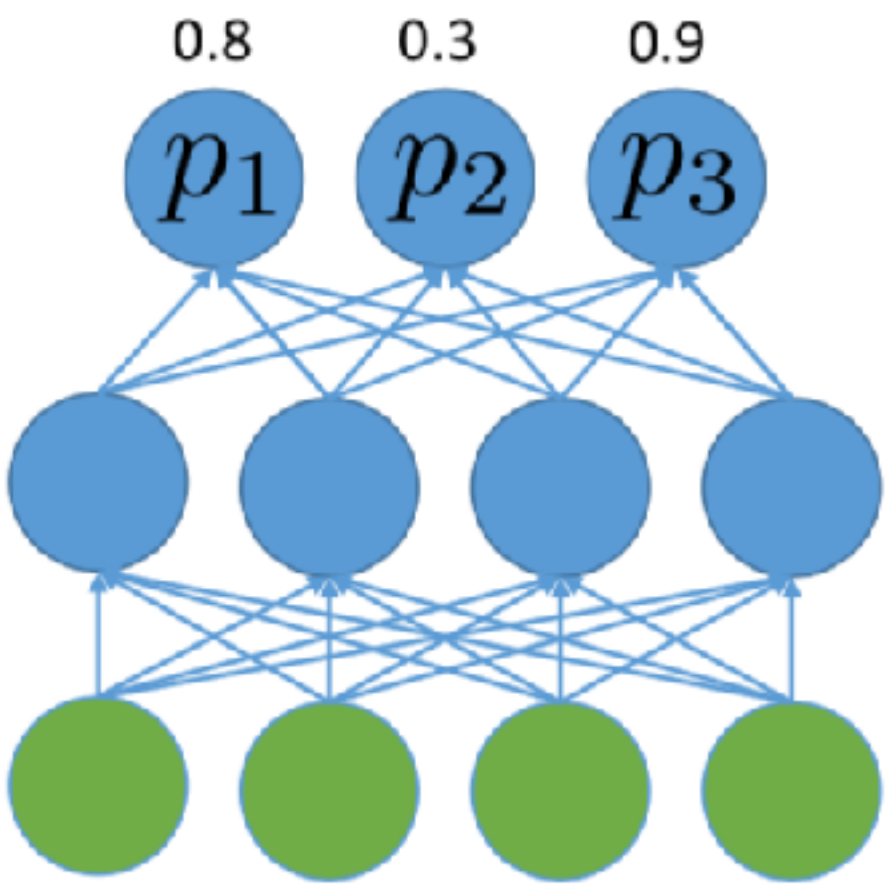
figures and example from Xu et al., ICML 2018



Logic as constraints

undirected StarAI approach and (soft) constraints

multi-class classification



Probability that constraint is satisfied

$$(1-x_1)(1-x_2)x_3+$$
$$(1-x_1)x_2(1-x_3)+$$
$$x_1(1-x_2)(1-x_3)$$

basis for SEMANTIC LOSS

(weighted model counting)

Logic as a regularizer

undirected StarAI approach and (soft) constraints

Semantic Loss:

- Use logic as constraints (very much like “propositional MLNs)
- Semantic loss

$$SLoss(T) \propto -\log \sum_{X \models T} \prod_{x \in X} p_i \prod_{\neg x \in X} (1 - p_i)$$

- Used as regulariser

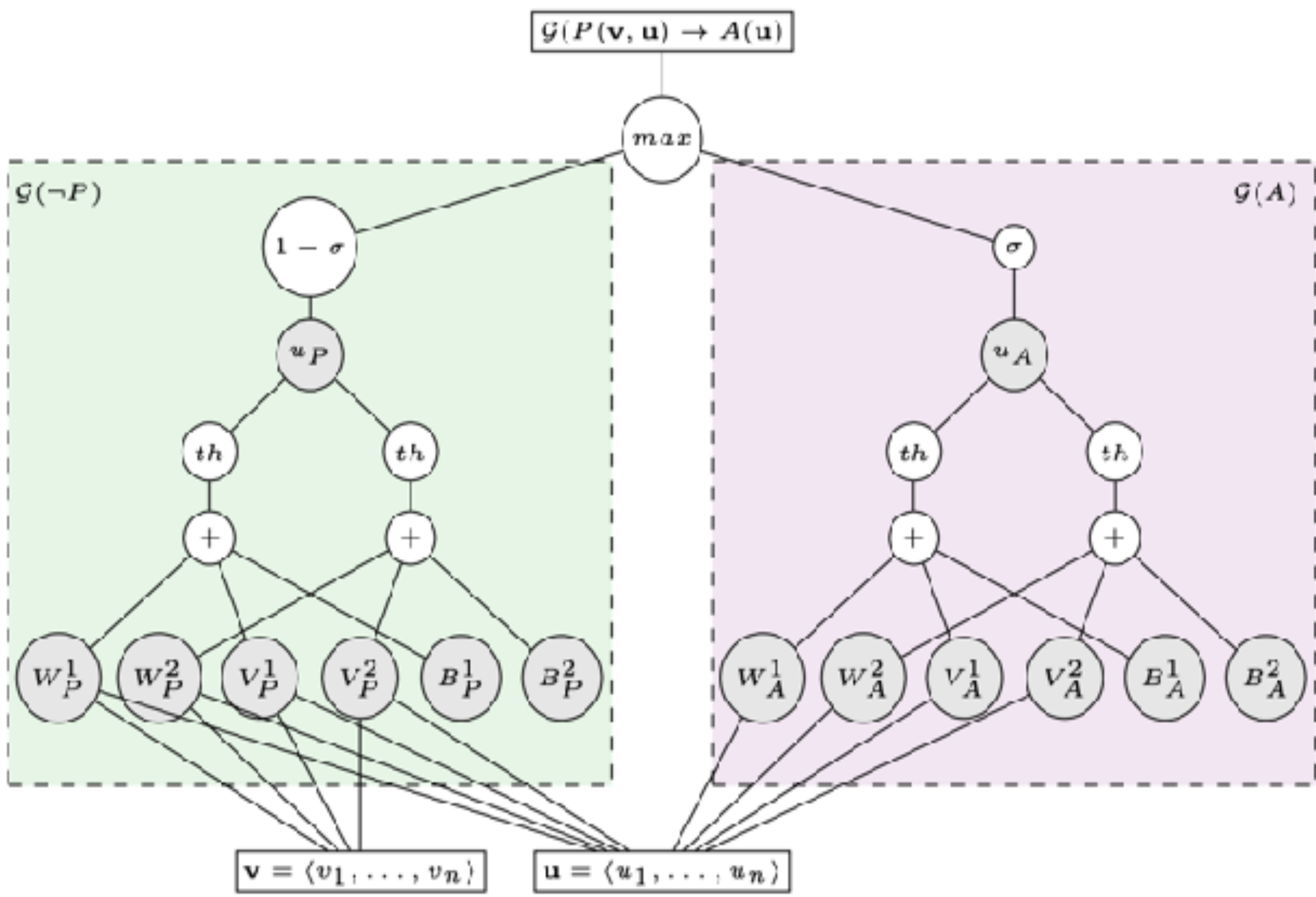
$$Loss = TraditionalLoss + w.SLoss$$

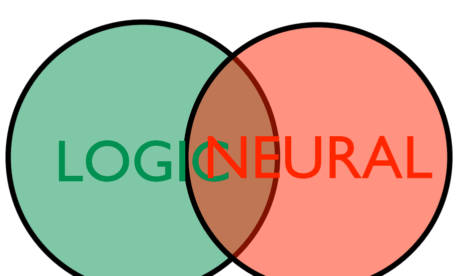
- Use weighted model counting , close to StarAI

Logic Tensor Networks

undirected StarAI approach and (soft) constraints

$$P(x, y) \rightarrow A(y), \text{ with } \mathcal{G}(x) = \mathbf{v} \text{ and } \mathcal{G}(y) = \mathbf{u}$$

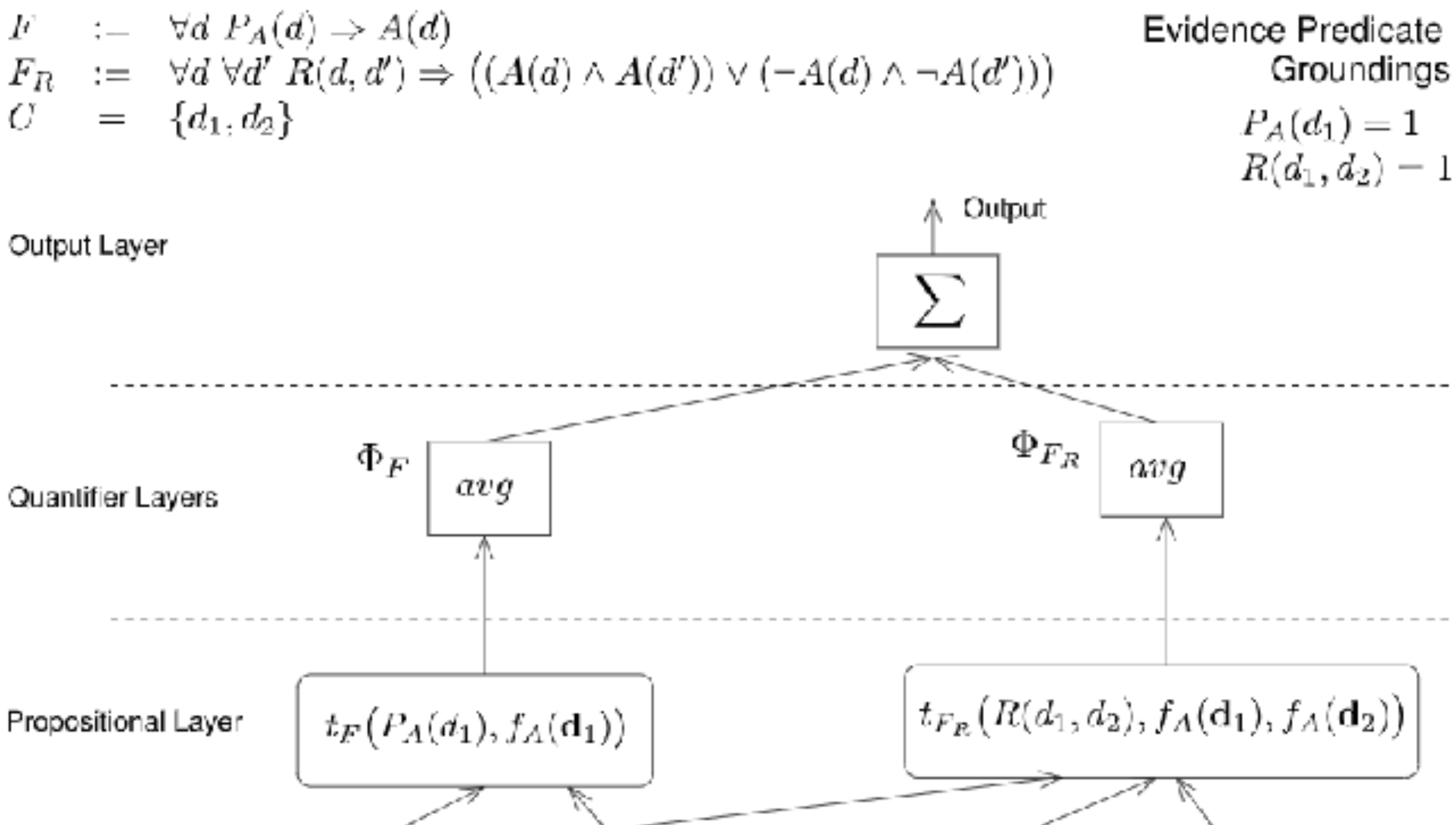




Serafini & Garcez

Semantic Based Regularization

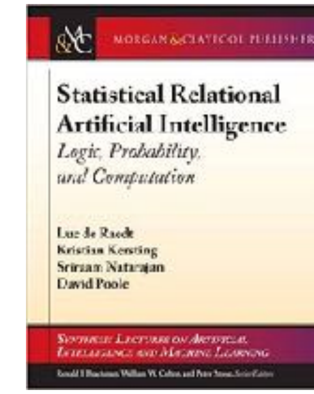
undirected StarAI approach and (soft) constraints



the logic is encoded in the network
how to reason logically ?

Diligenti et al. AIJ

Two types of Neural Symbolic Systems



Just like in StarAI

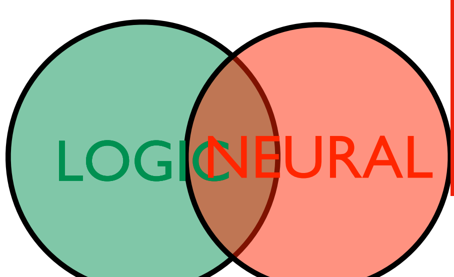
Logic as a kind of ***neural program***

directed StarAI approach and logic programs

Logic as the ***regularizer*** ***(reminiscent of Markov Logic Networks)***

undirected StarAI approach and (soft) constraints

Consequence :
the logic is encoded in the network
the ability to logically reason is lost
logic is not a special case

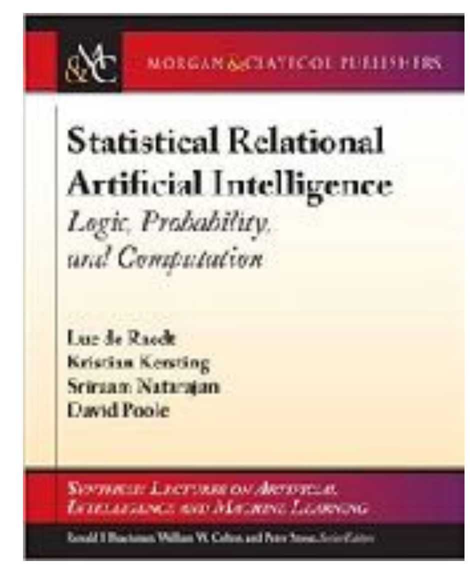


Key Message 1

StarAI and NeSy share similar problems and thus similar solutions apply

What do the numbers mean ?

Three possible choices:
Logic,
Probability &
Fuzzy



Just like in StarAI

Key Message 2

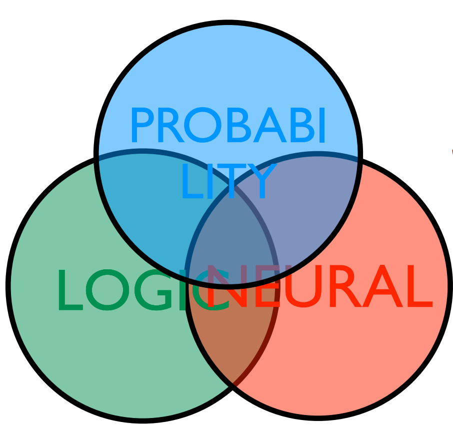
A different approach

A true integration T of X and Y should allow to reconstruct X and Y as special cases of T

Thus, Neural Symbolic approaches should have both logic and neural networks as special cases

Our approach: “an interface layer (<> pipeline) between neural & symbolic components” will be illustrated with DeepProbLog

See also [Manhaeve et al., NeurIPS 18; arXiv: 1907.08194]



Part 2 of the talk — illustration with DeepProbLog [NeurIPS 2018] and DeepStochLog [AAAI 2022]

Two types of probabilistic models

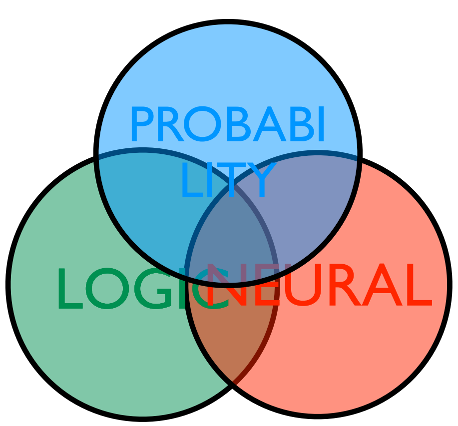
- Based on a random graph model
  - Bayesian Nets and ProbLog
- Based on a random walk model
  - Probabilistic grammars and Stochastic Logic Programs

DeepProbLog

DeepProbLog = Probability + Logic + Neural Network

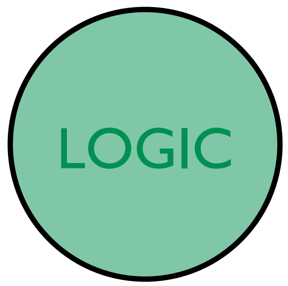
DeepProbLog = ProbLog + Neural Network

Related work in NeSy	DeepProbLog
Logic is made less expressive	Full expressivity is retained
Logic is pushed into the neural network	Maintain both logic and neural network
Fuzzy logic	Probabilistic logic programming
Language semantics unclear	Clear semantics



PART 2

FROM



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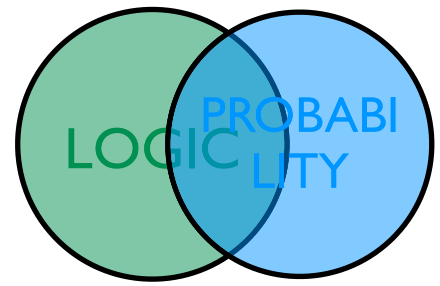
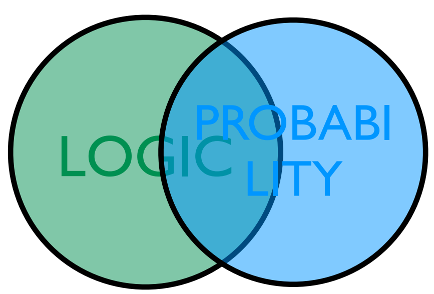


ProbLog

FROM



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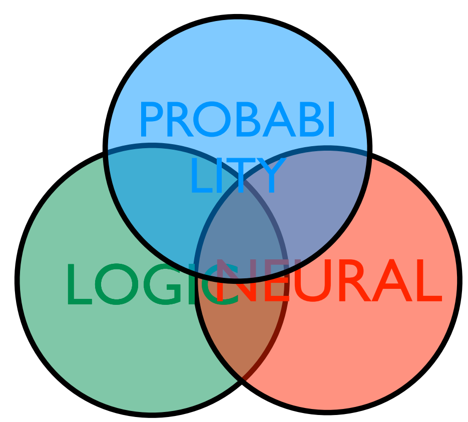


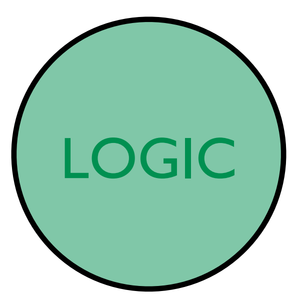
DeepProbLog

a logic programming perspective

PART 2 A

From Prolog to ProbLog

FROM



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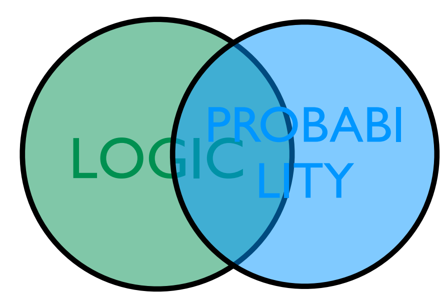

Logic Programs

as in the programming language Prolog

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earthquake.
hears_alarm(john).

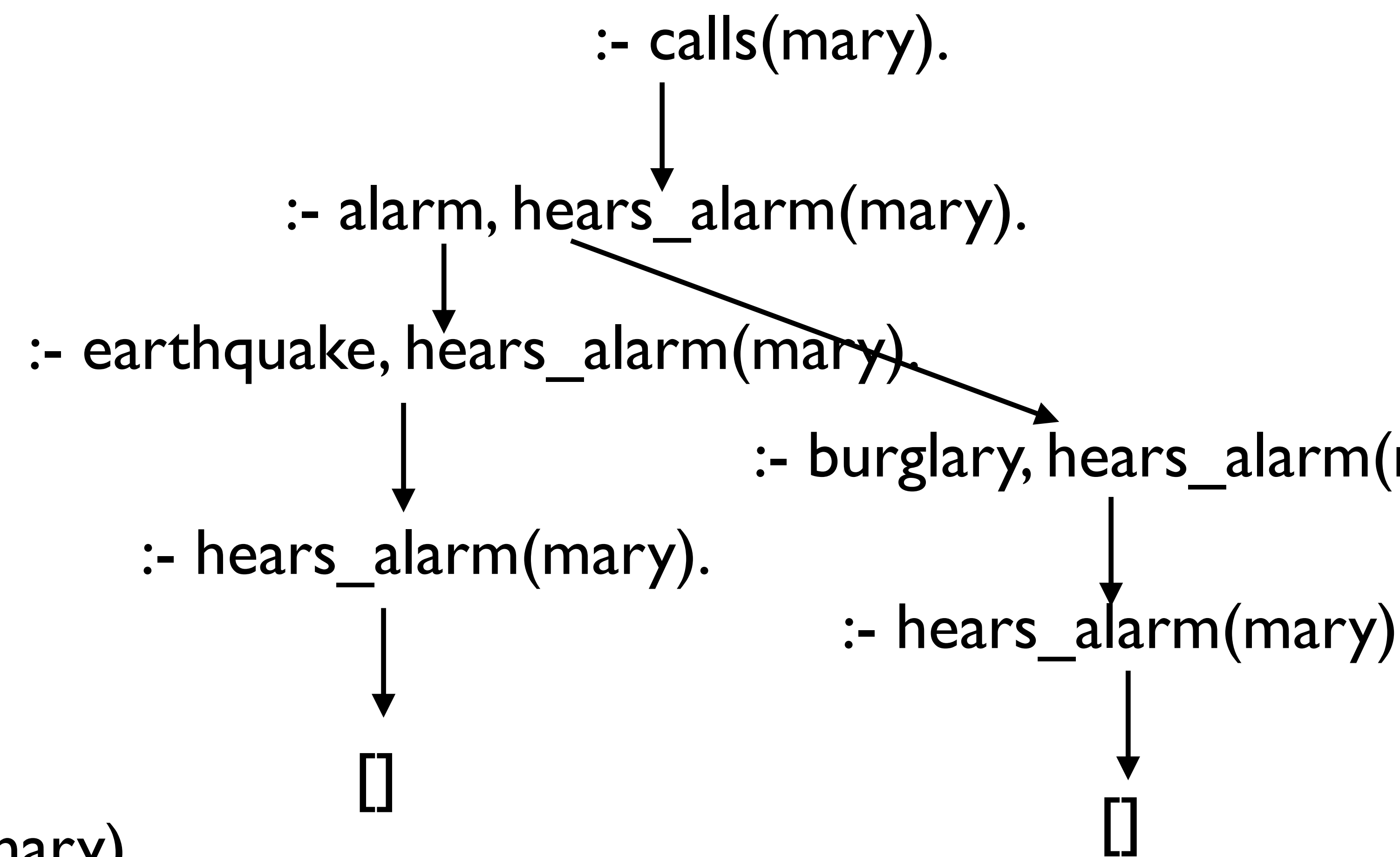
alarm :– earthquake.

alarm :– burglary.

calls(mary) :– alarm, hears_alarm(mary).

calls(john) :– alarm, hears_alarm(john).

Two proofs (by refutation)



A proof-theoretic view

LOGIC

Probabilistic Logic Programs

as in the probabilistic programming language ProbLog

Propositional logic program

0.1 :: burglary.
0.3 ::hears_alarm(mary).

Probabilistic facts

0.05 ::earthquake.
0.6 ::hears_alarm(john).

alarm :– earthquake.
alarm :– burglary.

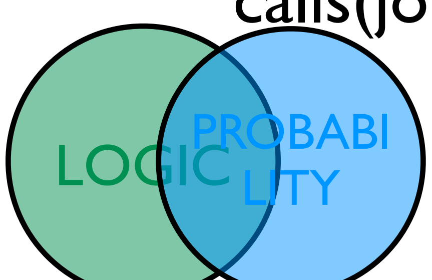
calls(mary) :– alarm, hears_alarm(mary).
calls(john) :– alarm, hears_alarm(john).

Key Idea (Sato & Poole)
the distribution semantics:

unify the basic concepts in logic and probability:

random variable ~ propositional variable

an interface between logic and probability



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0.1 :: burglary.
0.3 ::hears_alarm(mary).

0.05 ::earthquake.
0.6 ::hears_alarm(john).

alarm :– earthquake.

alarm :– burglary.

calls(mary) :– alarm, hears_alarm(mary).

calls(john) :– alarm, hears_alarm(john).

Two proofs (by refutation)

:- alarm

:- burglary.   :- earthquake.

P=0.1   **P=0.05**

[]   []

Probability of one proof : $\prod_{f:fact \in Proof} P_f$

Probabilistic Logic Programs

as in the probabilistic programming language ProbLog

Propositional logic program

0.1 :: burglary.
0.3 ::hears_alarm(mary).

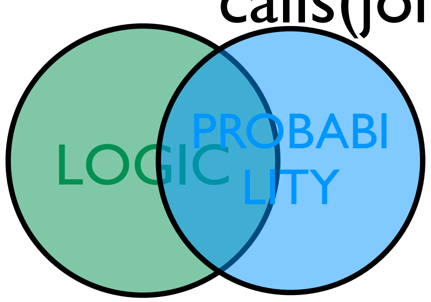
0.05 ::earthquake.
0.6 ::hears_alarm(john).

alarm :– earthquake.

alarm :– burglary.

calls(mary) :– alarm, hears_alarm(mary).

calls(john) :– alarm, hears_alarm(john).



Disjoint sum problem

:- alarm

:- burglary.

:- earthquake.

P=0.1

P=0.05

[]

[]

Probability of one proof : $\prod_{f:fact \in Proof} P_f$

P(alarm) = P(burg OR earth)
= P(burg) + P(earth) - P(burg AND earth)
=/= P(burg) + P(earth)

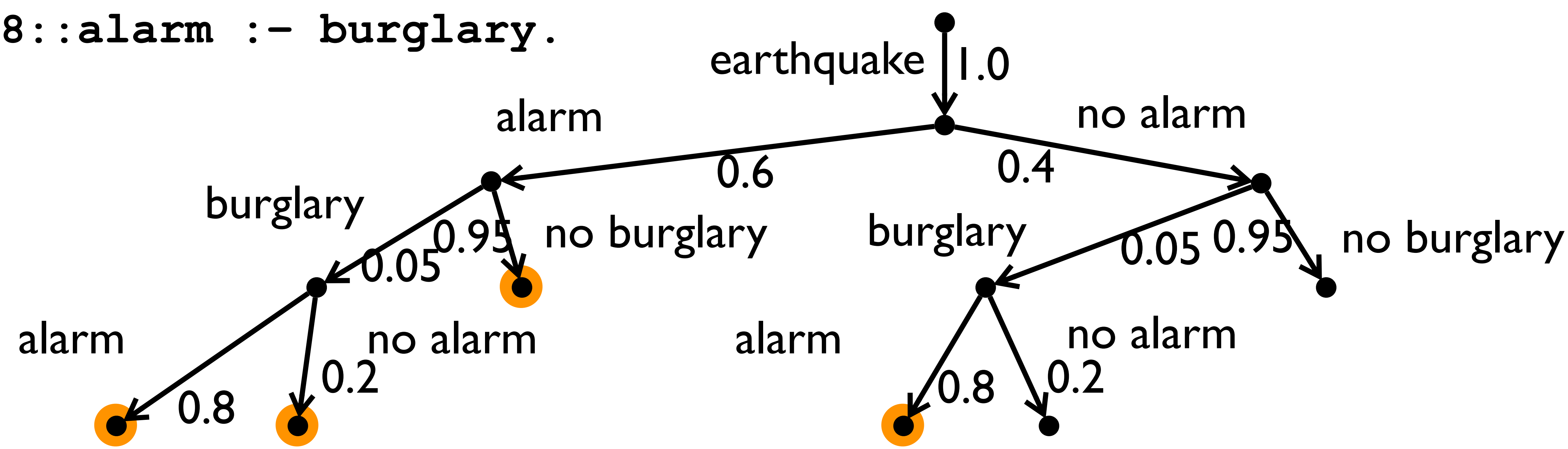
Probabilistic Logic Program Semantics

```
earthquake.
0.05::burglary.

0.6::alarm :- earthquake.
0.8::alarm :- burglary.
```

[Vennekens et al, ICLP 04]

probabilistic causal laws



P(alarm)=0.6×0.05×0.8+0.6×0.05×0.2+0.6×0.95+0.4×0.05×0.8

Probabilistic Logic Program Semantics

Propositional logic program

0.1 :: burglary.

0.05 :: earthquake.

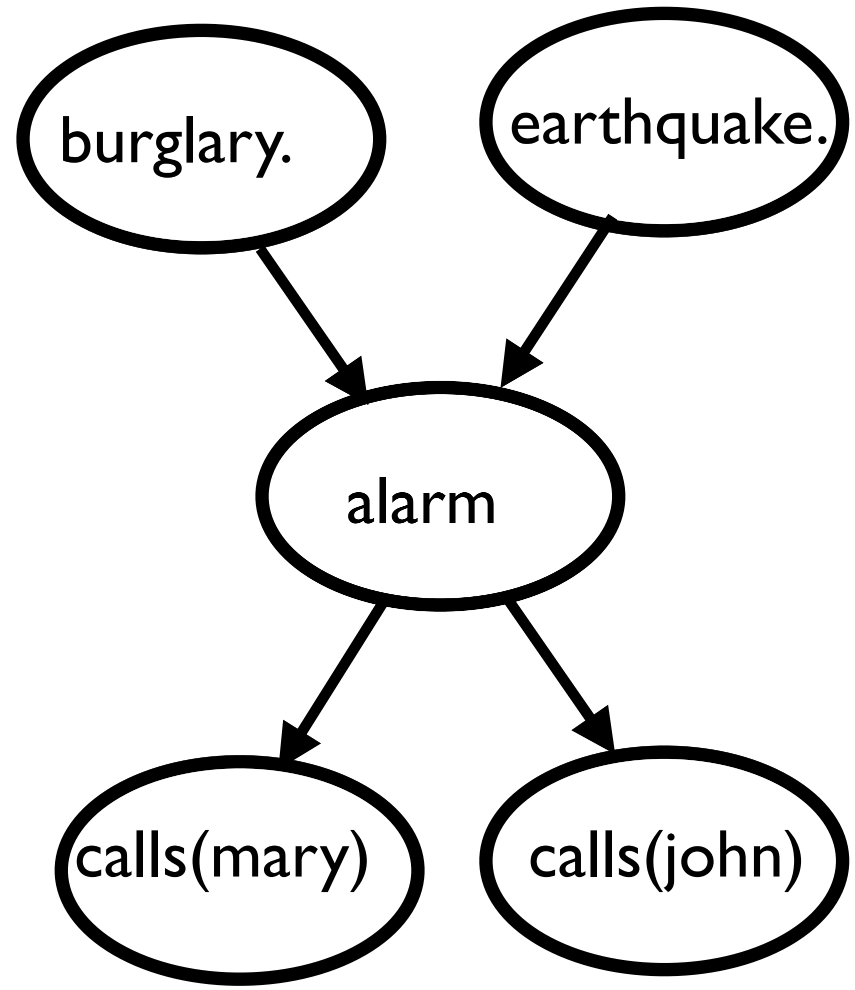
alarm :– earthquake.

alarm :– burglary.

0.7::calls(mary) :– alarm.

0.6::calls(john) :– alarm.

Bayesian Network

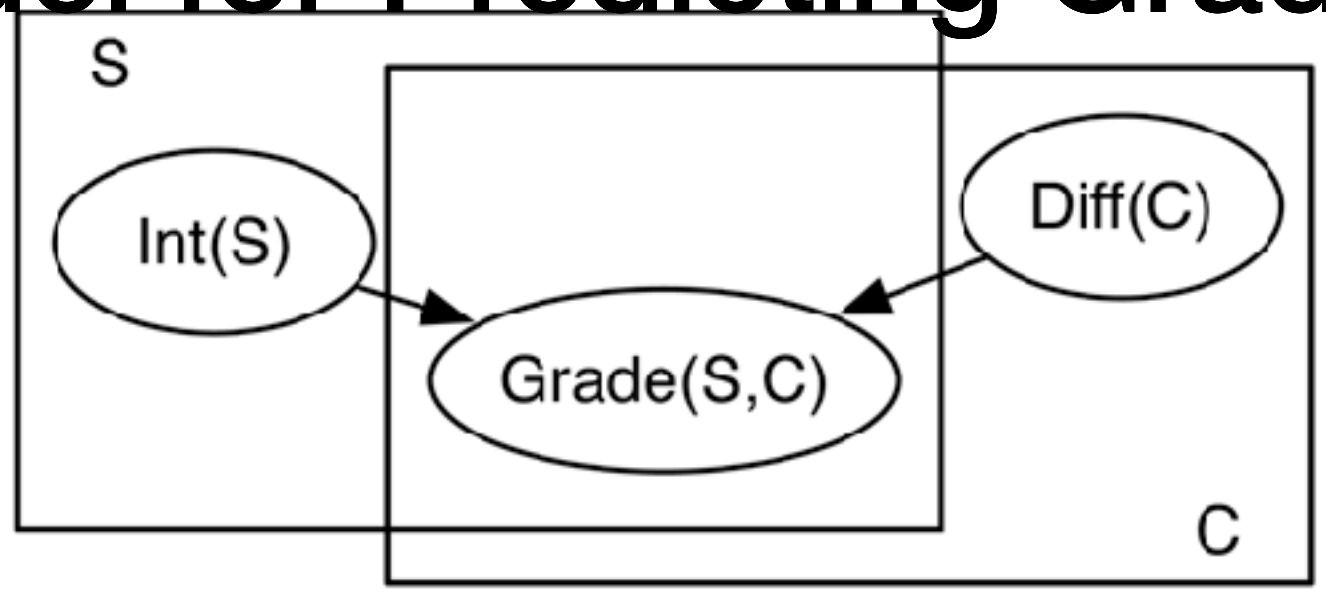


Bayesian net encoded as Probabilistic Logic Program
PLPs correspond to directed graphical models

ProbLog has both (directed) probabilistic graphic models, the programming language Prolog (and probabilistic databases) as special case

LOGIC PROBABILITY

Flexible and Compact Relational Model for Predicting Grades



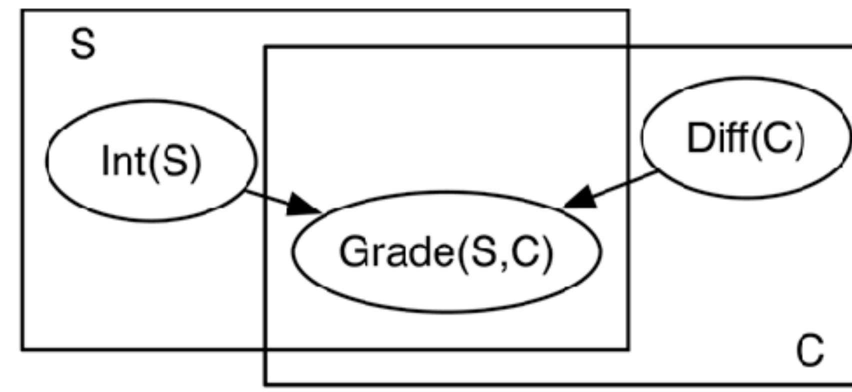
"Program" Abstraction:

- S, C logical variable representing students, courses
- the set of individuals of a type is called a population
- Int(S), Grade(S, C), D(C) are parametrized random variables

Grounding:

- for every student s, there is a random variable Int(s)
- for every course c, there is a random variable Di(c)
- for every s, c pair there is a random variable Grade(s,c)
- all instances share the same structure and parameters

Probabilistic Logic Programs



0.4 :: int(S) :- student(S).
0.5 :: diff(C):- course(C).

student(john). student(anna). student(bob).
course(ai). course(ml). course(cs).

gr(S,C,a) :- int(S), not diff(C).
0.3::gr(S,C,a); 0.5::gr(S,C,b);0.2::gr(S,C,c) :- int(S), diff(C).
0.1::gr(S,C,b); 0.2::gr(S,C,c); 0.2::gr(S,C,f) :-
student(S), course(C),
not int(S), not diff(C).
0.3::gr(S,C,c); 0.2::gr(S,C,f) :-
not int(S), diff(C).

ProbLog by example: Grading

```
unsatisfactory(S) :- student(S), grade(S,C,f).

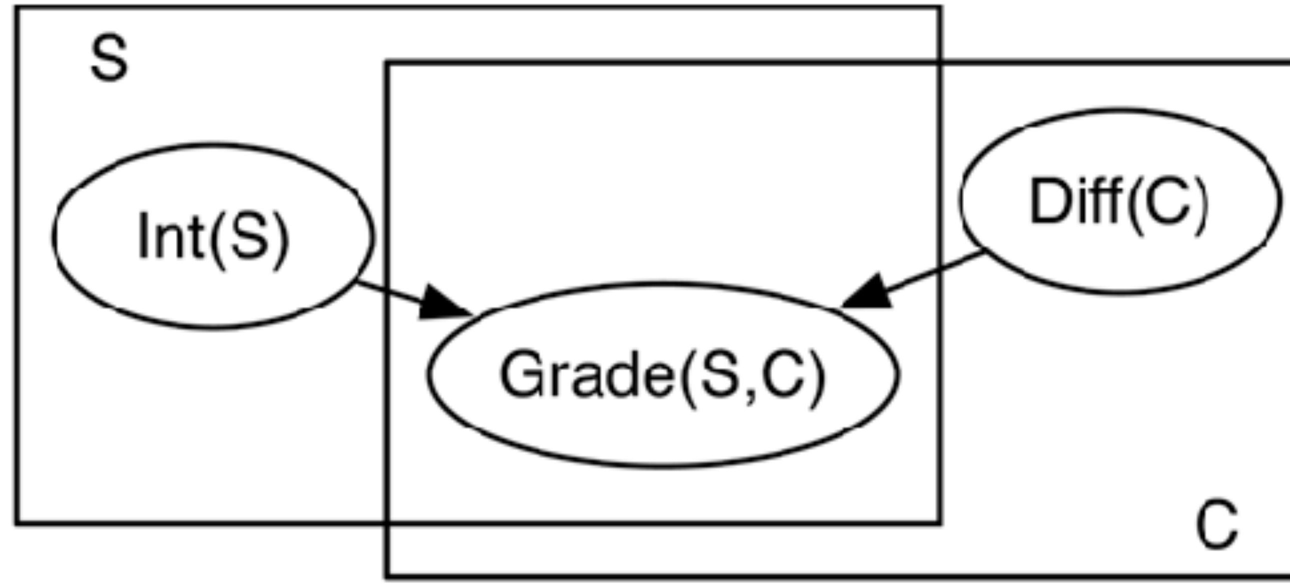
excellent(S):- student(S), not(grade(S,C1,G),below(G,a)), grade(S,C2,a).
```

```
0.4 :: int(S) :- student(S).
0.5 :: diff(C):- course(C).

student(john).  student(anna).  student(bob).
course(ai).     course(ml).     course(cs).

gr(S,C,a) :- int(S), not diff(C).
0.3::gr(S,C,a); 0.5::gr(S,C,b);0.2::gr(S,C,c) :-   int(S), diff(C).
0.1::gr(S,C,b); 0.2::gr(S,C,c); 0.2::gr(S,C,f) :-
        student(S), course(C),
        not int(S), not diff(C).
0.3::gr(S,C,c); 0.2::gr(S,C,f) :-  not int(S), diff(C).
```

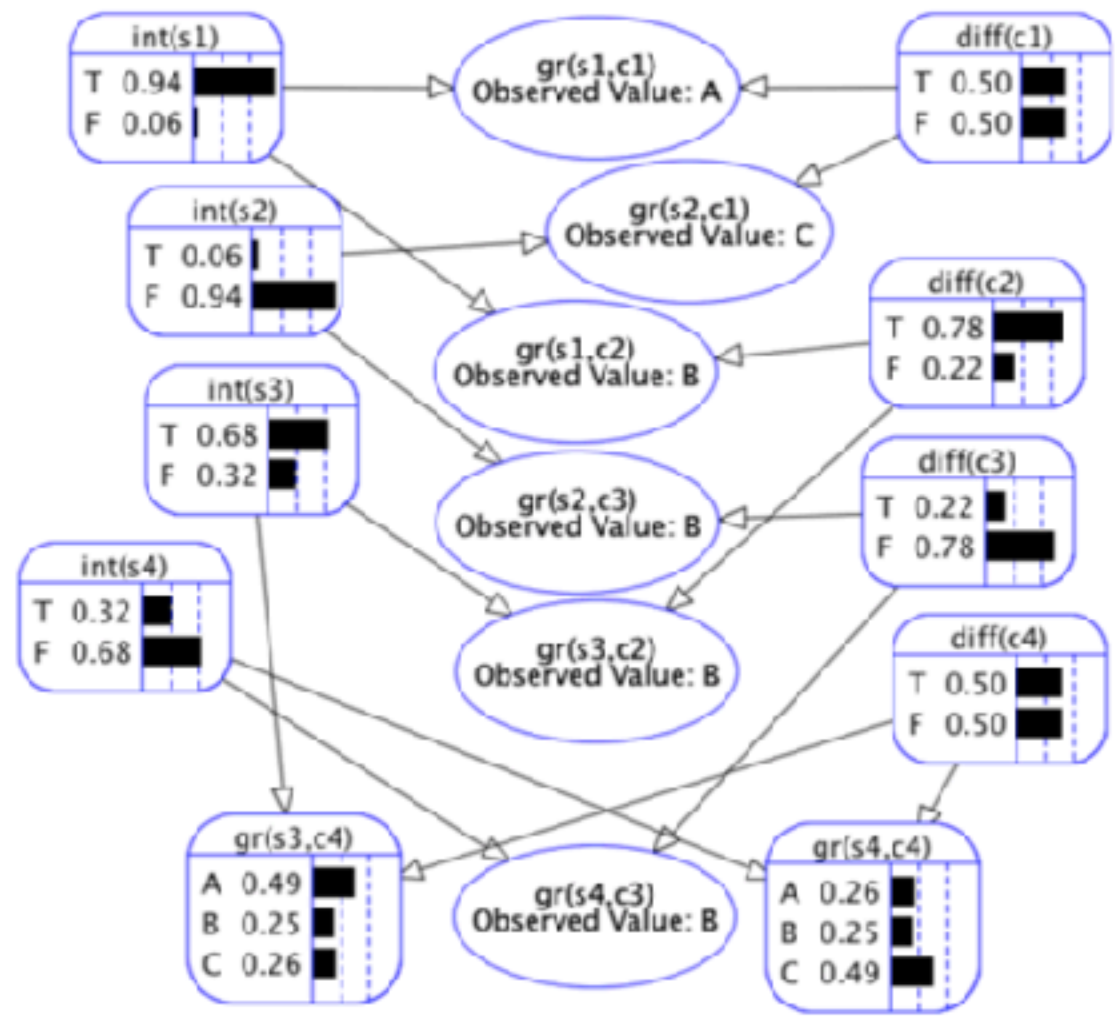
ProbLog by example: Grading



Shows relational structure

grounded model: replace variables by constants

Works for any number of students / classes (for 1000 students and 100 classes, you get 101100 random variables); still only few parameters



With SRL / PP

build and learn compact models,

from one set of individuals - > other sets;

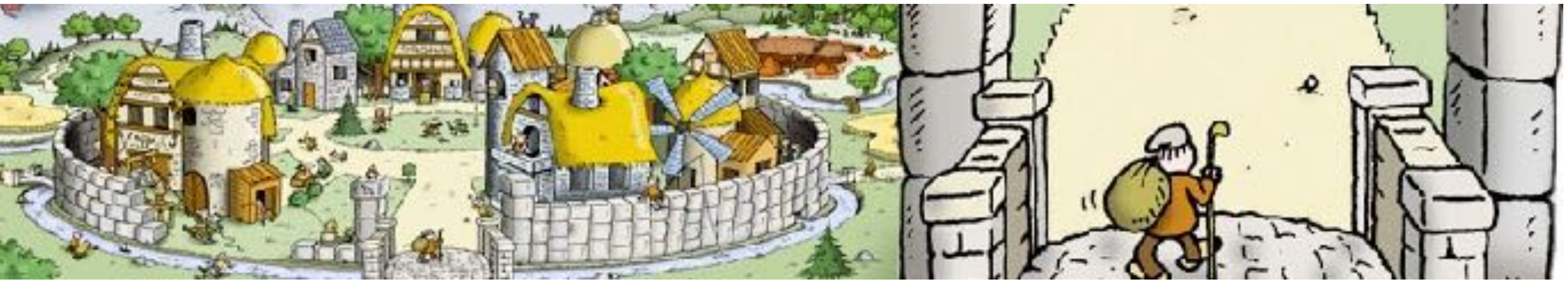
reason also about exchangeability,

build even more complex models,

incorporate background knowledge

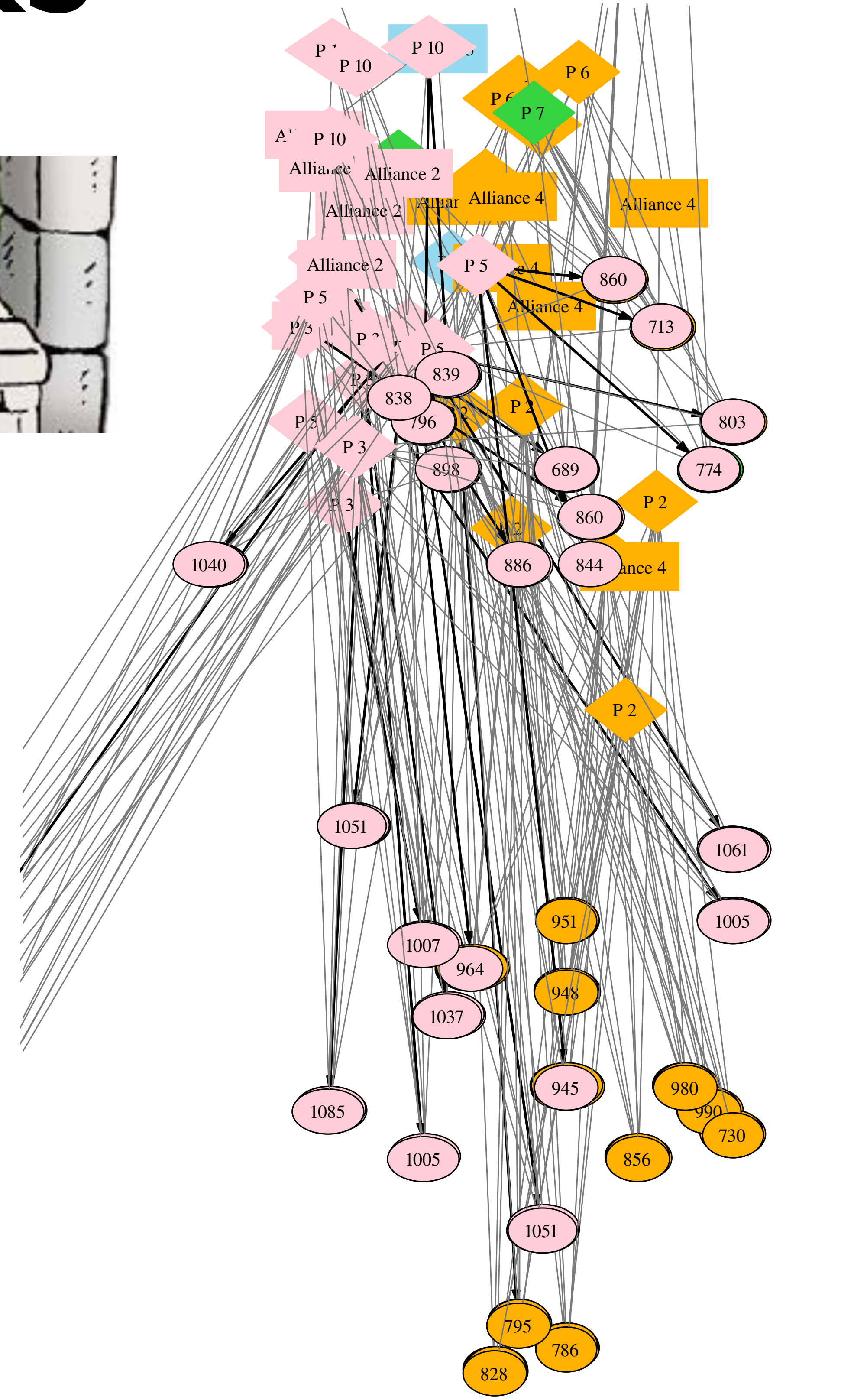
ProbLog applications

Dynamic networks

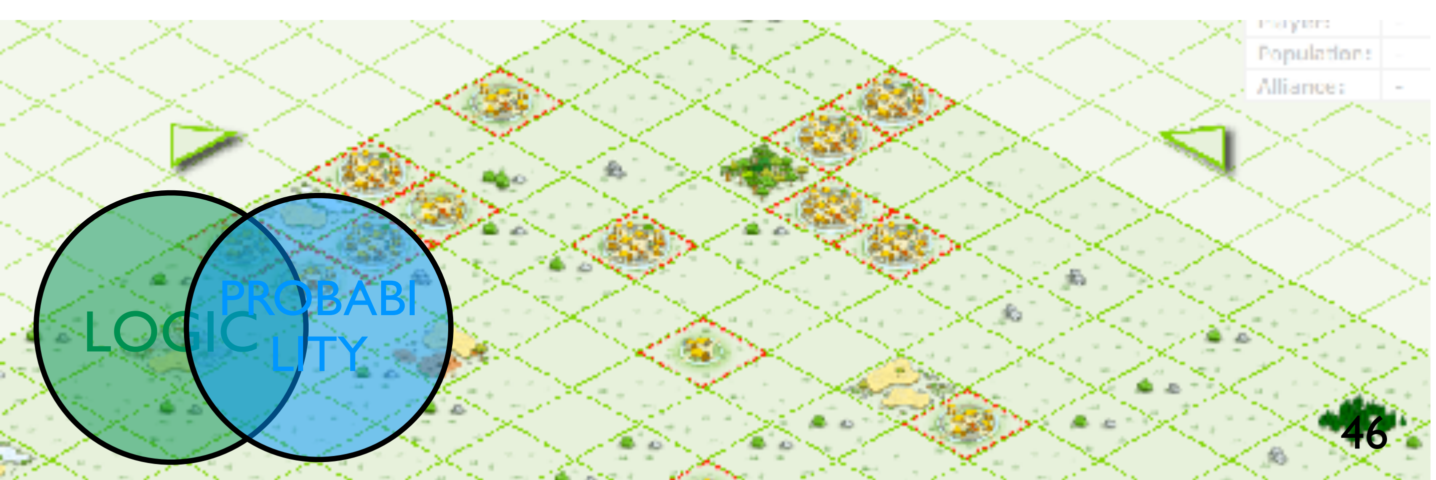


Travian: A massively multiplayer real-time strategy game

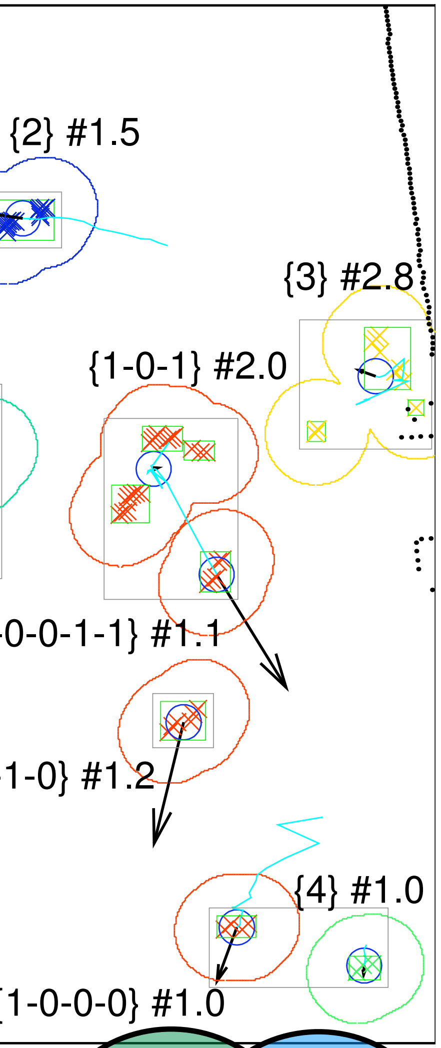
Can we build a model
of this world ?
Can we use it for playing
better ?

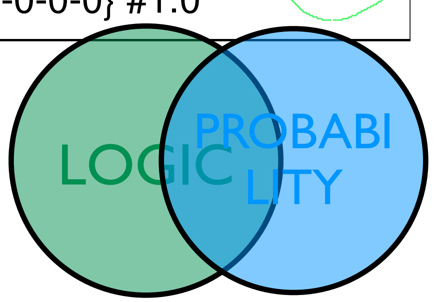


[Thon et al, MLJ 11]



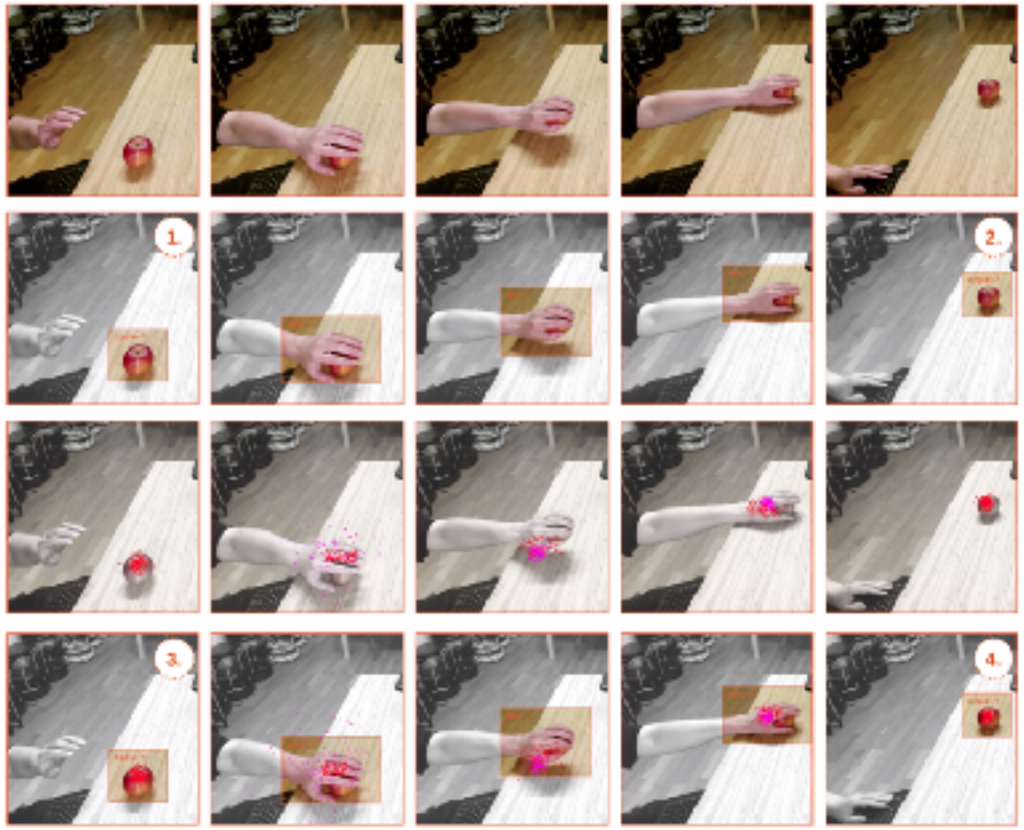
Activity analysis and tracking





- Track people or objects over time? Even if temporarily hidden?
- Recognize activities?
- Infer object properties?

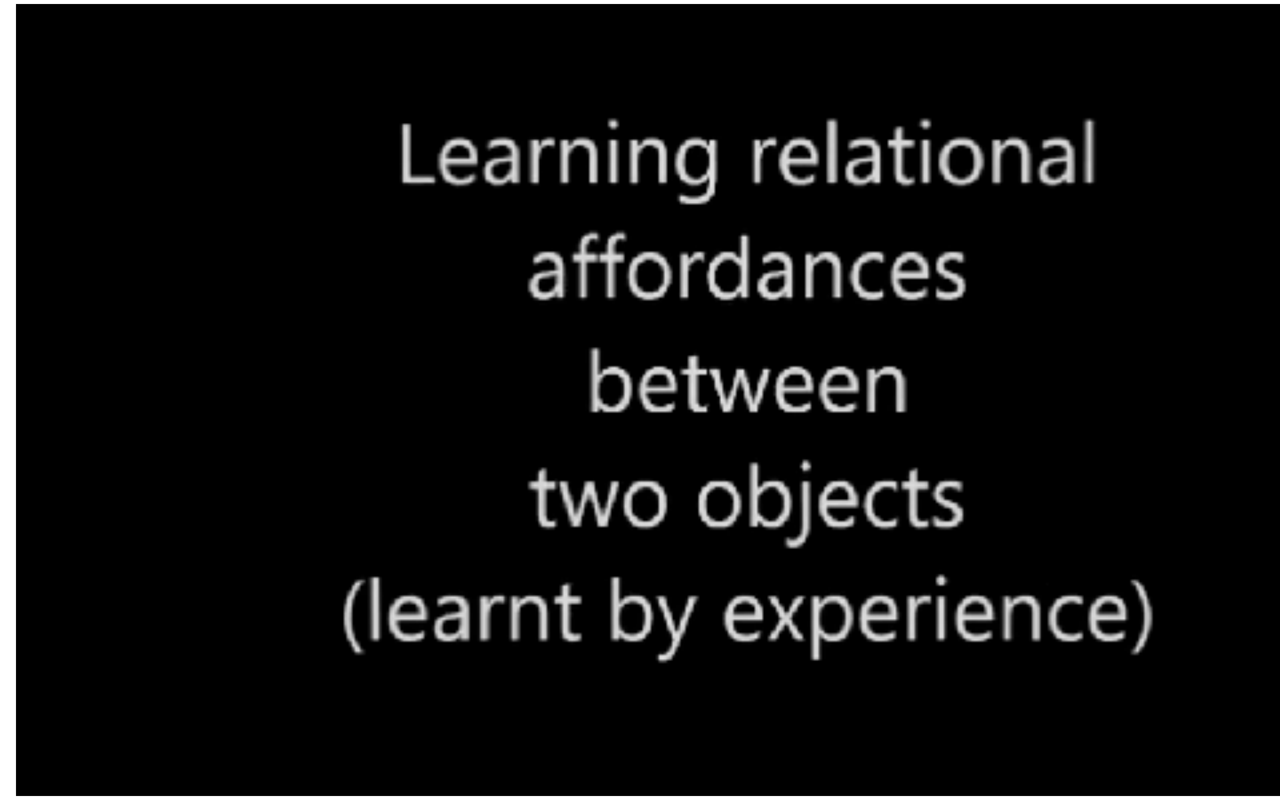
[Skarlatidis et al, TPLP 14; Nitti et al, IROS 13, ICRA 14, MLJ 16]



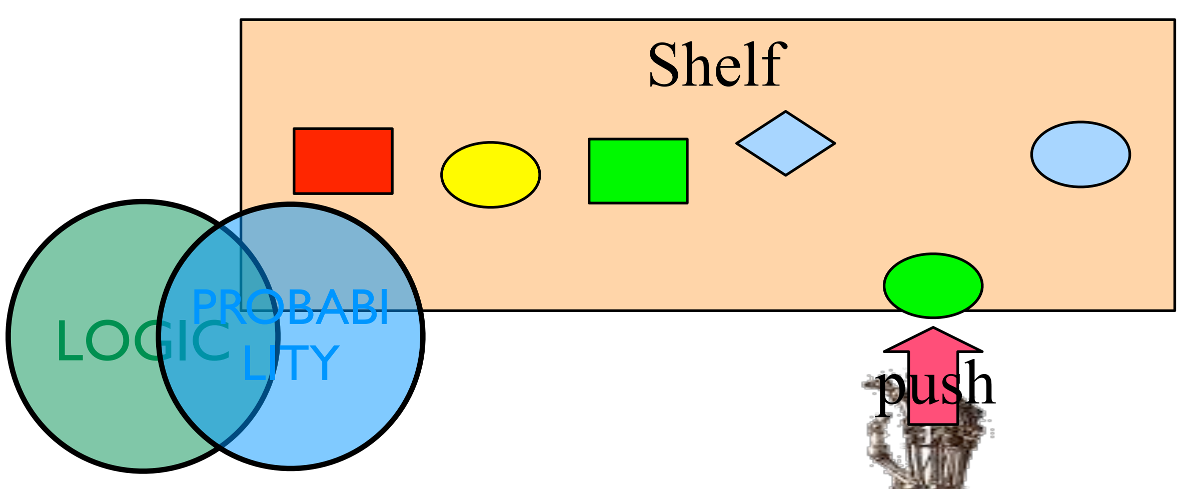
[Persson et al, IEEE Trans on Cogn. & Dev. Sys. 19; IJCAI 20]

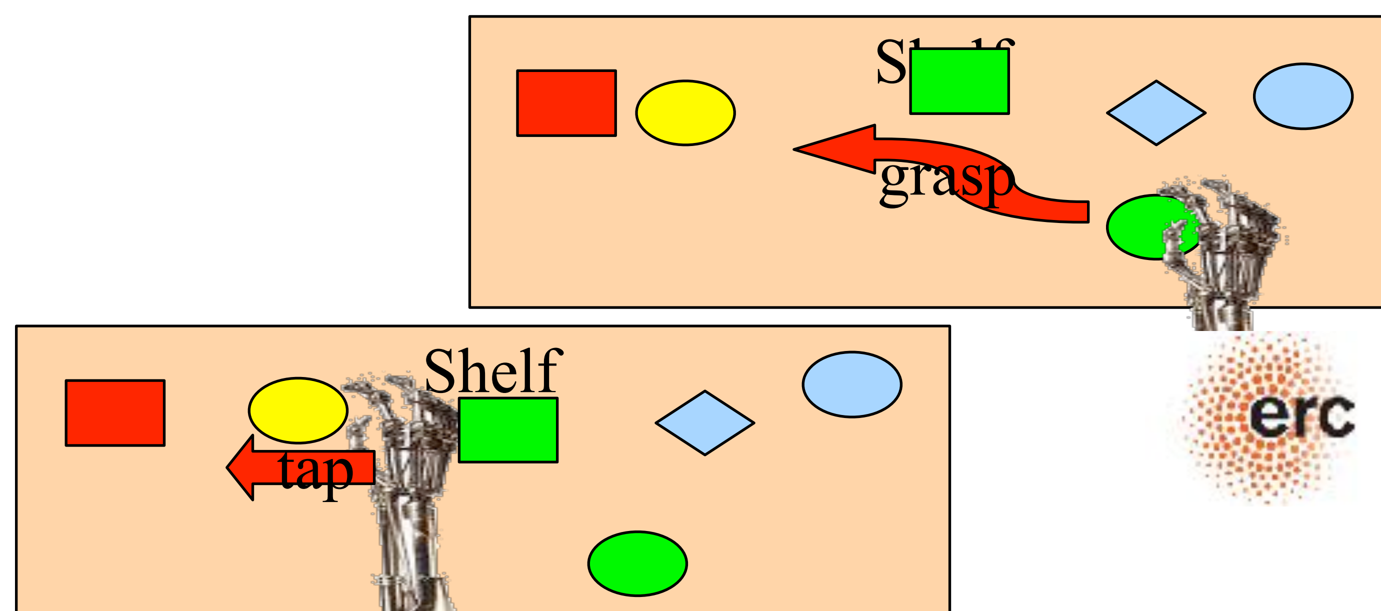
Learning relational affordances

similar to probabilistic Strips (with continuous distributions)



Moldovan et al. ICRA 12, 13, 14; Auton. Robots 18





Biology

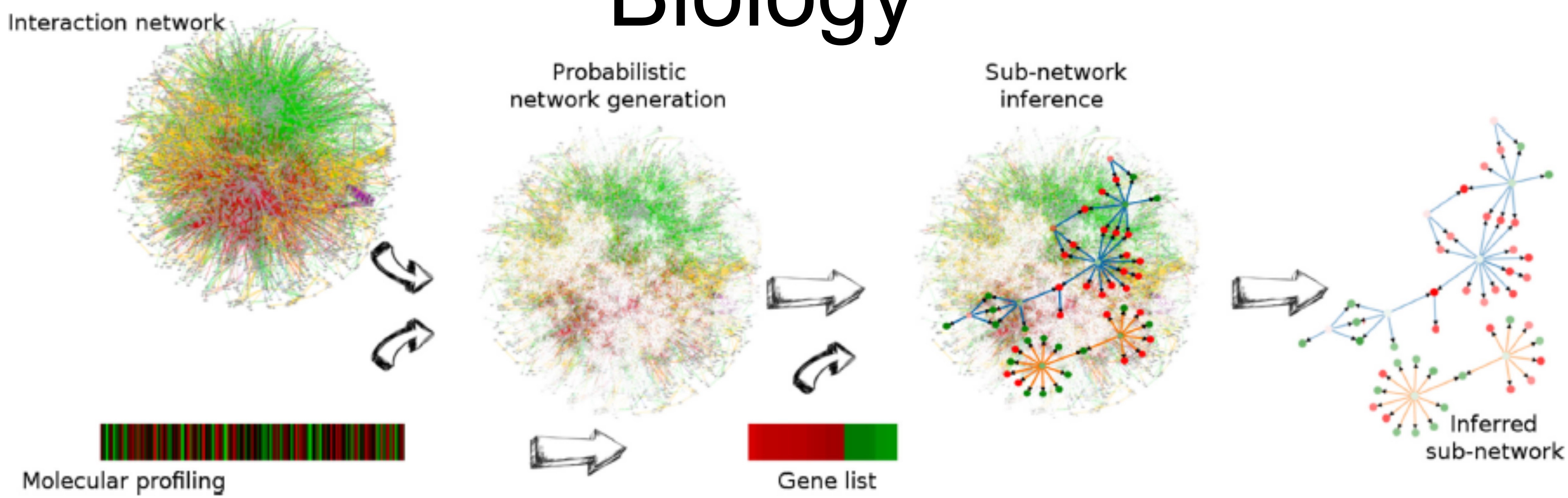


Figure 1. Overview of PheNetic, a web service for network-based interpretation of 'omics' data. The web service uses as input a genome wide interaction network for the organism of interest, a user generated molecular profiling data set and a gene list derived from these data. Interaction networks for a wide variety of organisms are readily available from the web server. Using the uploaded user-generated molecular data the interaction network is converted into a probabilistic network: edges receive a probability proportional to the levels measured for the terminal nodes in the molecular profiling data set. This probabilistic interaction network is used to infer the sub-network that best links the genes from the gene list. The inferred sub-network provides a trade-off between linking as many genes as possible from the gene list and selecting the least number of edges.

- Causes: Mutations
  - All related to similar phenotype
- Effects: Differentially expressed genes
- 27 000 cause effect pairs

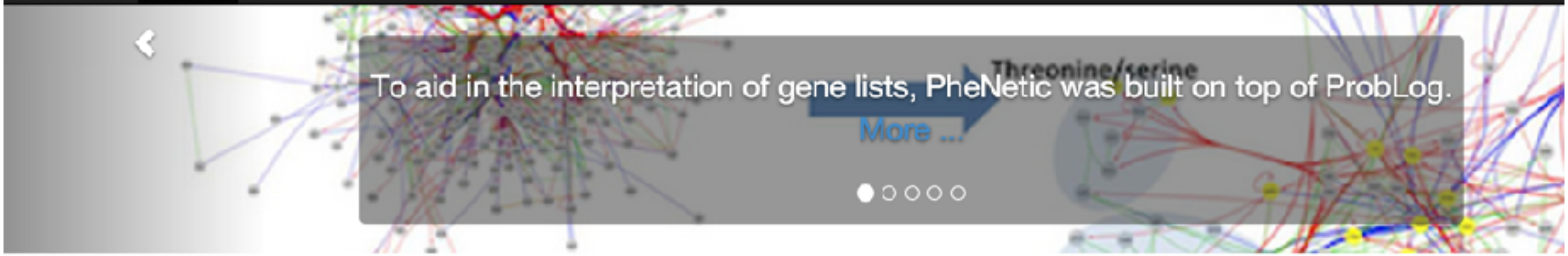
- Interaction network:
  - 3063 nodes
    - Genes
    - Proteins
  - 16794 edges
    - Molecular interactions
    - Uncertain

- Goal: connect causes to effects through common subnetwork
  - = Find mechanism
- Techniques:
  - DTProbLog
  - Approximate inference

LOGIC PROBABILITY

[De Maeyer et al., Molecular Biosystems 13, NAR 15] [Gross et al. Communications Biology, 19] https://dtai.cs.kuleuven.be/problog/

ProbLog | Home | Download | Publications | Help | People | Interactive



Introduction.

Probabilistic logic programs are logic programs in which some of the facts are annotated with probabilities.

ProbLog is a tool that allows you to intuitively build programs that do not only encode **complex interactions** between a large sets of **heterogenous components** bu **uncertainties** that are present in real-life situations.

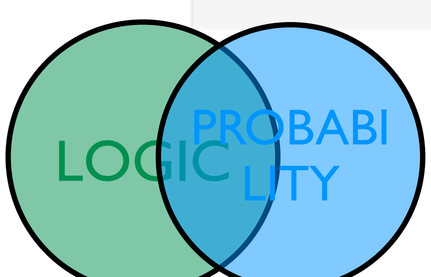
The engine tackles several tasks such as computing the marginals given evidence and learning from (partial) interpretations. ProbLog is a suite of efficient algorithms tasks. It is based on a conversion of the program and the queries and evidence to a weighted Boolean formula. This allows us to reduce the inference tasks to well-s weighted model counting, which can be solved using state-of-the-art methods known from the graphical model and knowledge compilation literature.

The Language. Probabilistic Logic Programming.

ProbLog makes it easy to express complex, probabilistic models.

```
0.3::stress(X) :- person(X).
0.2::influences(X,Y) :- person(X), person(Y).

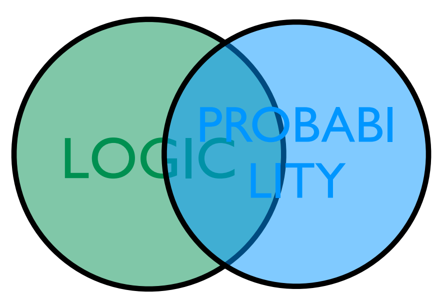
smokes(X) :- stress(X).
smokes(X) :- friend(X,Y), influences(Y,X), smokes(Y).
```



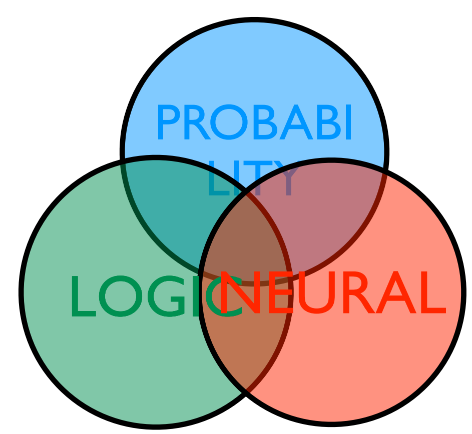
PART 2 B

From ProbLog to DeepProbLog

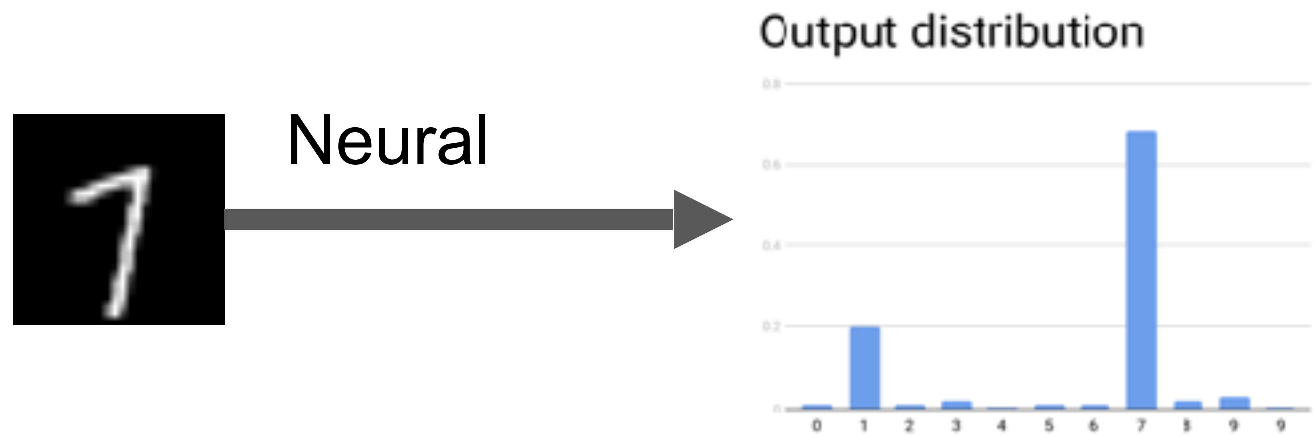
FROM



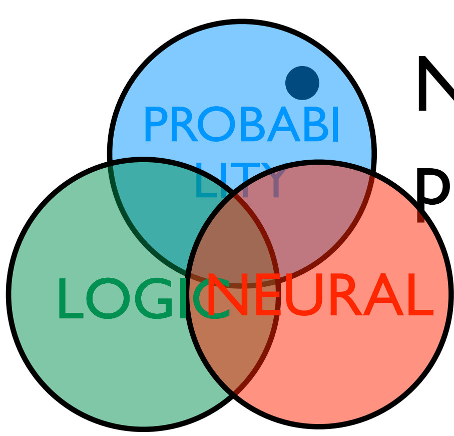
TO



Neural predicate



- Neural networks have uncertainty in their predictions
- A normalized output can be interpreted as a probability distribution
- Neural predicate models the output as probabilistic facts
- No changes needed in the probabilistic host language



Key Idea DeepProbLog

unify the basic concepts in logic and neural networks:

neural predicate ~ neural net

an interface between logic and neural nets

The neural predicate

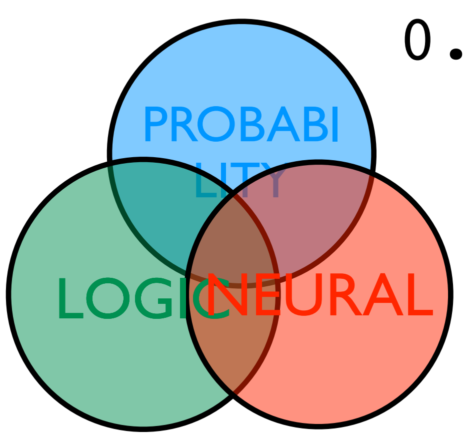
The output of the neural network is probabilistic facts in DeepProbLog

Example:

`nn(mnist_net, [X], Y, [0 ... 9] ) :: digit(X,Y).`

Instantiated into a (neural) Annotated Disjunction:

0.04::digit( ,0) ; 0.35::digit( ,1) ; ... ;
0.53::digit( ,7) ; ... ; 0.014::digit( ,9).



DeepProbLog exemplified: MNIST addition

Task: Classify pairs of MNIST digits with their sum

8

4

11

Benefit of DeepProbLog:

- Encode addition in logic
- Separate addition from digit classification

```
nn(mnist_net, [X], Y, [0 ... 9] ) :: digit(X,Y).

addition(X,Y,Z) :- digit(X,N1), digit(Y,N2), Z is N1+N2.
```

Examples:

addition( ,  ,8), addition( ,  ,4), addition( ,  ,11), …

DeepProbLog exemplified: MNIST addition

Task: Classify pairs of MNIST digits with their sum

Benefit of DeepProbLog:

- Encode addition in logic
- Separate addition from digit classification

```
nn(mnist_net, [X], Y, [0 ... 9] ) :: digit(X,Y).

addition(X,Y,Z) :- digit(X,N1), digit(Y,N2), Z is N1+N2.

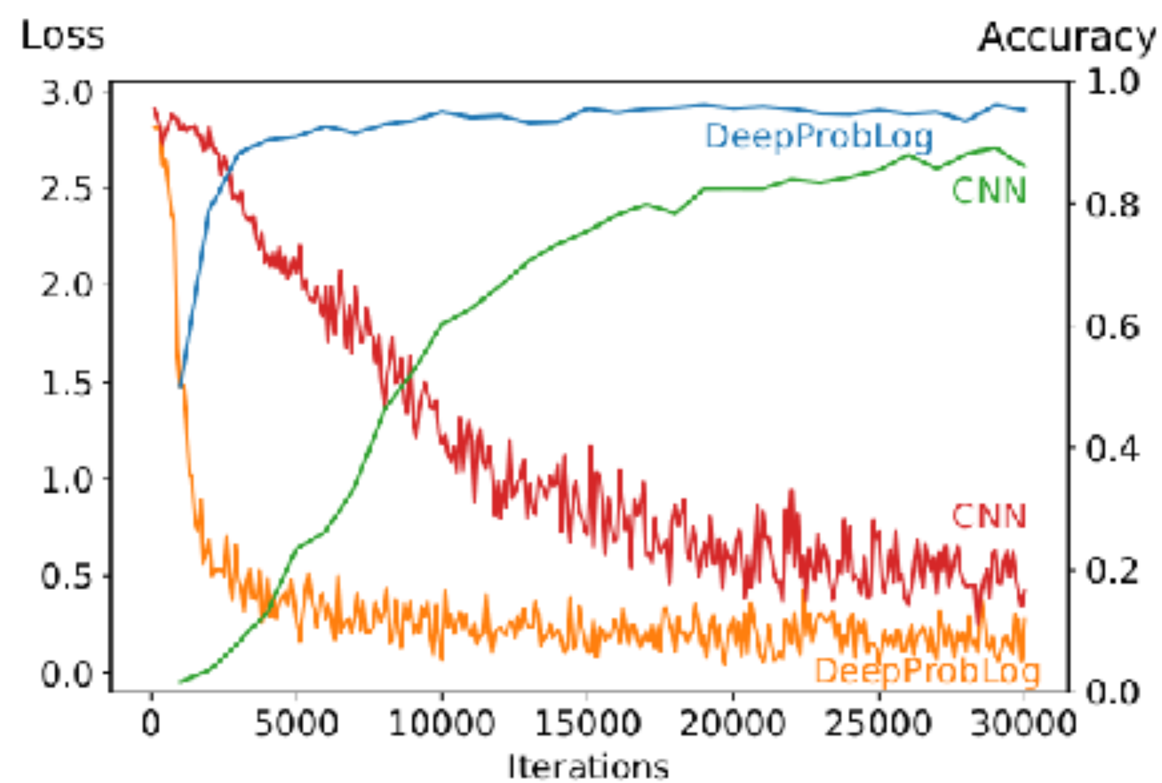
addition( , ,8) :- digit( ,N1), digit( ,N2), 8 is N1 + N2.
```

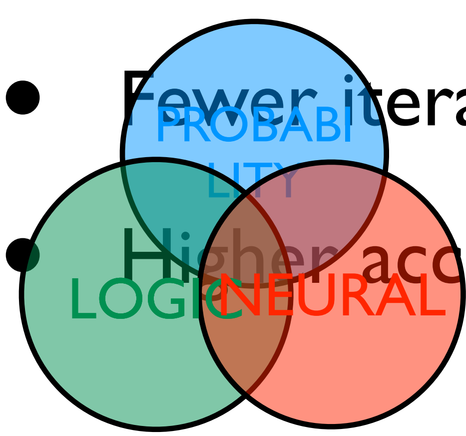
Examples:

```
addition( , ,8), addition( , ,4), addition( , ,11), …
```

MNIST Addition

- Pairs of MNIST images, labeled with sum
- Baseline: CNN
  - Classifies concatenation of both images into classes 0 ...18
- DeepProbLog:
  - CNN that classifies images into 0 … 9
  - Two lines of DeepProblog code
- Result:
- Fewer iterations necessary
- Higher accuracy achieved





Example

Learn to classify the sum of pairs of MNIST digits

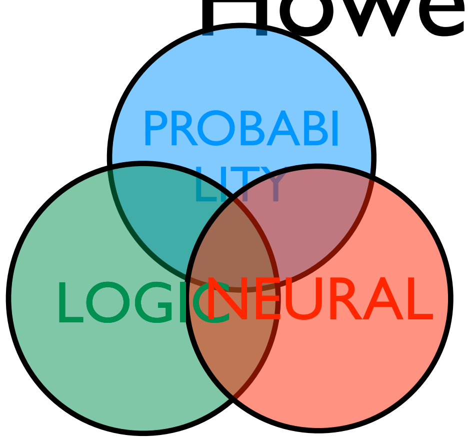
Individual digits are not labeled!

E.g. ( ,  , 8)

Could be done by a CNN: classify the concatenation of both images into 19 classes

However:

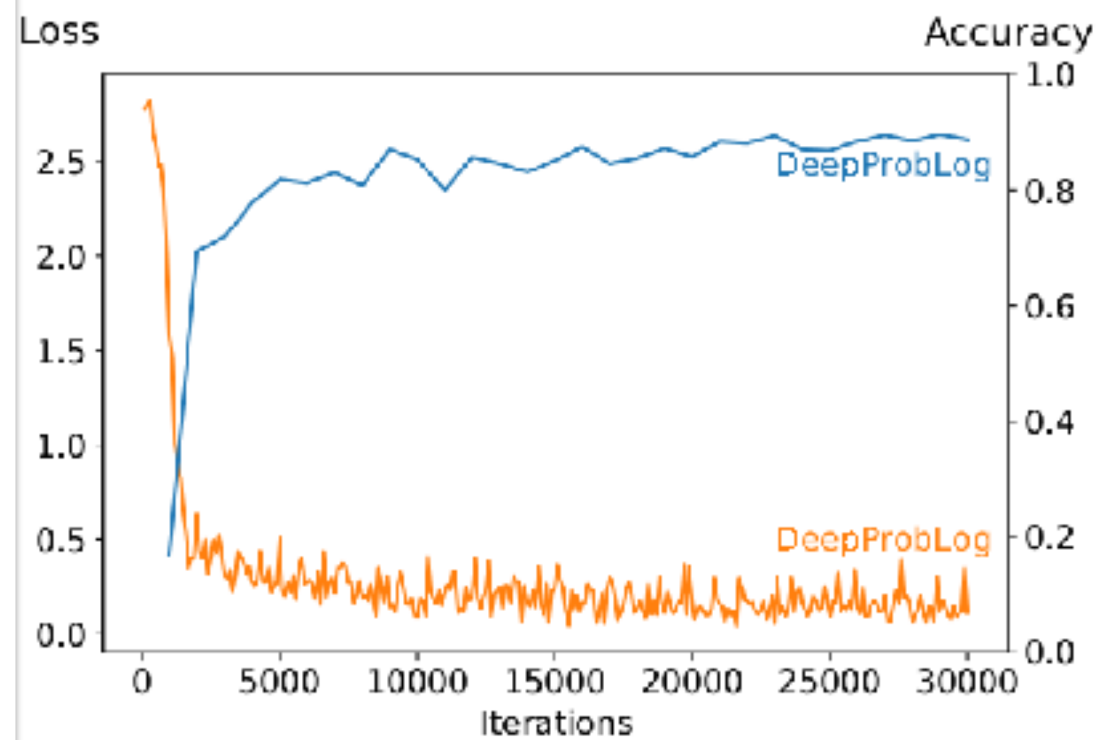
= ?



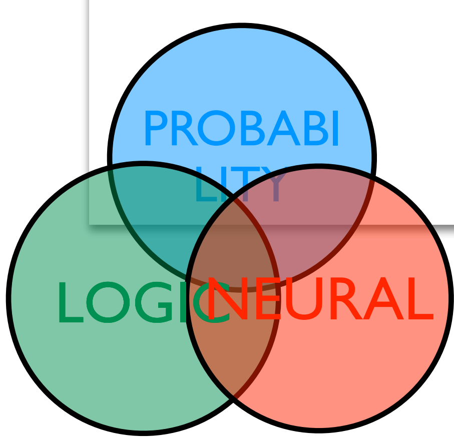
Multi-digit MNIST addition with MNIST

```
number ( [ ] , Result , Result ) .
number ( [H | T ] , Acc , Result) :–
  digit(H, Nr ), Acc2 is Nr +10*Acc ,
  number ( T , Acc2 , Result ) .
number (X,Y) :– number (X, 0 ,Y ) .

multiaddition(X, Y, Z ) :–
     number (X, X2 ) ,
     number (Y, Y2 ) ,
     Z is X2+Y2 .
```



(b) Multi-digit (**T2**)



Noisy Addition

```
nn(classifier, [X], Y, [0 .. 9]) :: digit(X,Y).
t(0.2) :: noisy.

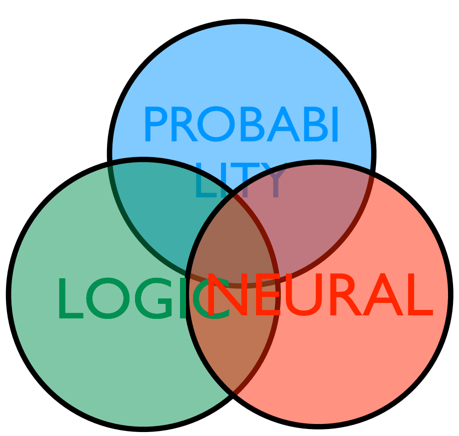
1/19 :: uniform(X,Y,0) ; ... ; 1/19 :: uniform(X,Y,18).

addition(X,Y,Z) :- noisy, uniform(X,Y,Z).
addition(X,Y,Z) :- \+noisy, digit(X,N1), digit(Y,N2), Z is N1+N2.
```

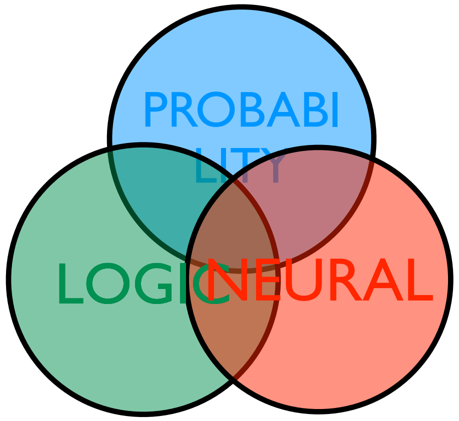
(a) The DeepProbLog program.

	Fraction of noise					
	0.0	0.2	0.4	0.6	0.8	1.0
Baseline	93.46	87.85	82.49	52.67	8.79	5.87
DeepProbLog	97.20	95.78	94.50	92.90	46.42	0.88
DeepProbLog w/ explicit noise	96.64	95.96	95.58	94.12	73.22	2.92
Learned fraction of noise	0.000	0.212	0.415	0.618	0.803	0.985

Table 3: The accuracy on the test set for **T4**.



Inference & Learning



ProbLog Inference

Answering a query in a ProbLog program happens in four steps

1. Grounding the program w.r.t. the query
2. **Rewrite the ground logic program into a propositional logic formula**
3. Compile the formula into an arithmetic circuit
4. Evaluate the arithmetic circuit

```
0.1 :: burglary.
0.5 :: hears_alarm(mary).

0.2 :: earthquake.
0.4 :: hears_alarm(john).

alarm :– earthquake.

alarm :– burglary.
calls(mary) :– alarm, hears_alarm(mary).

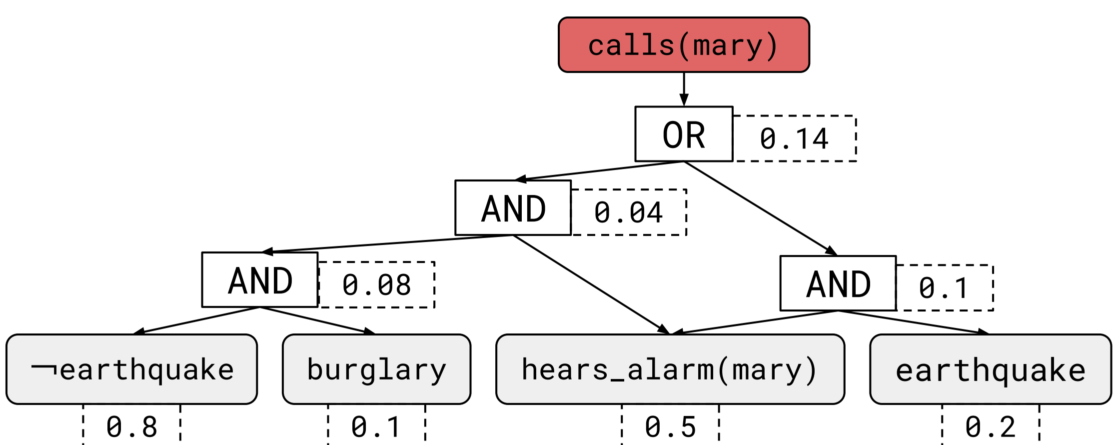
calls(john) :– alarm, hears_alarm(john).
```

$$\text{calls(mary)} \leftrightarrow \text{hears\_alarm(mary)} \land (\text{burglary} \lor \text{earthquake})$$

ProbLog Inference

Answering a query in a ProbLog program happens in four steps

1. Grounding the program w.r.t. the query
2. Rewrite the ground logic program into a propositional logic formula
3. **Compile the formula into an arithmetic circuit (knowledge compilation)**
4. Evaluate the arithmetic circuit

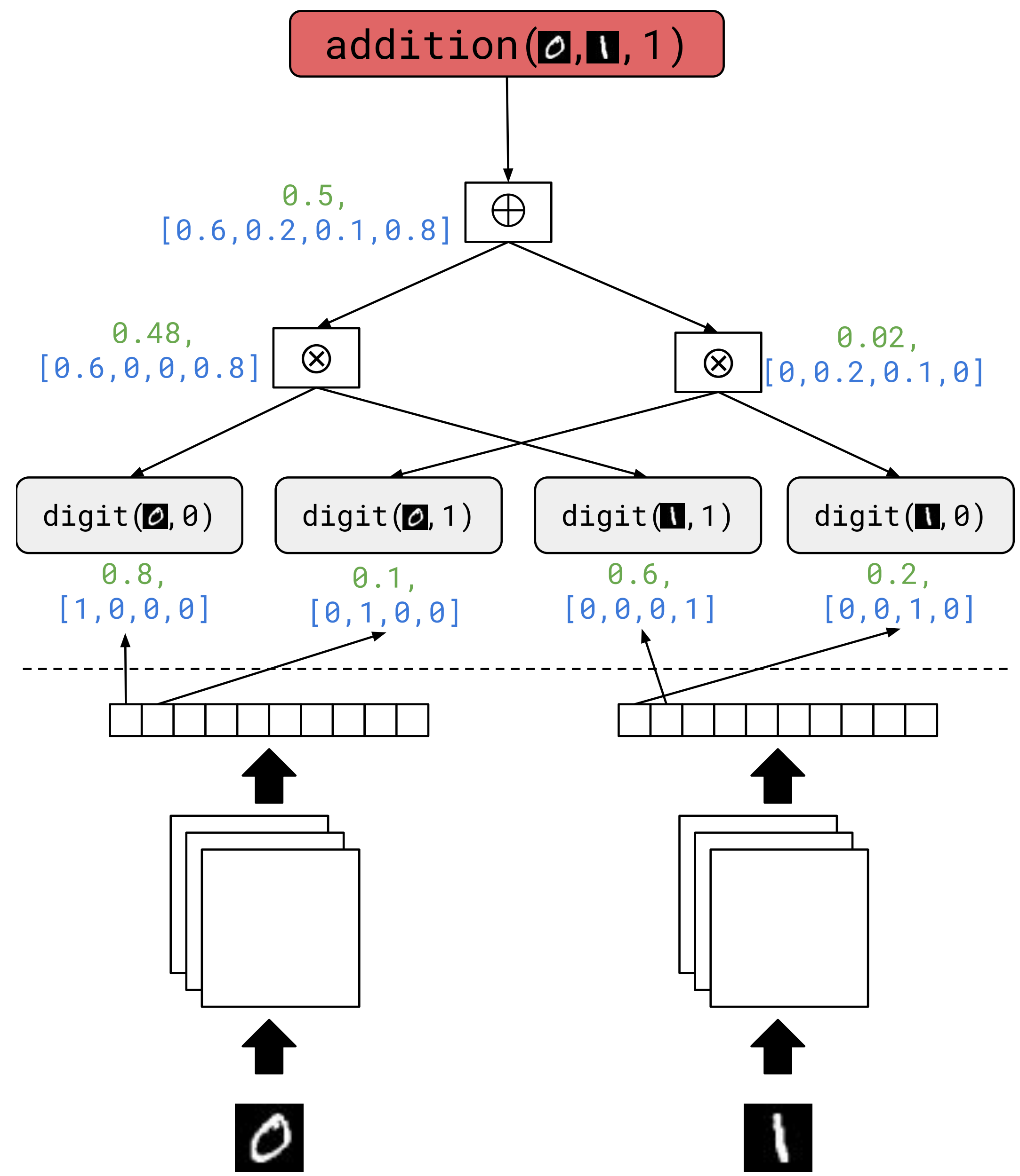


$$\text{calls(mary)} \leftrightarrow \text{hears\_alarm(mary)} \wedge (\text{burglary} \vee \text{earthquake})$$

Gradient Semiring

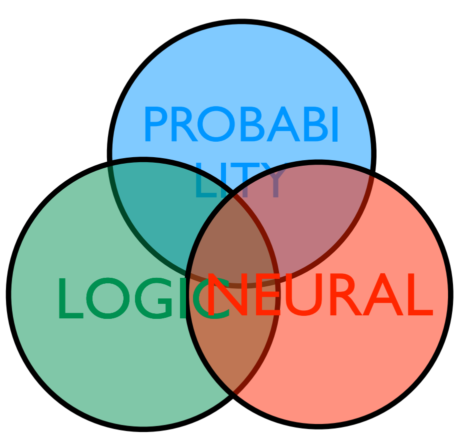
```
nn(mnist_net, [X], Y, [0 ... 9] ) ::
      digit(X,Y).

addition(X,Y,Z) :-
   digit(X,N1),
   digit(Y,N2),
   Z is N1+N2.
```

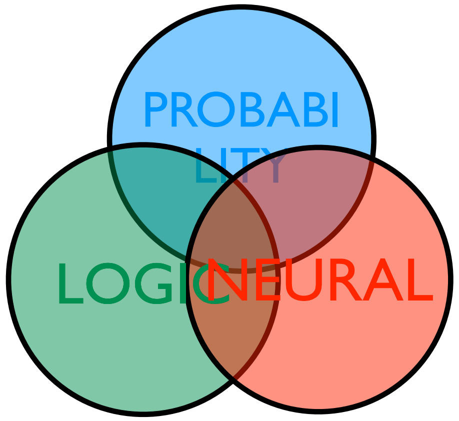


The ACs are differentiable
and there is an interface
with the neural nets

(Pretty elegant in ProbLog
we use the "gradient" semi-ring)



Experiments



Program Induction/Sketching

In Neural Symbolic methods

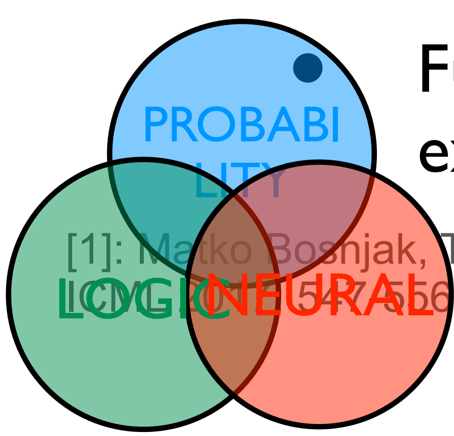
- Rule Induction — work with templates

  P(X) :- R(X,Y), Q(Y)

- and have the "predicate" variables / slots P,Q, R determined by the NN
- Simpler form, fill just a few slots / holes

Approach similar to '*Programming with a Differentiable Forth Interpreter*' [1] ∂4

- Partially defined Forth program with slots / holes
- Slots are filled by neural network (encoder / decoder)
- Fully differentiable interpreter: NNs are trained with input / output examples



[1]: Matko Bosnjak, Tim Rocktäschel, Jason Naradowsky, Sebastian Riedel: Programming with a Differentiable Forth Interpreter. ICML 2017: 547-556

Example DeepProbLog

neural predicate

```
hole(X,Y,X,Y):-
    swap(X,Y,0).

hole(X,Y,Y,X):-
    swap(X,Y,1).
```

bubble sort

```
bubble([X],[],X).
bubble([H1,H2|T],[X1|T1],X):-
    hole(H1,H2,X1,X2),
    bubble([X2|T],T1,X).

bubblesort([],L,L).

bubblesort(L,L3,Sorted) :-
    bubble(L,L2,X),
    bubblesort(L2,[X|L3],Sorted).

sort(L,L2) :- bubblesort(L,[],L2).
```

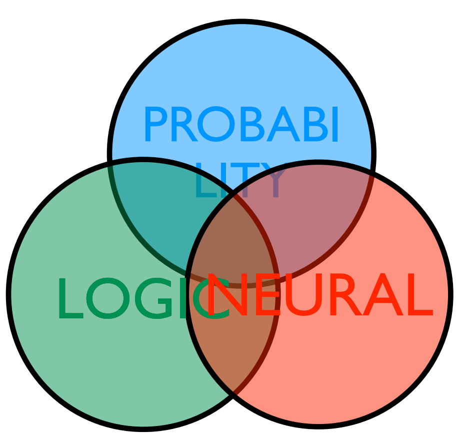
		Sorting: Training length					Addition: training length		
	Test Length	2	3	4	5	6	2	4	8
$\partial 4$ [Bošnjak et al., 2017]	8	100.0	100.0	49.22	–	–	100.0	100.0	100.0
	64	100.0	100.0	20.65	–	–	100.0	100.0	100.0
DeepProbLog	8	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	64	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

(a) Accuracy on the sorting and addition problems (results for $\partial 4$ reported by Bošnjak et al. [2017]).

Training length ⟶	2	3	4	5	6
$\partial 4$ on GPU	42 s	160 s	–	–	–
$\partial 4$ on CPU	61 s	390 s	–	–	–
DeepProbLog on CPU	11 s	14 s	32 s	114 s	245 s

(b) Time until 100% accurate on test length 8 for the sorting problem.

Table 1: Results on the Differentiable Forth experiments

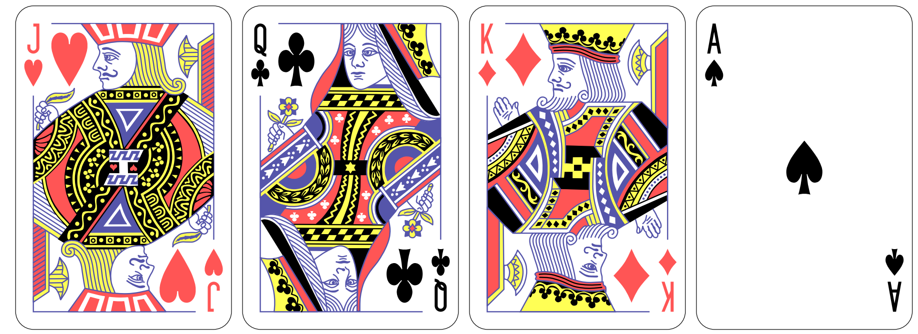


Tasks[1]

- Sorting
  - Sort lists of numbers using Bubble sort
  - Hole: Swap or don't swap when comparing two numbers
- Addition
  - Add two numbers and a carry
  - Hole: What is the resulting digit and carry on each step
  - (Note: not MNIST digits, but actual numbers)
- Word Algebra Problems
  - E.g. "Ann has 8 apples. She buys 4 more. She distributes them equally among her 3 kids. How many apples does each child receive?
  - Hole: Sequence of permuting, swapping and performing operations on the three numbers

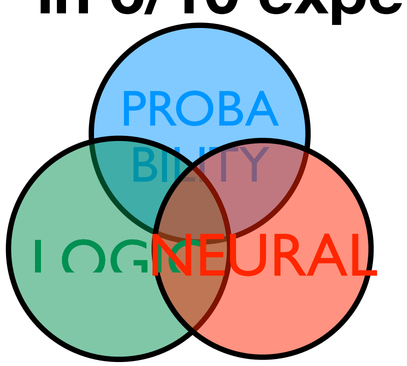
[1]: Matko Bošnjak, Tim Rocktäschel, Jason Naradowsky, Sebastian Riedel: Programming with a Differentiable Forth Interpreter. ICML 2017: 547-556

Simplified Poker



- dealing with uncertainty
- ignore suits and just with A, J, Q and K
- two players, two cards, and one community card
  - train the neural network to recognize the four cards
  - reason probabilistically about the non-observed card
  - learn the distribution of the unlabeled community card
- `0.8 :: poker([Q♡, Q♢, A♢, K♣], loss)` `poker([Q♡, Q♢, A♢, K♣], A♢, loss).`

in 6/10 experiments



Distribution	Jack	Queen	King	Ace
Actual	0.2	0.4	0.15	0.25
Learned	0.203 ± 0.002	0.396 ± 0.002	0.155 ± 0.003	0.246 ± 0.002

Table 8: The results for the Poker experiment (**T9**).

Neural Theorem Prover

Towards Neural Theorem Proving at Scale

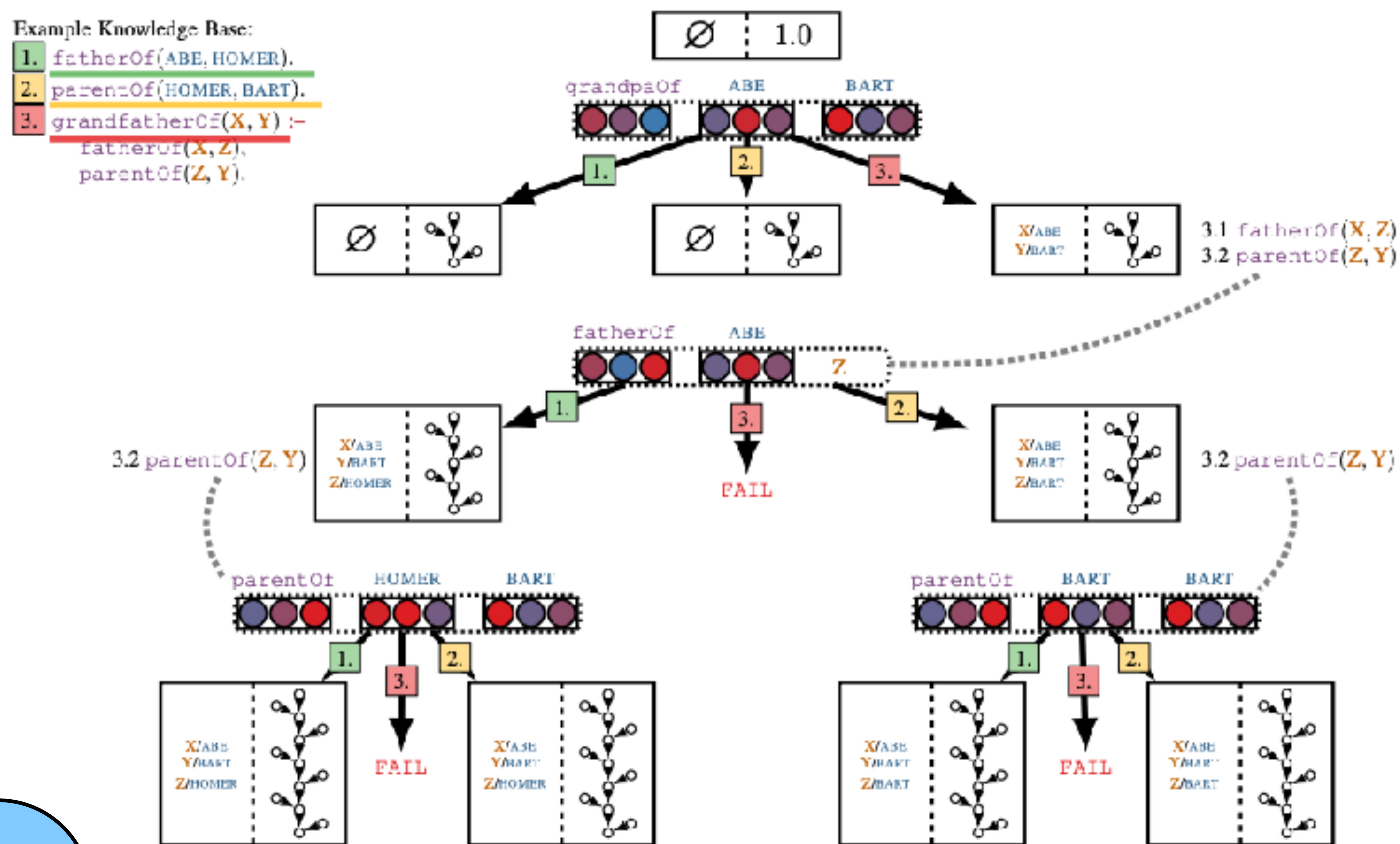
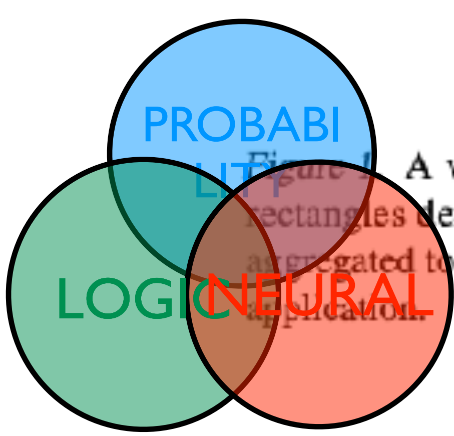


Figure 1: A visual depiction of the NTP' recursive computation graph construction, applied to a toy KB (top left). Dash-separated rectangles denote proof states (left: substitutions, right: proof score -generating neural network). All the non-FAIL proof states are aggregated to obtain the final proof success (depicted in Figure 2). Colours and indices on arrows correspond to the respective KB rule application.



Minervini Bosnjak Rocktäschel Riedel

Soft Unification

- NTP : "grandpa" **softly unifies** with "grandfather", as embeddings are close
- DeepProblog : define

softunification(X,Y) :- embed(X,EX), embed(Y,EY), rbf(EX,EY).

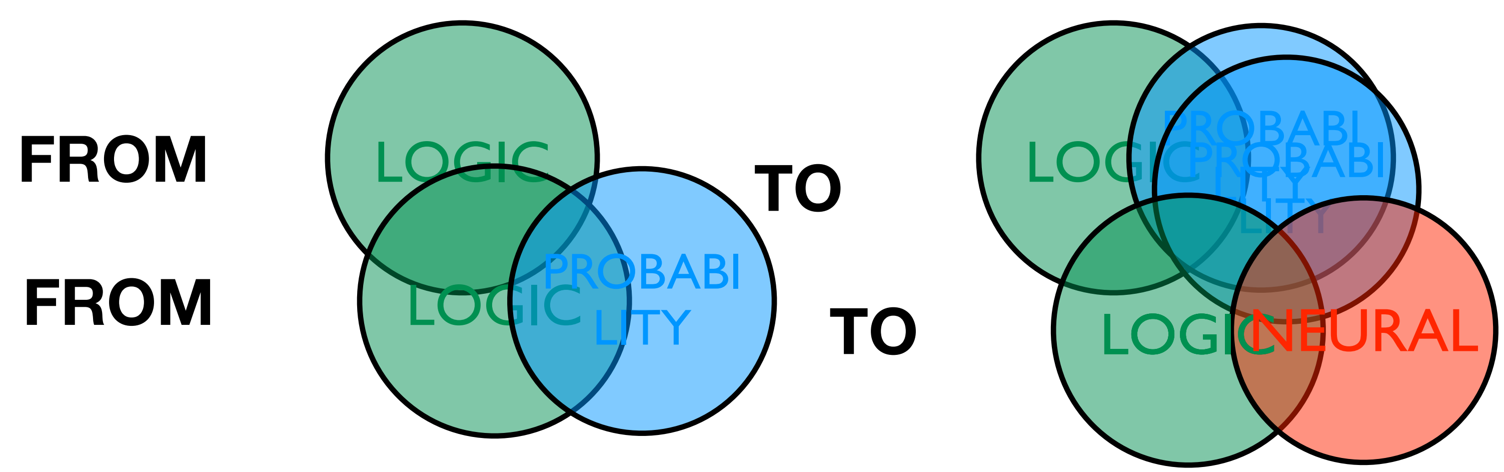
softunification(X,Y) returns 1 if X and Y unify

otherwise returns $exp(\frac{-||e_X - e_Y||_2}{2\mu^2})$

grandPaOf(X,Y) :- softunification(grandPaOf,R), R(X,Y).

PART 2c

From PCFGs to DeepStochLog



One NeSy Recipe

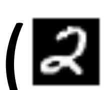
1. Take a symbolic (logic / rule based) representation
2. Turn the 0/1 True/False in Fuzzy or Probabilistic Interpretation
3. Interpret neural networks as logical predicates/functions
4. (The harder part): inference and learning

For instance:

map an MNIST image to a number

m() = 2

m as a neural network

mp(,2) =0.93 as a neural predicate

(with a fuzzy/prob. interpretation)

DeepStochLog

- Little sibling of DeepProbLog [Winters, Marra, et al AAAI 22]
- Based on a different semantics
  - probabilistic graphical models vs grammars
  - random graphs vs random walks
- Underlying StarAI representation is Stochastic Logic Programs (Muggleton, Cussens)
  - close to Probabilistic Definite Clause Grammars, ako probabilistic unification based grammar formalism
  - again the idea of neural predicates
- Scales better, is faster than DeepProbLog

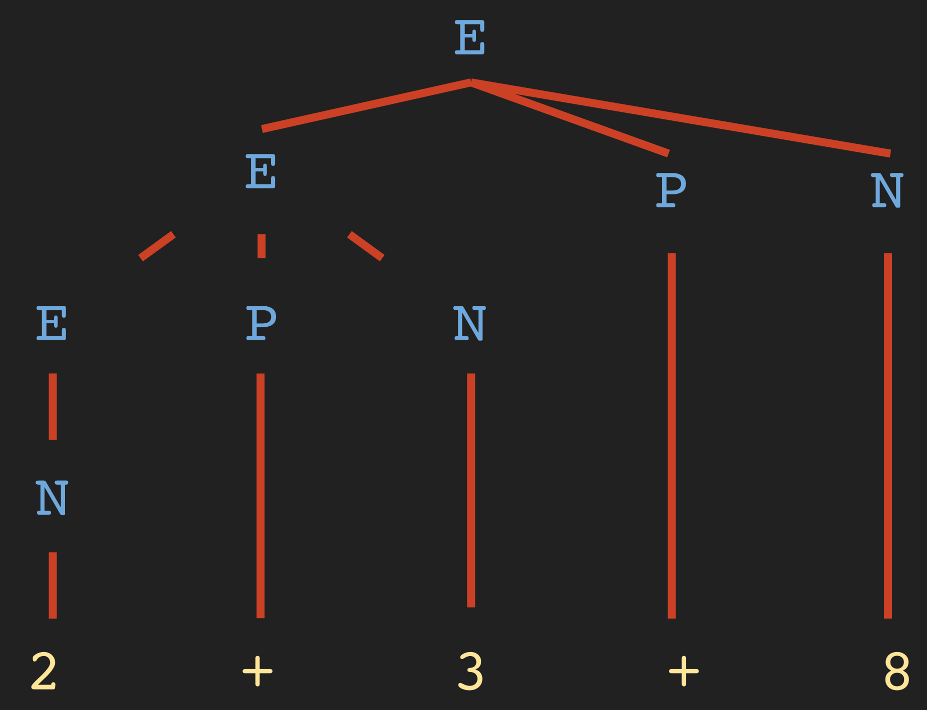
Neural Definite Clause Grammar

CFG: Context-Free Grammar

```
E --> N
E --> E, P, N

P --> ["+"]

N --> ["0"]
N --> ["1"]
...
N --> ["9"]
```



Useful for:

- Is sequence an element of the specified language?
- What is the *"part of speech"*-tag of a terminal
- Generate all elements of language

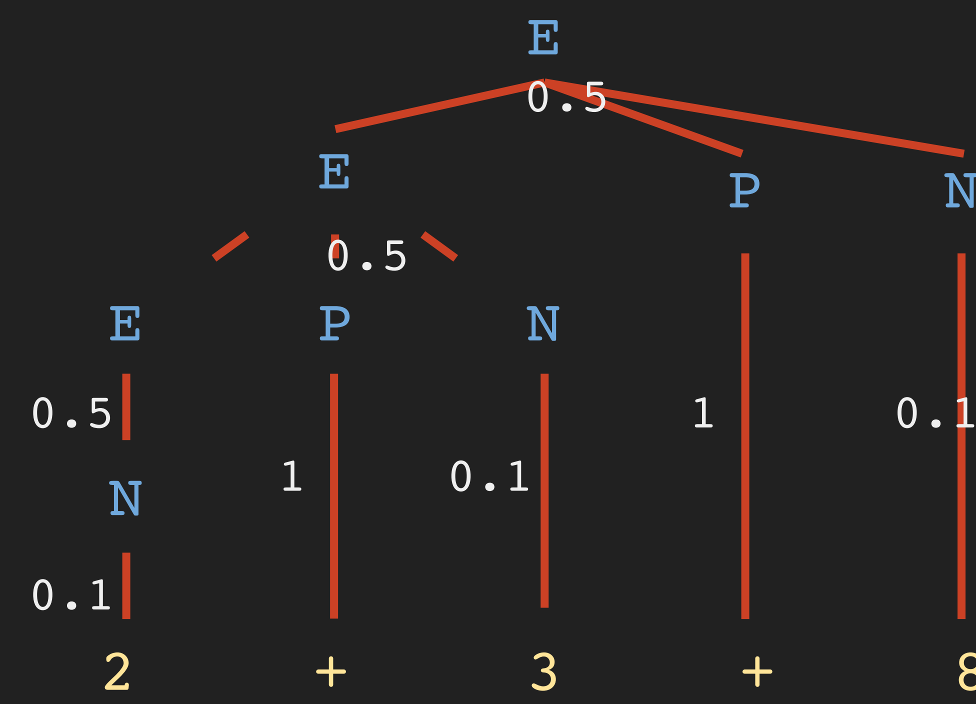
PCFG: Probabilistic Context-Free Grammar

```
0.5 :: E --> N
0.5 :: E --> E, P, N

1.0 :: P --> ["+"]

0.1 :: N --> ["0"]
0.1 :: N --> ["1"]
      …
0.1 :: N --> ["9"]
```

Always sums to 1 per non-terminal



*Probability of this parse = 0.5*0.5*0.5*0.1*1*0.1*1*0.1*

= 0.000125

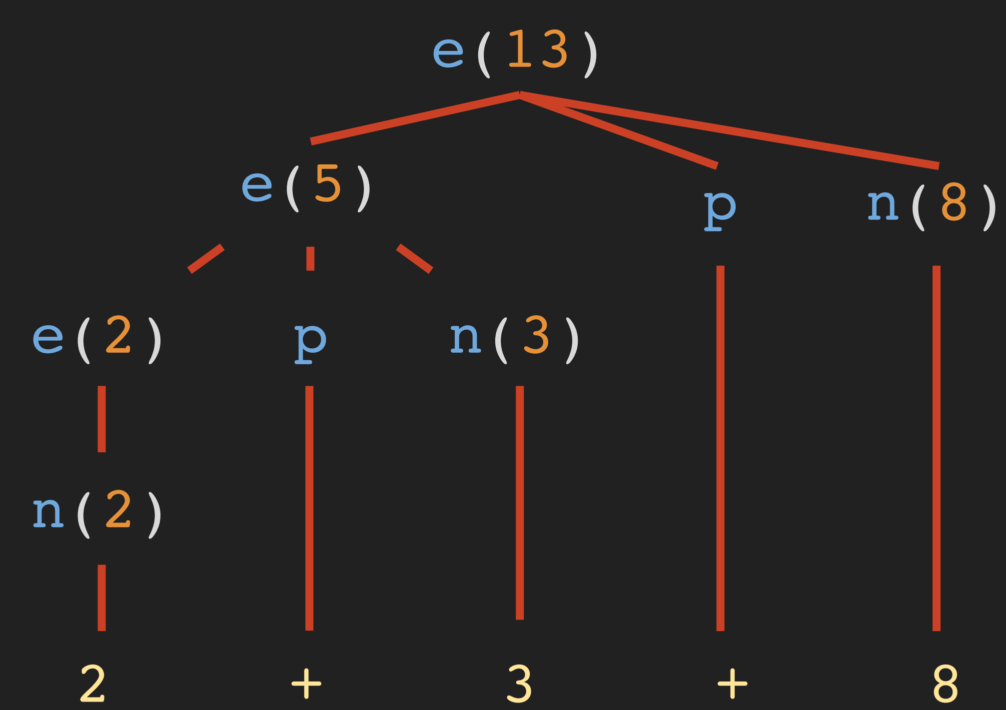
Useful for:

- What is the most likely parse for this sequence of terminals? *(useful for ambiguous grammars)*
- What is the probability of generating this string?

DCG: Definite Clause Grammar

```
e(N) --> n(N).
e(N) --> e(N1), p, n(N2),
         {N is N1 + N2}.
p    --> ["+"].

n(0) --> ["0"].
n(1) --> ["1"].
…
n(9) --> ["9"].
```



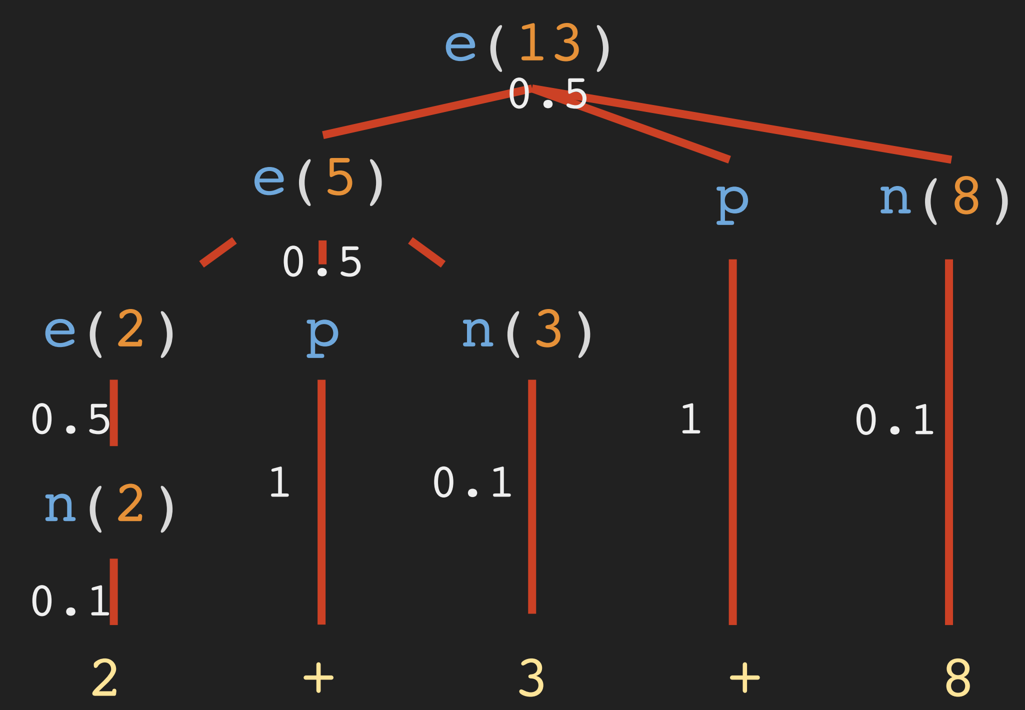
Useful for:

- Modelling more complex languages *(e.g. context-sensitive)*
- Adding constraints between non-terminals thanks to Prolog power *(e.g. through unification)*
- Extra inputs & outputs aside from terminal sequence *(through unification of input variables)*

SDCG: Stochastic Definite Clause Grammar

```
0.5 :: e(N) --> n(N).
0.5 :: e(N) --> e(N1), p, n(N2),
                {N is N1 + N2}.
1.0 :: p    --> ["+"].

0.1 :: n(0) --> ["0"].
0.1 :: n(1) --> ["1"].
       …
0.1 :: n(9) --> ["9"].
```



*Probability of this parse = 0.5*0.5*0.5*0.1*1*0.1*1*0.1*
= 0.000125

Useful for:

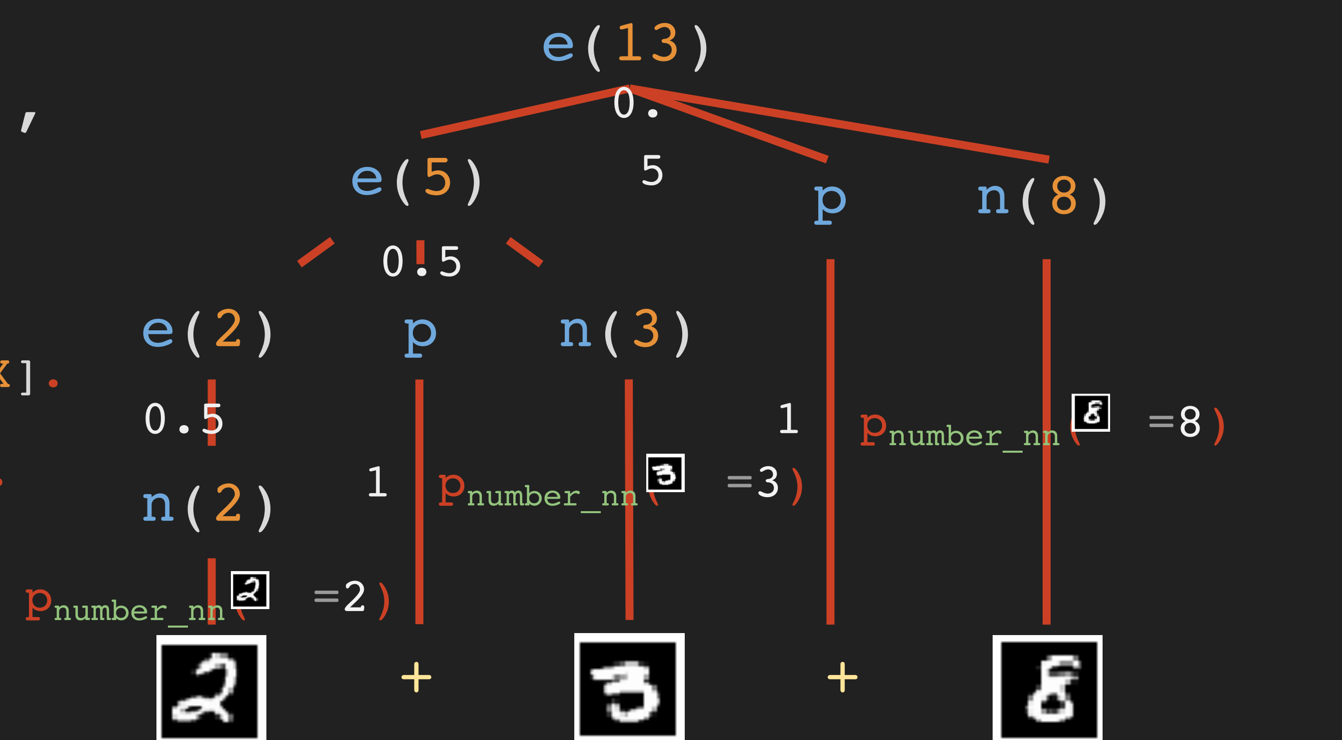
- Same benefits as PCFGs give to CFG *(e.g. most likely parse)*
- But: loss of probability mass possible due to failing derivations

NDCG: Neural Definite Clause Grammar (= DeepStochLog)

```
0.5 :: e(N) --> n(N).
0.5 :: e(N) --> e(N1), p, n(N2),
                {N is N1 + N2}.
1.0 :: p    --> ["+"].

nn(number_nn,[X],[Y],[digit]) :: n(Y) --> [X].

digit(Y) :- member(Y,[0,1,2,3,4,5,6,7,8,9]).
```



Probability of this parse =

0.5*0.5*0.5*p_{number_nn}(=2)*1*p_{number_nn}(=3)*1*p_{number_nn}(=8)

Useful for:

- Subsymbolic processing: e.g. tensors as terminals
- Learning rule probabilities using neural networks

DeepStochLog NDCG definition

$nn(m,[I_1,\dots,I_m],[O_1,\dots,O_L],[D_1,\dots,D_L])$ :: nt --> $g_1, \dots, g_n$.

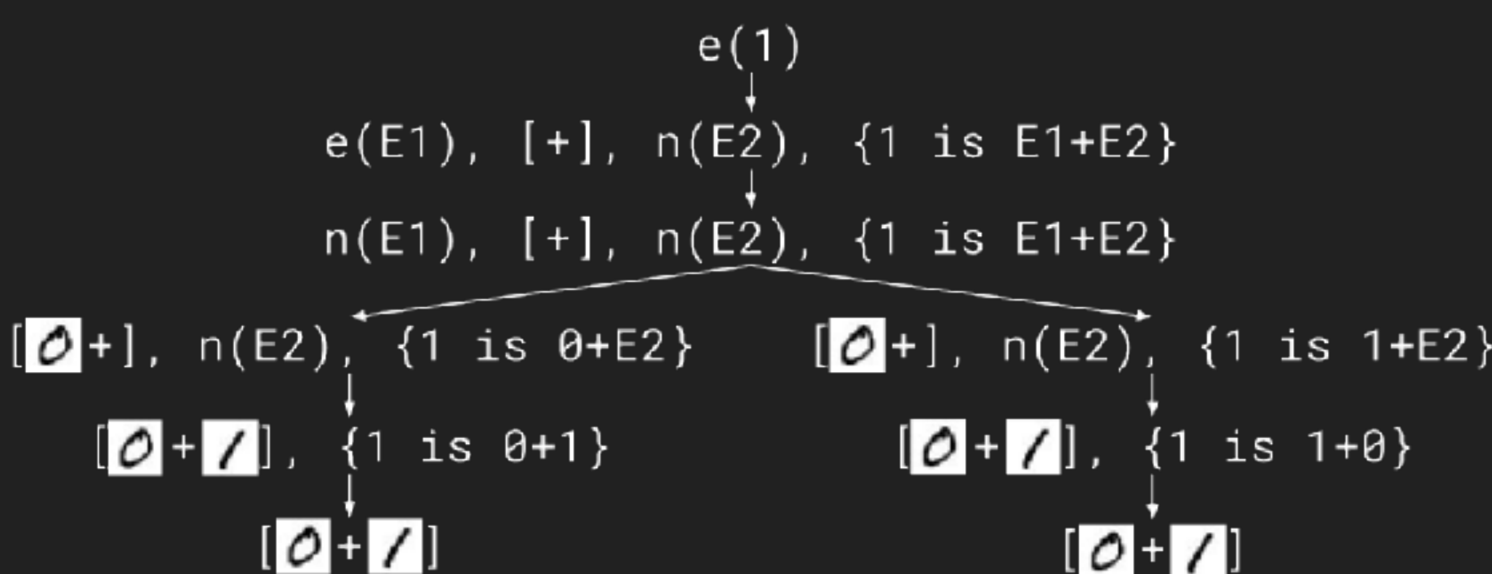
Where:

- nt is an atom
- $g_1, \dots, g_n$ are goals *(goal = atom or list of terminals & variables)*
- $I_1,\dots,I_m$ and $O_1,\dots,O_L$ are variables occurring in $g_1, \dots, g_n$ and are the inputs and outputs of m
- $D_1,\dots,D_L$ are the predicates specifying the domains of $O_1,\dots,O_L$
- m is a neural network mapping $I_1,\dots,I_m$ to probability distribution over $O_1,\dots,O_L$ *(= over cross product of $D_1,\dots,D_L$)*

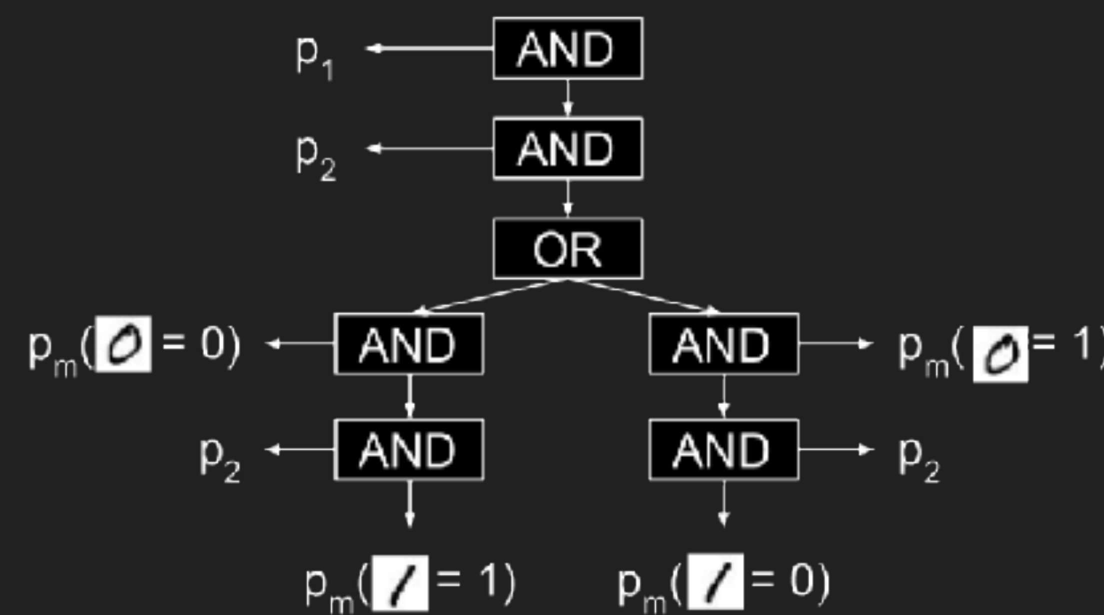
DeepStochLog Inference

Deriving probability of goal for given terminals in NDCG

Proof derivations d(e(1), [+])



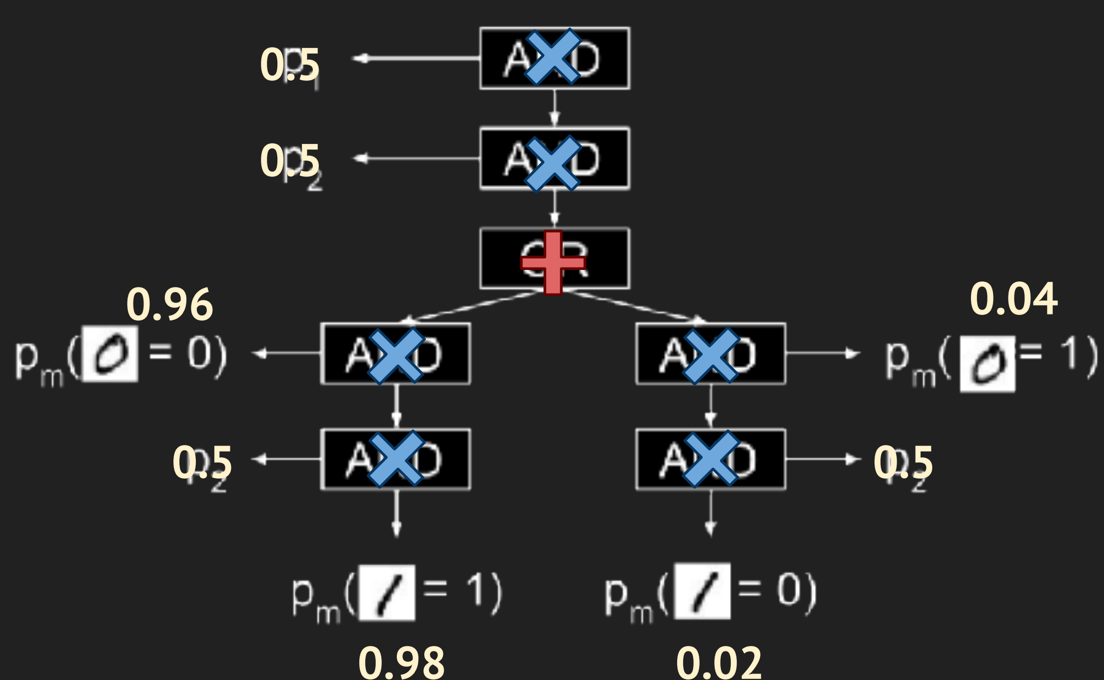
then turn it into and/or tree



And/Or tree + semiring for different inference types

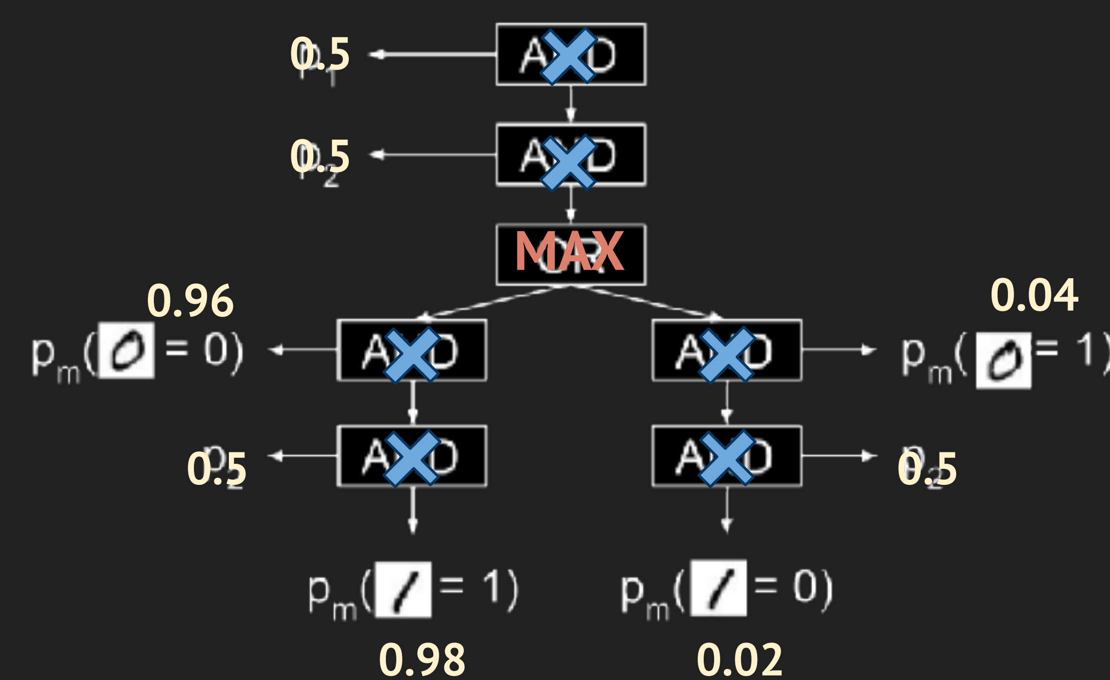
Probability of goal

P_G(derives(e(1), [, +,]) = 0.1141



Most likely derivation

d_{max}(e(1), [, +,]) = $argmax_{d(e(t))=[\ ,+,\ ]}$ P_G(d(e(1))) = [0,+,1]



Inference optimisation

Inference is optimized using

1. **SLG resolution**: Prolog tables the returned proof tree(s), and thus creates forest

→ Allows for reusing probability calculation results from intermediate nodes

Table 6: **Q4** Parsing time in seconds (**T2**). Comparison of the DeepStochLog with and without tabling (SLD vs SLG resolution).

Lengths	**# Answers**	**No Tabling**	**Tabling**
1	10	0.067	0.060
3	95	0.081	0.096
5	1066	3.78	0.95
7	10386	30.42	10.95
9	68298	1494.23	132.26
11	416517	timeout	1996.09

1. **Batched network calls**: Evaluate all the required neural network queries first

→ Very natural for neural networks to evaluate multiple instances at once using batching & less overhead in logic & neural network communication

Research questions

Q1: Does DeepStochLog reach state-of-the-art predictive performance on neural-symbolic tasks?

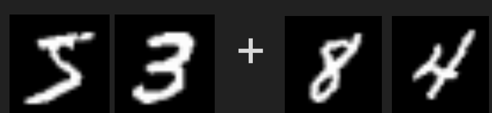
Q2: How does the inference time of DeepStochLog compare to other neural-symbolic frameworks and what is the role of tabling?

Q3: Can DeepStochLog handle larger-scale tasks?

Q4: Can DeepStochLog go beyond grammars and encode more general programs?

Mathematical expression outcome

T1: Summing MNIST numbers with pre-specified # digits



= 137

T2: Expressions with images representing operator or single digit number.

= 19

Table 1: The test accuracy (%) on the MNIST addition (**T1**).

	Number of digits per number (N)			
Methods	1	2	3	4
NeurASP	97.3 ± 0.3	93.9 ± 0.7	timeout	timeout
DeepProbLog	97.2 ± 0.5	95.2 ± 1.7	timeout	timeout
DeepStochLog	97.9 ± 0.1	96.4 ± 0.1	94.5 ± 1.1	92.7 ± 0.6

Table 2: The accuracy (%) on the HWF dataset (**T2**).

	Expression length			
Method	1	3	5	7
NGS	90.2 ± 1.6	85.7 ± 1.0	91.7 ± 1.3	20.4 ± 37.2
DeepProbLog	90.8 ± 1.3	85.6 ± 1.1	timeout	timeout
DeepStochLog	90.8 ± 1.0	86.3 ± 1.9	92.1 ± 1.4	94.8 ± 0.9

Performance comparison

Table 7: Inference times in milliseconds for DeepStochLog, DeepProbLog and NeurASP on task **T1** for variable number lengths.

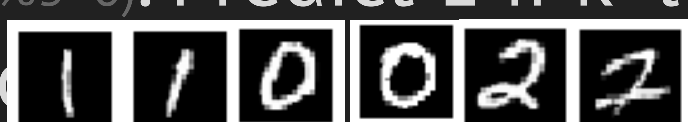
Numbers Length	1	2	3	4
DeepStochLog	1.3 ± 0.9	2.3 ± 0.4	4.0 ± 0.4	5.7 ± 1.8
DeepProbLog	13.5 ± 3.0	36.0 ± 0.5	199.7 ± 14.0	timeout
NeurASP	9.2 ± 1.4	85.7 ± 22.6	158.2 ± 47.7	timeout

Classic grammars, but with MNIST images as terminals

T3: Well-formed brackets as input (without parse). Task: predict parse.

→ parse = ( ) (( ) ( ))

T4: inputs are strings $a^k b^l c^m$ (or permutations of [a,b,c], and (k+l+m) %3=0). Predict 1 if k=l=m,



= 1

= 0

Table 3: The parse accuracy (%) on the well-formed parentheses dataset (**T3**).

Method	Maximum expression length 10	14	18
DeepProbLog	100.0 ± 0.0	99.4 ± 0.5	99.2 ± 0.8
DeepStochLog	100.0 ± 0.0	100.0 ± 0.0	100.0 ± 0.0

Table 4: The accuracy (%) on the $a^n b^n c^n$ dataset (**T4**).

Method	Expression length 3-12	3-15	3-18
DeepProbLog	99.8 ± 0.3	timeout	timeout
DeepStochLog	99.4 ± 0.5	99.2 ± 0.4	98.8 ± 0.2

Natural way of expressing this grammar knowledge

```
brackets_dom(X) :- member(X, ["(",")"]).

nn(bracket_nn, [X], Y, brackets_dom) :: bracket(Y) -->
[X].


t(_) :: s --> s, s.

t(_) :: s --> bracket("("), s, bracket(")").

t(_) :: s --> bracket("("), bracket(")").
```

All power of Prolog DCGs (here: $a^n b^n c^n$)

```
letter(X) :- member(X, [a,b,c]).

0.5 :: s(0) --> akblcm(K,L,M),
                {K \= L; L \= M; M \= K},
                {K \= 0, L \= 0, M \= 0}.

0.5 :: s(1) --> akblcm(N,N,N).

akblcm(K,L,M) --> rep(K,A),
                  rep(L,B),
                  rep(M,C),
                  {A \= B, B \= C, C \= A}.

rep(0, _) --> [].
nn(mnist, [X], C, letter) :: rep(s(N), C) --> [X],
                                              rep(N,C).
```

Citation networks

T5: Given scientific paper set with only few labels & citation network, find all labels

Table 5: **Q3** Accuracy (%) of the classification on the test nodes on task **T5**

Method	Citeseer	Cora
ManiReg	60.1	59.5
SemiEmb	59.6	59.0
LP	45.3	68.0
DeepWalk	43.2	67.2
ICA	69.1	75.1
GCN	70.3	81.5
DeepProbLog	timeout	timeout
DeepStochLog	65.0	69.4

Word Algebra Problem

T6: natural language text describing algebra problem, predict outcome

E.g. "Mark has 6 apples. He eats 2 and divides the remaining among his 2 friends. How many apples did each friend get?"

Uses *"empty body trick"* to emulate SLP logic rules through SDCGs:

```
nn(m,[I_1,…,I_m],[O_1,…,O_L],[D_1,…,D_L]) :: nt --> [].
```

Enables fairly straightforward translation of DeepProbLog programs for a lot of tasks

DeepStochLog performs equally well as DeepProbLog: 96% accuracy

Challenges

- For NeSy, DeepProbLog and others
  - scaling up (in DeepProbLog — now has both approximate and exact inference — an A* like algorithm to find the best proofs [Manhaeve KR 21])
  - which models and which knowledge to use
  - real life applications
  - peculiarities of neural nets
  - dynamics / continuous
- This is an excellent area for starting researchers / PhDs

One NeSy Recipe

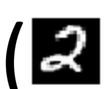
1. Take a symbolic (logic / rule based) representation
2. Turn the 0/1 True/False in Fuzzy or Probabilistic Interpretation
3. Interpret neural networks as logical predicates/functions
4. (The harder part): inference and learning

For instance:

map an MNIST image to a number

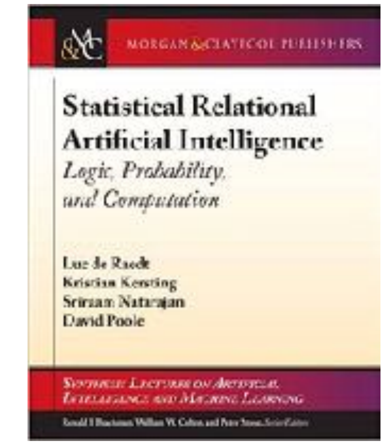
m() = 2

m as a neural network

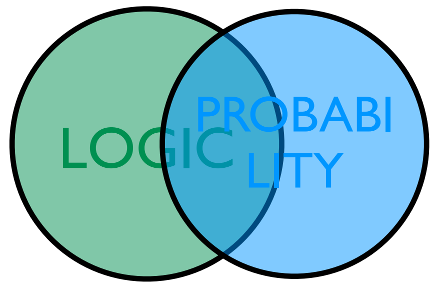
mp(,2) =0.93 as a neural predicate

(with a fuzzy/prob. interpretation)

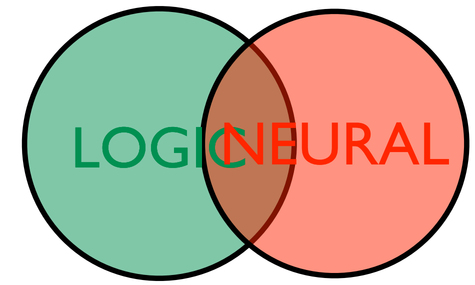
Key Message 1



FROM



TO



StarAI and NeSy share similar problems and thus similar solutions apply

Part 1 of the talk

See also [De Raedt et al., IJCAI 20]

Key Message 2

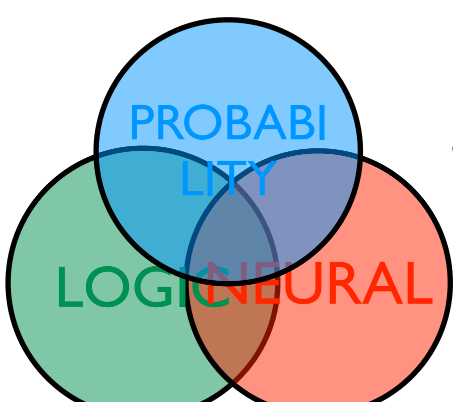
A different approach

A true integration T of X and Y should allow to reconstruct X and Y as special cases of T

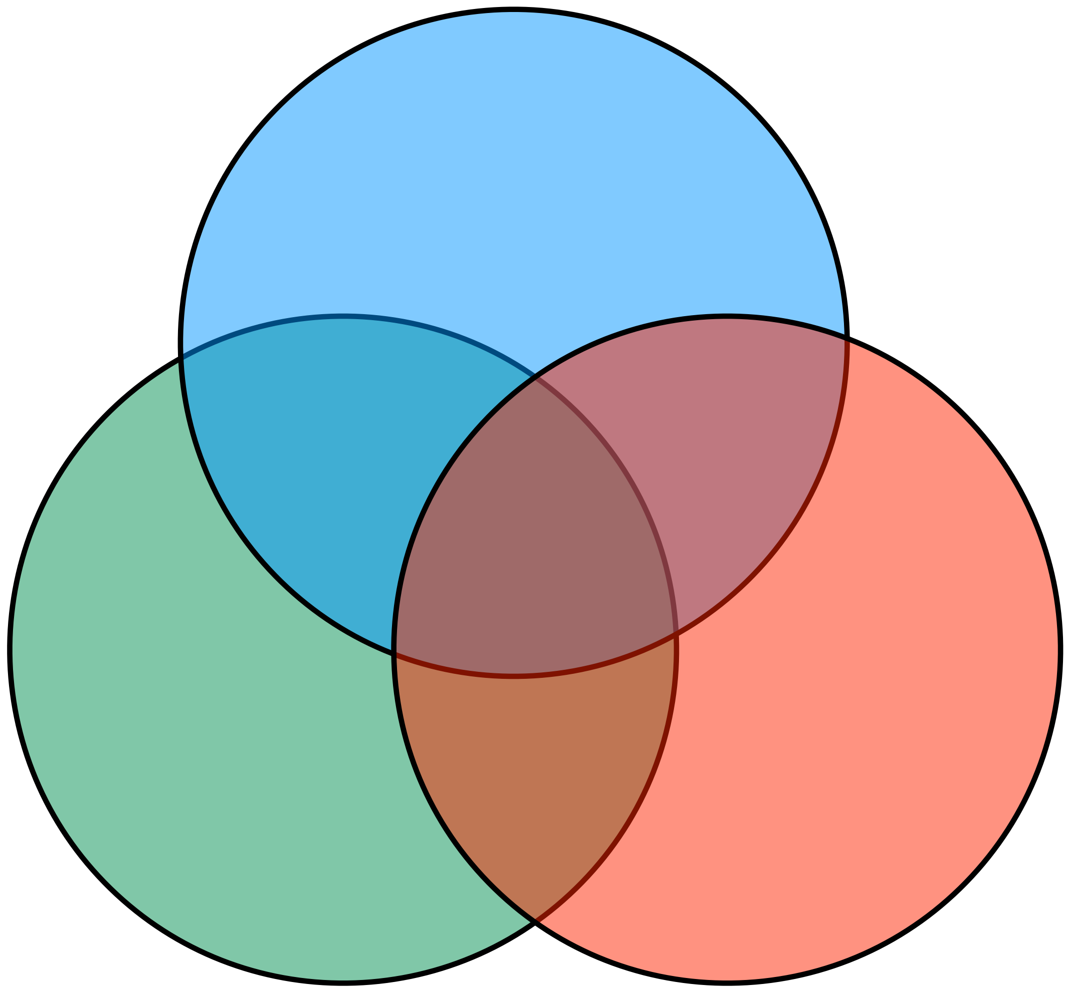
Thus, Neural Symbolic approaches should have both logic and neural networks as special cases

Our approach: "an interface layer (<> pipeline) between neural & symbolic components" will be illustrated with DeepProbLog

See also [Manhaeve et al., NeurIPS 18; arXiv: 1907.08194]



Part 2 of the talk — illustration with DeepProbLog [NeurIPS 2018] and DeepStochLog [AAAI 2022]



THANKS