

Machine Learning-Supported Decomposition Algorithms for a Large Scale Hub Location Problem

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**Deutsche Post DHL
Group**



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What we'll discuss

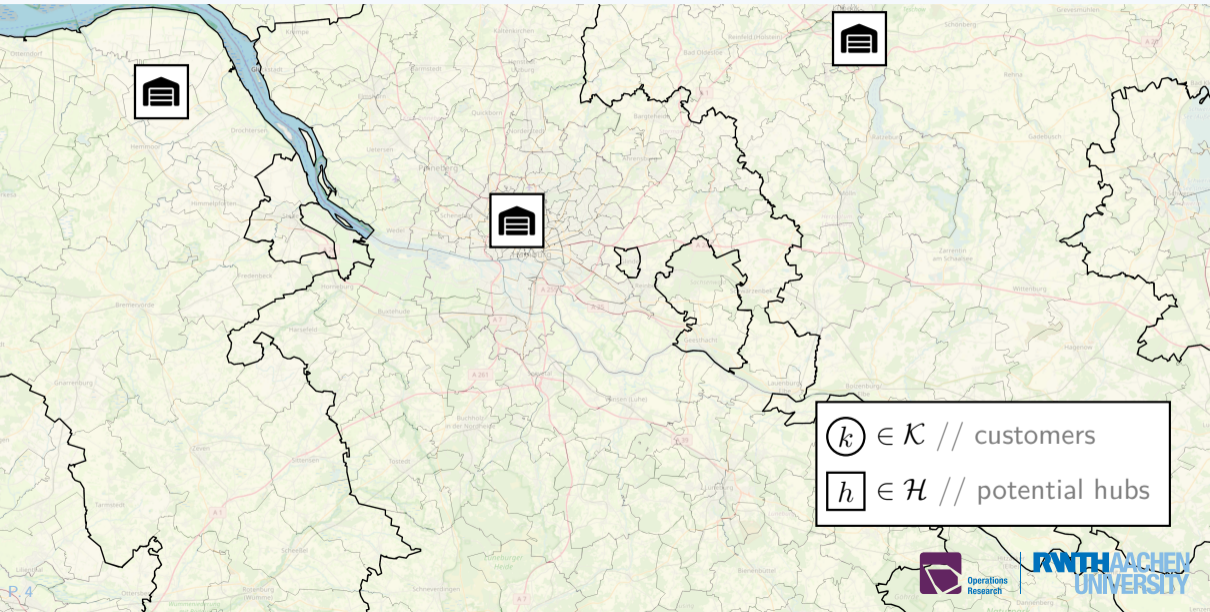
- ▶ a very decomposable problem
- ▶ column generation approach
- ML to learn good heuristics
- ▶ benders decomposition approach
- replacing subproblem with a learned representation?

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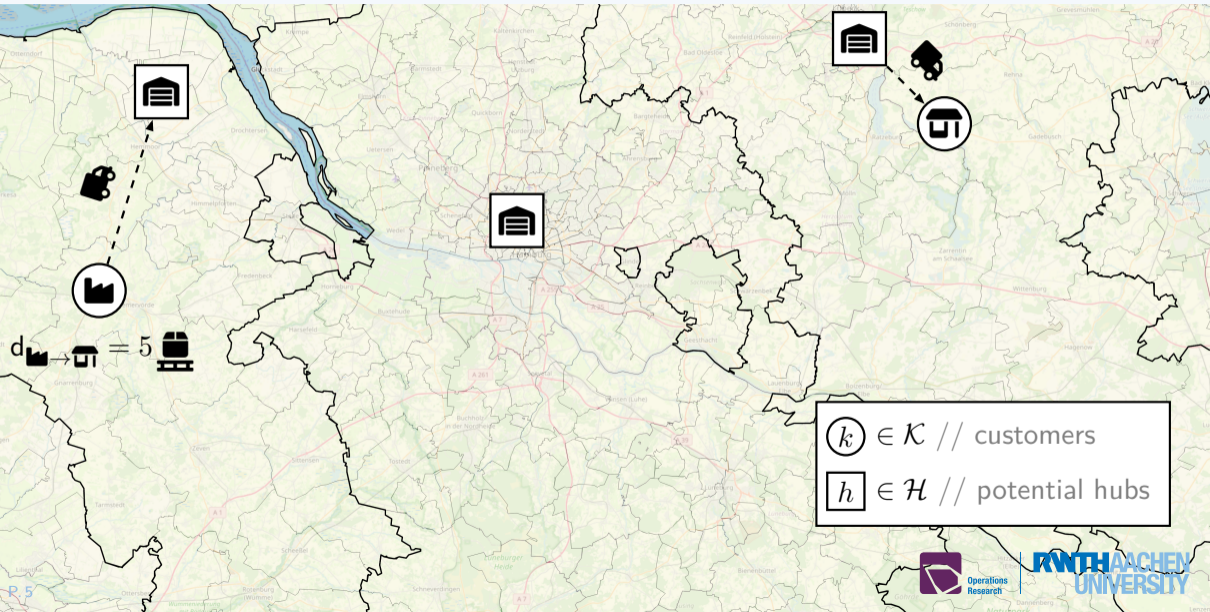
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⚠ Disclaimer: Rough ideas - i bring questions, not answers

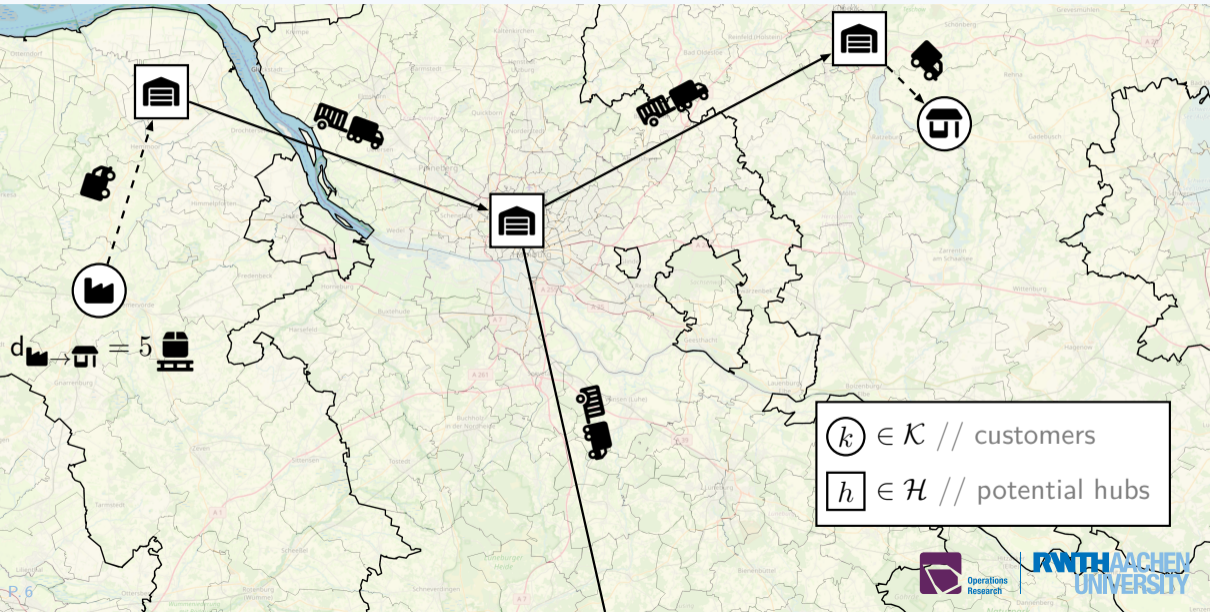
Goal: Support Strategic Location Planning in Freight Networks



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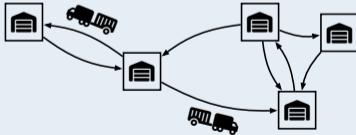


What's in a Problem?

Hub (Facility) Location



Line-Haul (Service Network Design)



Pick-Up and Delivery (Vehicle Routing)

- ▶ operational
- ▶ unpredictable
- ▶ hard to solve

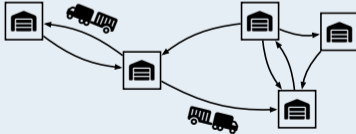


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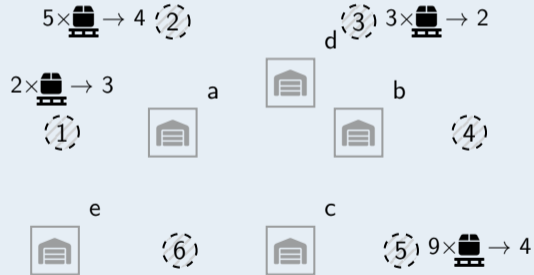


Decisions in Hub Location and Service Network Design

Decisions

- ▶ facilities to open
- ▶ assignments
- ▶ paths for goods
- ▶ truck services

Example

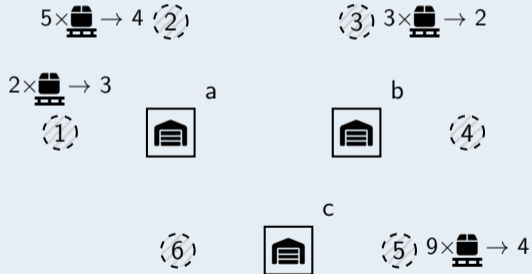


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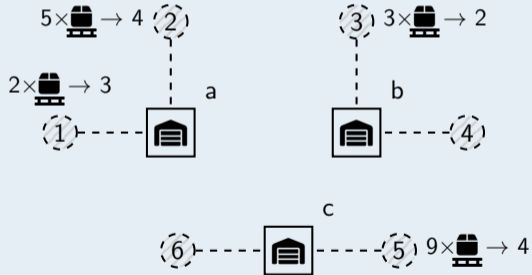


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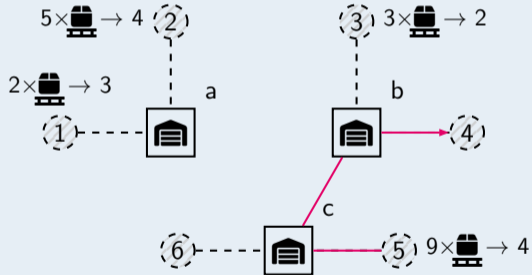


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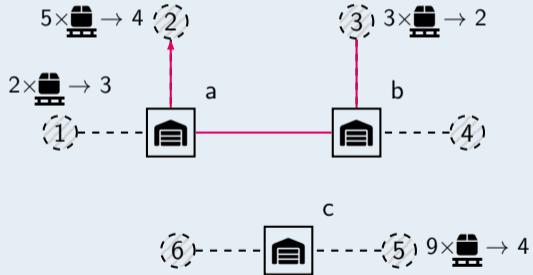


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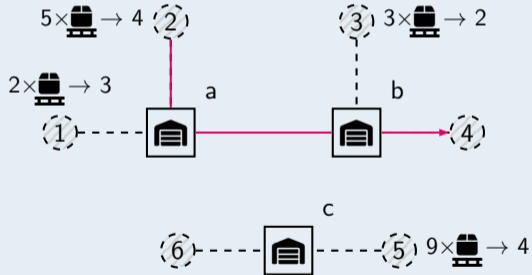


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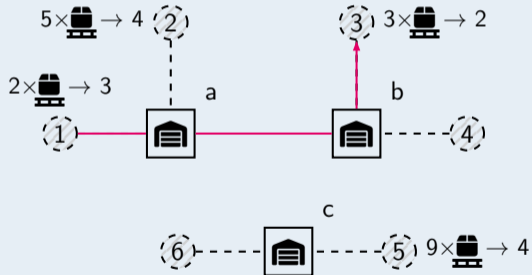


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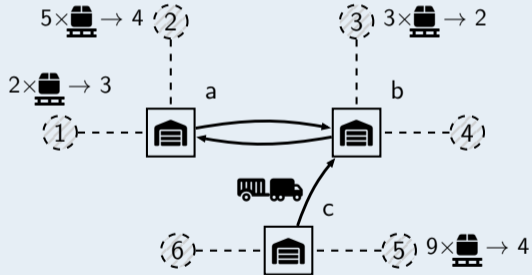


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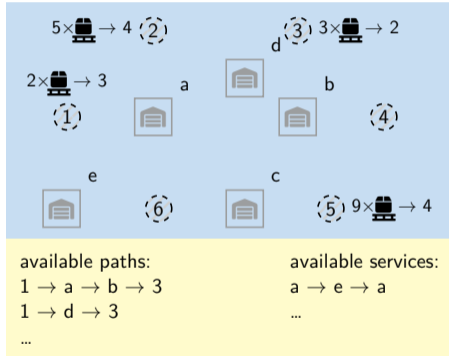
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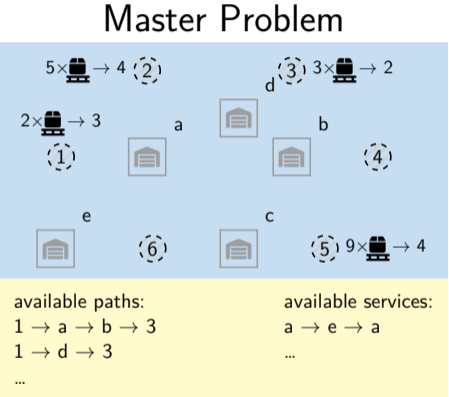
Speeding Up Column Generation with Learned Heuristics

Master Problem

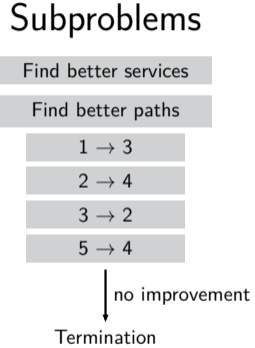


CG generally: Lübbecke et al. 2005,
for hub location with services: Rothenbächer et al. 2016

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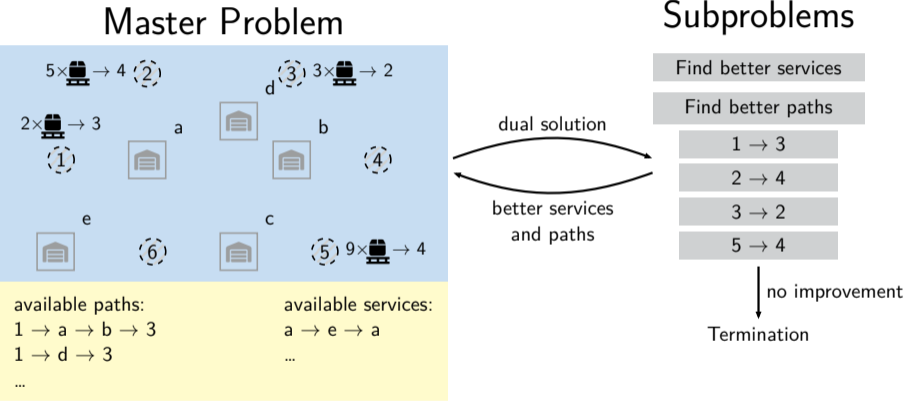


dual solution



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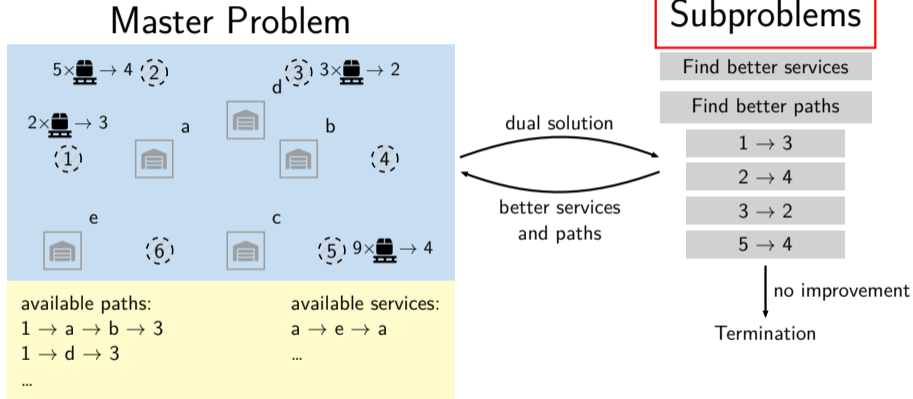
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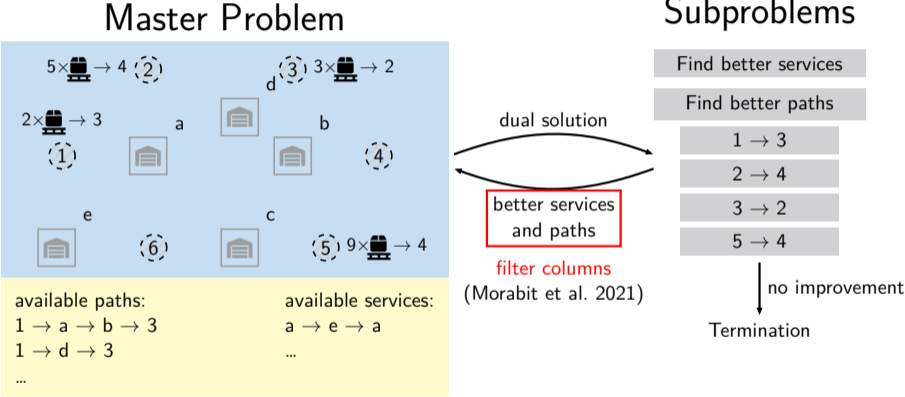
Speeding Up Column Generation with Learned Heuristics

solve heuristically (Morabit et al. 2022)



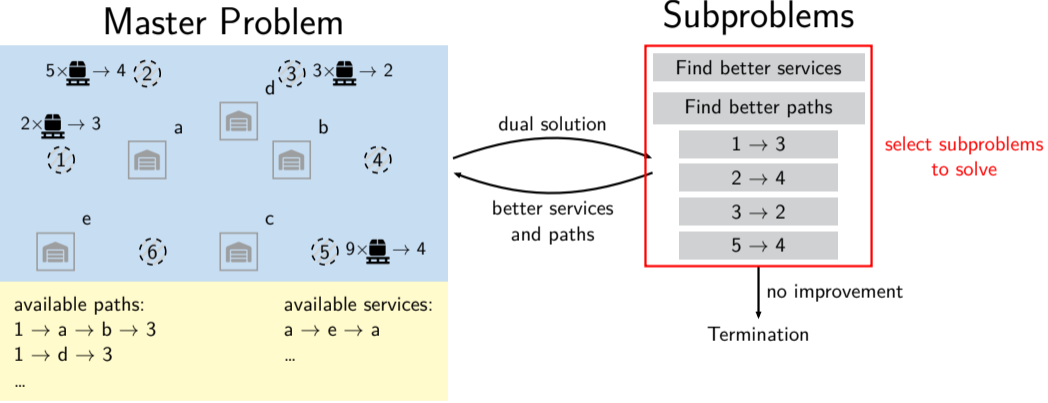
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Master Problem

decisions:

- facilities to open
- assignments

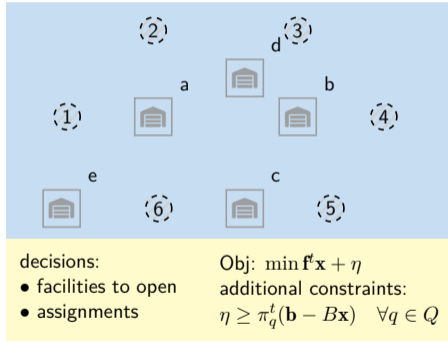
Obj: $\min \mathbf{f}^t \mathbf{x} + \eta$

additional constraints:

$$\eta \geq \pi_q^t (\mathbf{b} - B\mathbf{x}) \quad \forall q \in Q$$

A Bit More Out There: Approximate Benders Cuts?

Master Problem



primal solution \mathbf{x}', η'

Subproblem

With $\mathbf{h} = \mathbf{b} - B\mathbf{x}$ solve
 $z^*(\mathbf{h}) = \min \{ \mathbf{c}^t \mathbf{y} \mid A\mathbf{y} \geq \mathbf{h}, \mathbf{y} \geq 0 \}$



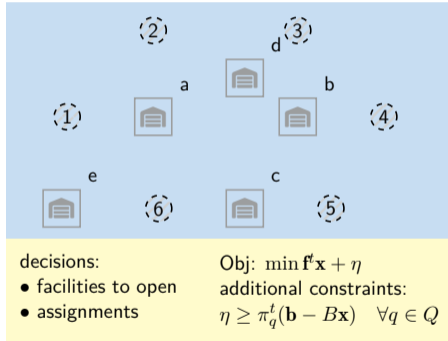
$\eta' = z^*$

Termination

Benders decomposition: Benders 1962

A Bit More Out There: Approximate Benders Cuts?

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new constraint,
 $Q \leftarrow q'$

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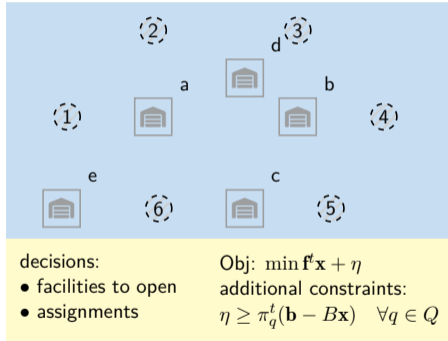


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A Bit More Out There: Approximate Benders Cuts?

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Subproblem

Note: $\pi_{qi} = \frac{\partial z^*}{\partial h_i}(\mathbf{h}')$

Train model $\psi(\mathbf{h}) \approx z^*(\mathbf{h})?$

\Rightarrow 'approximate' benders cut







$\eta' = z^*$

Termination






What we've seen and where to go




- ▶ ML can support decomposition methods
 - ▶ CG: learn better heuristics - still an exact method
 - ▶ Benders: could replace subproblem - now a heuristic
- ▶ (partially) open questions:
 - ▶ appropriate ML models
 - ▶ generating sufficient training data
 - ▶ for benders: can models scale well?

References I

-  Alumur, Sibel A. et al. (2021). “Perspectives on modeling hub location problems”. In: *European Journal of Operational Research* 291.1, pp. 1–17.
-  Benders, J F (1962). “Partitioning procedures for solving mixed-variables programming problems”. In: *Numerische Mathematik* 4.1, pp. 238–252.
-  Contreras, Ivan, Jean-françois Cordeau, and Gilbert Laporte (2011). “Benders Decomposition for Large-Scale Uncapacitated Hub Location”. In: 59.6, pp. 1477–1490.
-  Contreras, Ivan, Juan A. Díaz, and Elena Fernández (2011). “Branch and price for large-scale capacitated hub location problems with single assignment”. In: *INFORMS Journal on Computing* 23.1, pp. 41–55.
-  Contreras, Ivan and Morton E. O’Kelly (2019). “Hub Location Problems”. In: *Location Science*. Springer International Publishing, pp. 327–363.
-  Lübbecke, Marco E. and Jacques Desrosiers (2005). “Selected topics in column generation”. In: *Operations Research* 53.6, pp. 1007–1023.

References II

-  Meier, J. Fabian and Uwe Clausen (2018). “Solving single allocation hub location problems on euclidean data”. In: *Transportation Science* 52.5, pp. 1141–1155.
-  Morabit, Mouad, Guy Desaulniers, and Andrea Lodi (2021). “Machine-Learning–Based Column Selection for Column Generation”. In: *Transportation Science* 55.4, pp. 815–831.
-  — (2022). “Machine-learning-based arc selection for constrained shortest path problems in column generation”. In: pp. 1–32.
-  O’Kelly, Morton E. (1990). “A quadratic integer program for the location of interacting hub facilities”. In: *European Journal of Operational Research* 46.3, pp. 409–411.
-  Rothenbächer, Ann Kathrin, Michael Drexl, and Stefan Irnich (2016). “Branch-and-price-and-cut for a service network design and hub location problem”. In: *European Journal of Operational Research* 255.3, pp. 935–947.

-  Wandelt, Sebastian et al. (2021). “An efficient and scalable approach to hub location problems based on contraction”. In: *Computers and Industrial Engineering* 151, p. 106955.
-  Yang, Xiao, Nathalie Bostel, and Pierre Dejax (2019). “A MILP model and memetic algorithm for the Hub Location and Routing problem with distinct collection and delivery tours”. In: *Computers and Industrial Engineering* 135, pp. 105–119.
-  Zetina, Carlos Armando, Ivan Contreras, and Sachin Jayaswal (2015). “An Exact Algorithm for Large-Scale Non-convex Quadratic Facility Location”. In: pp. 1–32.