



Machine learning-based algorithms for dynamic patient scheduling problems with uncertainty

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November 4th 2022 - Focus Period Linköping

OPTIMIZATION IN THE HEALTHCARE DOMAIN

Time to tackle health-care wait times in Canada

— *Mackenzie Moir, Bacchus Barua*

[https://nationalpost.com > news > canada > why-canada...](https://nationalpost.com/news/canada/why-canada...)

Why Canada's hospital capacity was so easily overwhelmed ...

Jan 17, 2022 — The latest numbers from the OECD show Canada with just one hospital bed for every 400 citizens, a ratio that put us in the bottom tier of OECD ...

[https://uwaterloo.ca > sites > files > uploads > files > ...](https://uwaterloo.ca/sites/files/uploads/files/...)

SeniorS and HouSing: THE CHALLENGE aHead

As Canada's aging population grows, and as Canadians live longer, there is an overwhelming desire among seniors to "age in place" or "age at home."

Canada Research Chair in Analytics and Logistics in Healthcare

Mental health among healthcare workers in Canada during the COVID-19 pandemic

Release date: 2021-02-02

Optimizing treatment planning and delivery in healthcare

[https://cmajnews.com > 2020/08/28 > access-qa-1095895](https://cmajnews.com/2020/08/28/access-qa-1095895)

How can Canada improve worsening wait times? - CMAJ News

Aug 28, 2020 — Simpson: Canadians have accepted that some delay is reasonable in order to have an equitable health care system, and that's not necessarily a ...

[https://hospitalnews.com > Topics > Health Care P...](https://hospitalnews.com/Topics/Health-Care-P...)

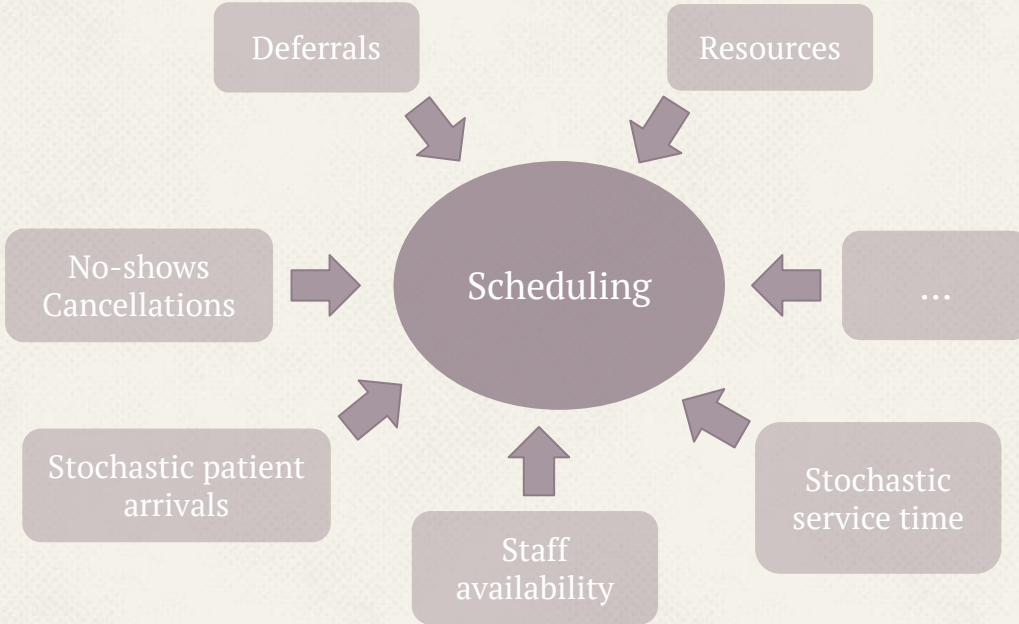
Canada ranks last on number of hospital beds, wait times

For example, Canada ranks 26th (out of 28 countries) for the number of doctors (2.8 per 1,000 people) and 26th (out of 27) for the number of hospital beds ...



HANALOG

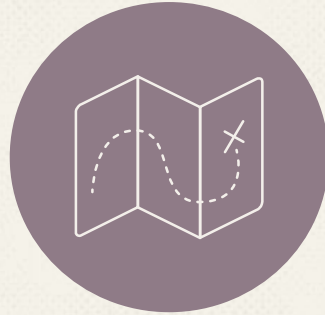
UNCERTAINTY IN HEALTHCARE SCHEDULING



- Dynamic problems
- Online decisions

- Stochastic optimization
- MDP - (approximate) dynamic programming
- Simulation-based

Machine-learning based approaches



Radiotherapy scheduling for cancer treatments



Cancer incidence (2020)

- *10.000.000 deaths*
- *1 out of 6 deaths*



Challenges

- 
- *Growing and aging population*



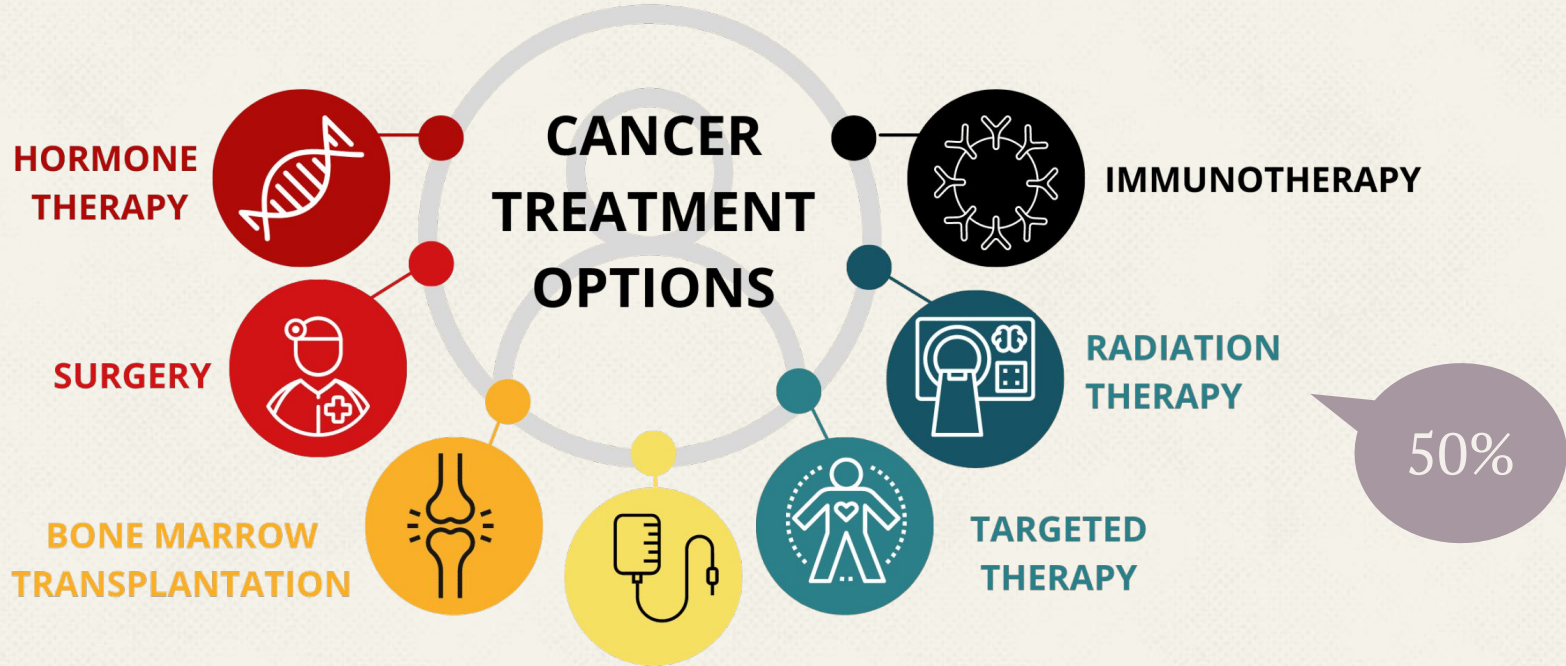
How to reduce death rates?

- 
- *Early detection*
 - *Early treatment*



Optimize treatment scheduling

CANCER TREATMENT



Optimizing radiotherapy treatment schedules to reduce patients' waiting times

RADIOTHERAPY TREATMENT

- Linear accelerators (**linac**)
- **Fraction**: a small dose of radiation
- Treatment plan
 - Multiple consecutive fractions
 - Fraction duration

Patient scheduling with
multi-appointments,
multi-resources



LITERATURE

- Markov Decision Process & Approximate Dynamic Programming (ADP)
 - Patrick et al. 2008
 - Saure et al. 2012, 2020 (3 linacs)
 - Gocgun 2018
- Stochastic Programming
 - Legrain et al. 2015 (2 linacs)

CHUM - CENTRE HOSPITALIER DE L'UNIVERSITÉ DE MONTRÉAL

10 linacs

5 generics
4 specialized
1 cyberknife

4400 consultations

3500 new patients

40.000 fractions

(2019)

GRAY

<https://www.graysuite.com/>

2021 INNOVE-ACTION
BEST STARTUP AWARD

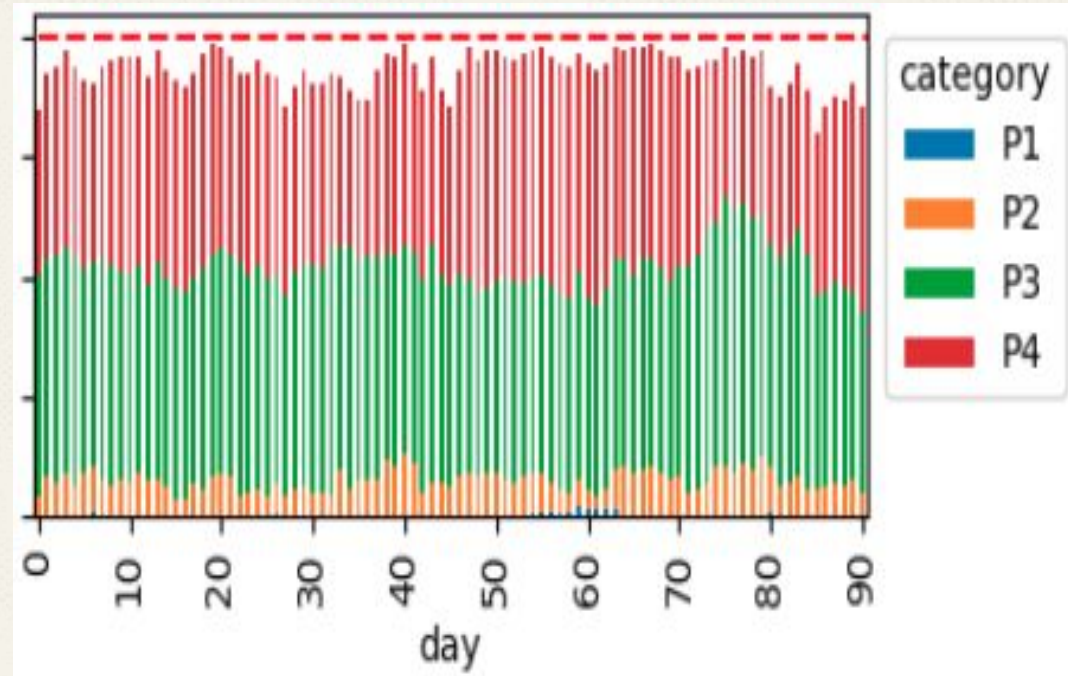
CHUM - 2019

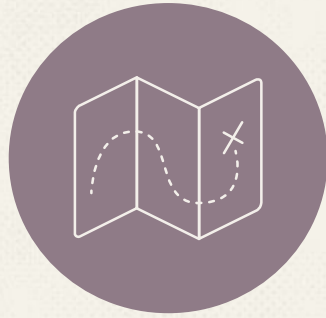
| | Category | Percentages (%) | Treatment deadline (days) | Percentage of overdue treatment (%) | Average waiting time (days) |
|------------|----------|-----------------|---------------------------|-------------------------------------|-----------------------------|
| Palliative | P1 | 0.4 | 1 | 14.29 | 1.09 |
| | P2 | 27.2 | 3 | 79.89 | 6.91 |
| Curative | P3 | 41.4 | 14 | 74.55 | 18.11 |
| | P4 | 31.0 | 28 | 29.89 | 22.59 |

Objective: minimizing overdue treatment and waiting time

THE MAIN CHALLENGES

- Preserving linac capacity
 - Reserved capacity vs occupancy rate?
- A prediction-based approach
 - Learn to delay low-priority patients

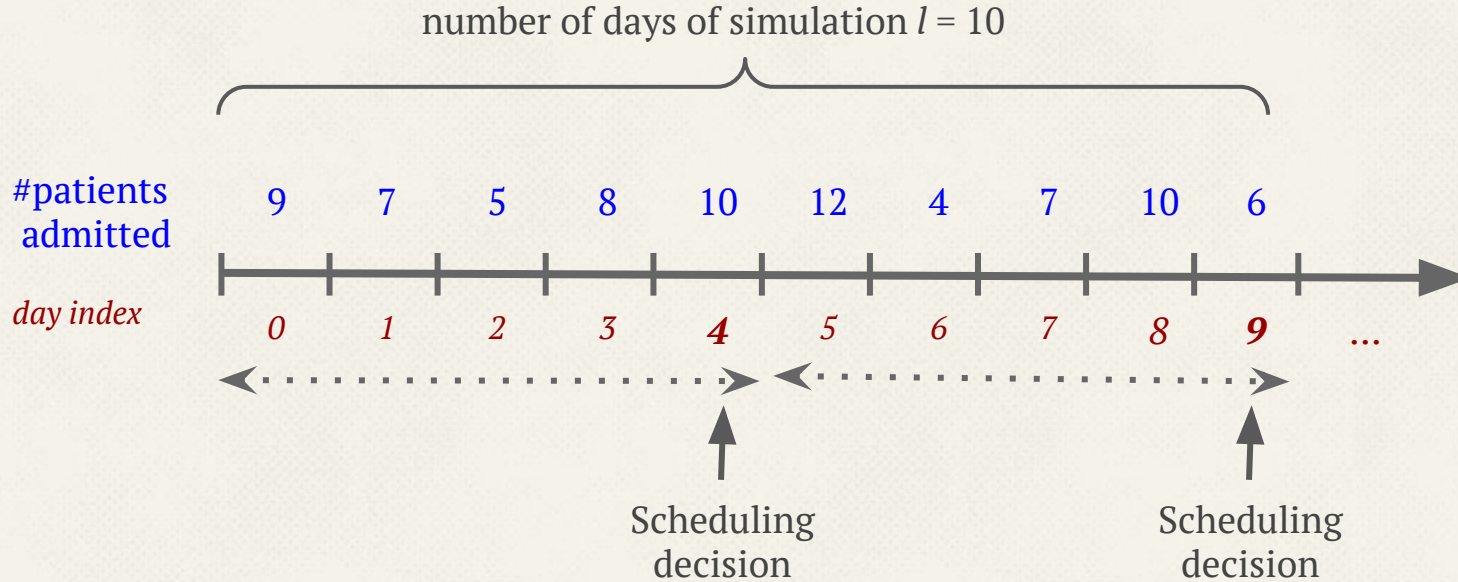




Scheduling strategies

- *Batch scheduling*
- *Offline scheduling*
- *Online scheduling*
- *Prediction-based approach*

BATCH SCHEDULING



Palliative patients: schedule at arrival

AN INTEGER PROGRAMMING MODEL FOR BATCH SCHEDULING

$$x_{tl}^i = \begin{cases} 1 & \text{if patient } i \text{ receives their treatment on day } t, \text{ linac } l \\ 0 & \text{otherwise} \end{cases}$$

minimize $\sum_{i \in \hat{\mathcal{P}}} \sum_{t \in \mathcal{T}, t > a_i} \sum_{l \in \mathcal{L}} \omega_1 (t - a_i) \log(t - a_i + 1) x_{tl}^i$ waiting time

+ $\sum_{i \in \hat{\mathcal{P}}} \sum_{t \in \mathcal{T}, t > d_i} \sum_{l \in \mathcal{L}} \omega_2 (t - d_i) \log(t - d_i + 1) x_{tl}^i$ overdue time

IP MODEL

$$\sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}} x_{tl}^i = 1$$

assignment constraint

$$\forall i \in \hat{\mathcal{P}}$$

$$x_{tl}^i = 0$$

ready date

$$\forall i \in \hat{\mathcal{P}}, l \in \mathcal{L}, t \in \{0, \dots, r_i - 1\}$$

$$\sum_{i \in \hat{\mathcal{P}}} \sum_{t' = \max\{0, t - I_i + 1\}}^t p_i x_{t'l}^i \leq \hat{C}_l^t$$

capacity constraints

$$\forall t \in \mathcal{T}, l \in \mathcal{L}$$

$$\sum_{i \in \mathcal{P}^c} \sum_{t' \in \{t - I_i + 1, \dots, t\}} p_i x_{t'l}^i \leq \max\{0, \hat{C}_l^t - \gamma C_l^t\}$$

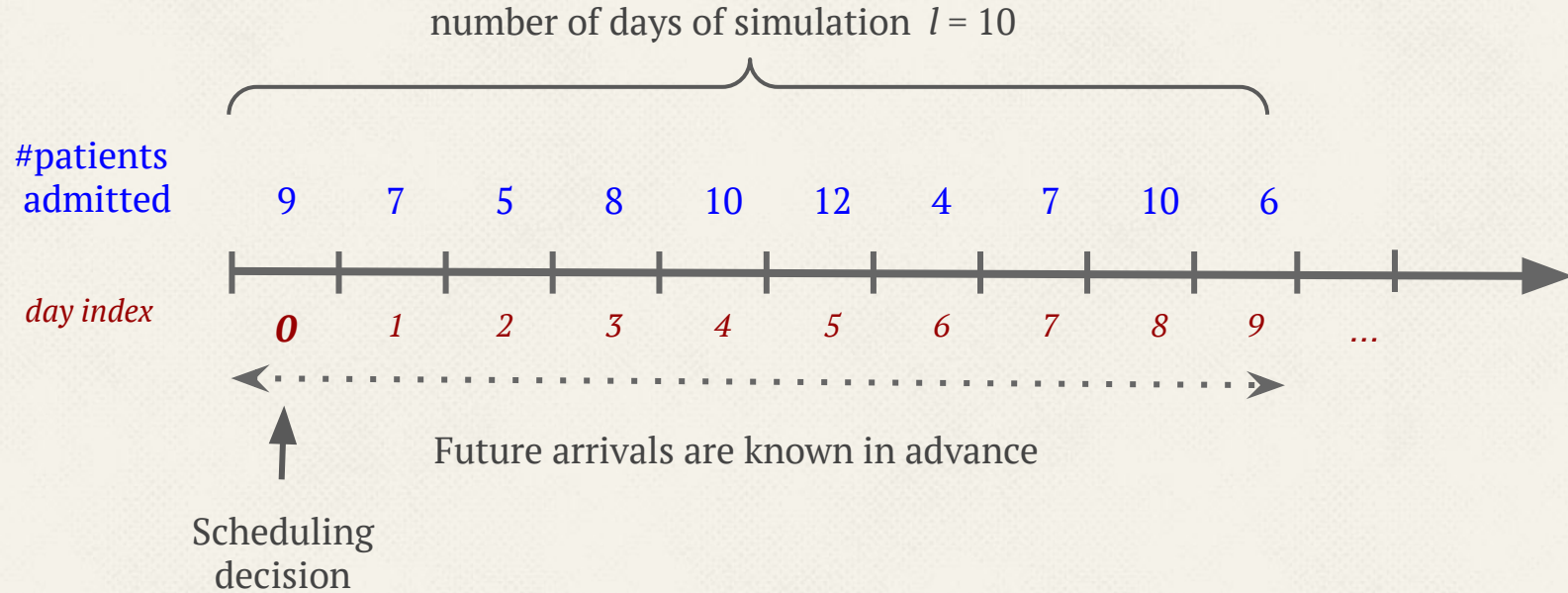
reserved capacity

$$\forall t \in \mathcal{T}, l \in \mathcal{L}$$

$$x_{tl}^i \in \{0, 1\}$$

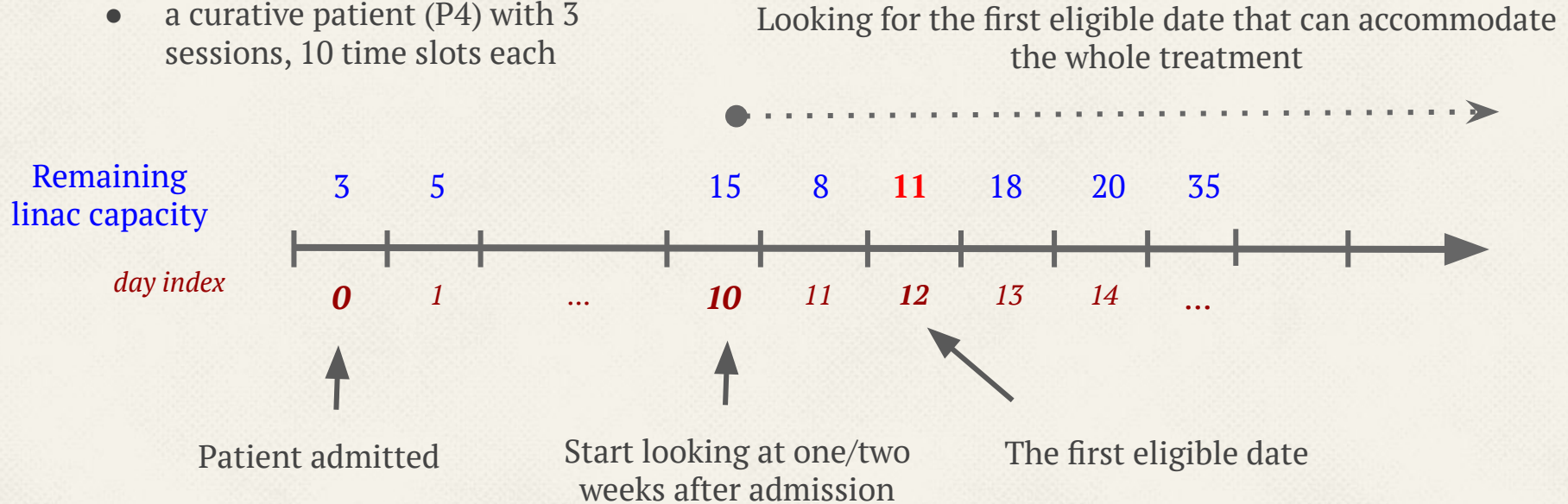
$$\forall i \in \hat{\mathcal{P}}, t \in \mathcal{T}, l \in \mathcal{L}$$

OFFLINE SCHEDULING - THE PERFECT SCENARIO



ONLINE SCHEDULING WITH A GREEDY HEURISTIC

- 1 linac, capacity 120 time slots
- a curative patient (P4) with 3 sessions, 10 time slots each



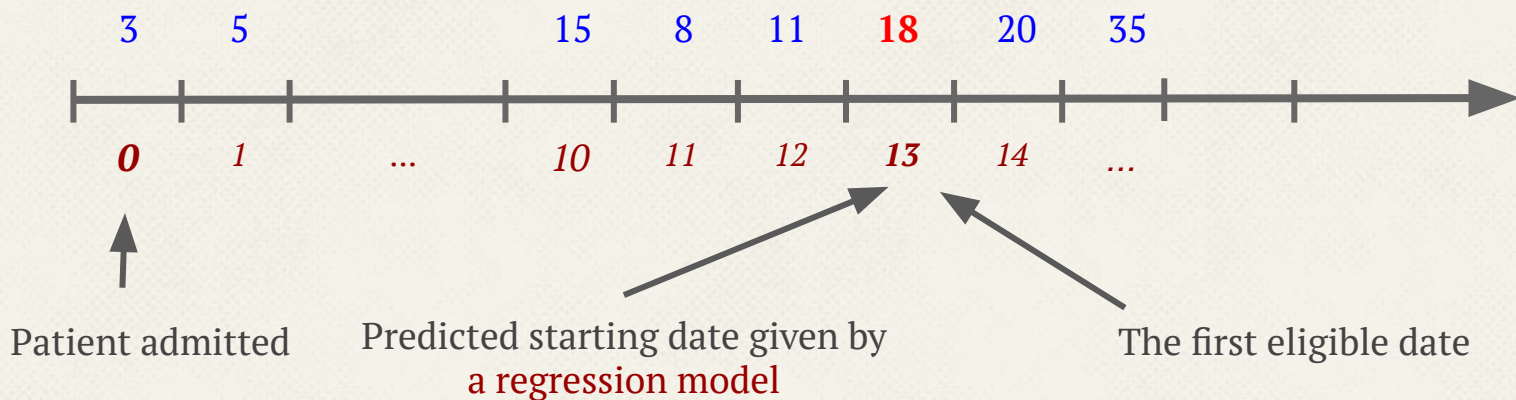
PREDICTION-BASED SCHEDULING

- 1 linac, capacity 120 time slots
- a curative patient with 3 sessions, 10 time slots each

Looking for the first eligible date that can accommodate the whole treatment

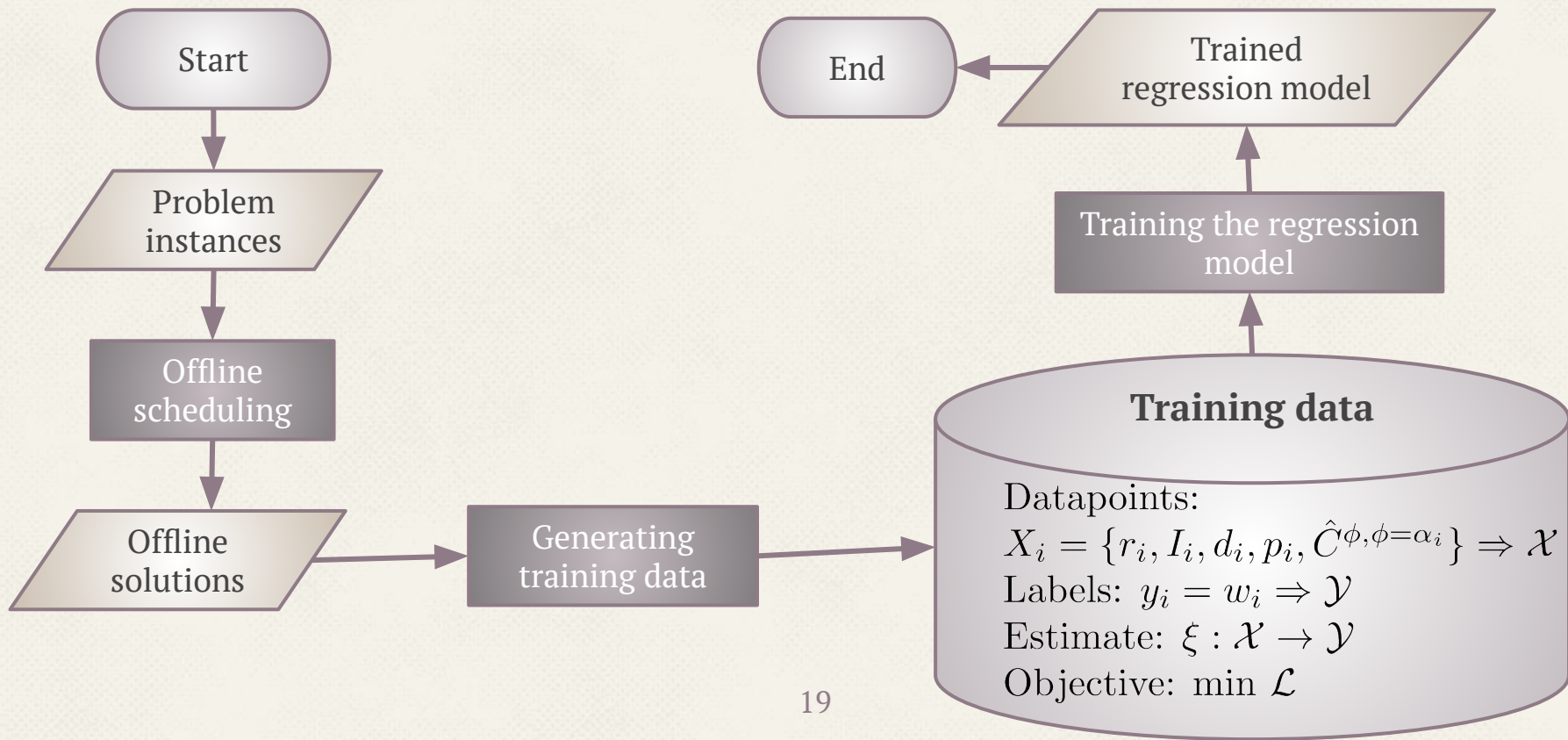
Remaining
linac capacity

day index



How do we predict a “good” starting date for a patient?

TRAINING THE REGRESSION MODEL





Numerical results

- *Data generation*
- *Model selection*
- *Results on simulated data*
- *Results on real data*
- *Explainability with SHAP values*

DATA GENERATION

- **Patient arrivals:** Poisson distribution
- **Treatment plans:** based on historical data
- **Instance setting**
 - Number of linacs
 - Arrival rate (average daily number of patients)
- For each instance setting: 500 instances
 - 400 for training the regression model
 - 100 for testing

PREDICTION MODELS

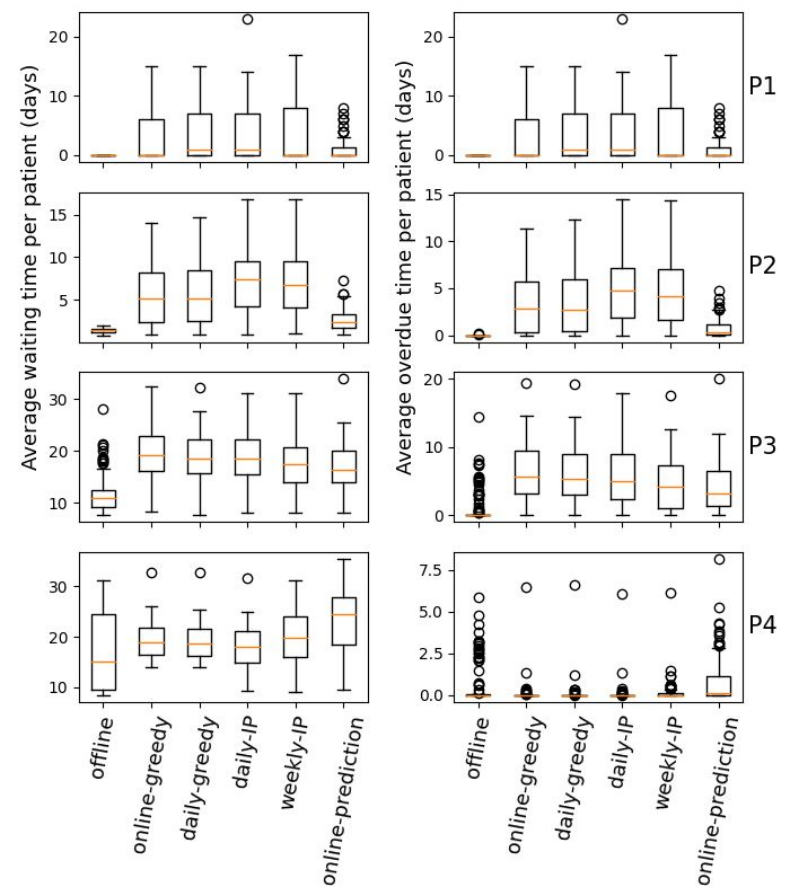
| | Training time | Training | | Testing | |
|---------------|---------------|-------------|-------------|-------------|-------------|
| | | MSE | MAE | MSE | MAE |
| MLP | 116.19 | 3.45 | 1.32 | 3.33 | 1.29 |
| SGD | 0.35 | 6.06 | 1.84 | 5.61 | 1.77 |
| Lasso | 0.44 | 5.97 | 1.81 | 5.52 | 1.74 |
| ElasticNet | 0.25 | 6.26 | 1.85 | 5.83 | 1.8 |
| SVR | 43.16 | 3.19 | 1.07 | 3.12 | 1.07 |
| Decision Tree | 0.84 | 2.41 | 0.48 | 6.59 | 1.4 |
| Random forest | 51 | 0.38 | 0.39 | 2.64 | 1.03 |
| XGBoost | 7.71 | 0.96 | 0.66 | 2.44 | 0.97 |

SCHEDULING STRATEGIES

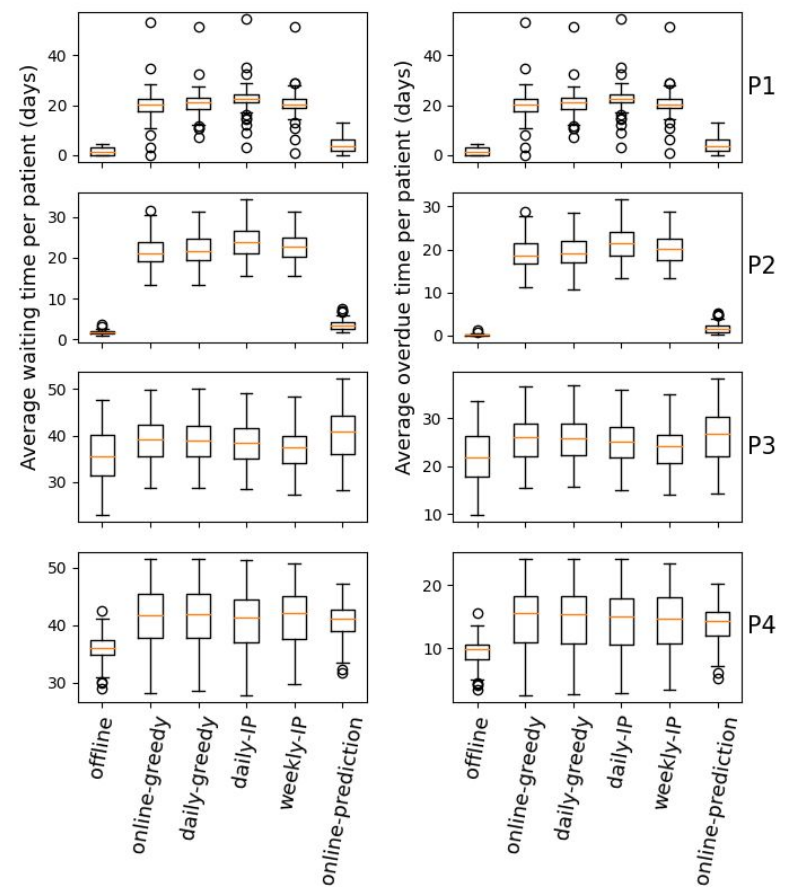
| | Scheduling strategy | Scheduling palliative patients | Scheduling curative patients |
|-------------------|-------------------------|---|------------------------------|
| Batch scheduling | Offline | Scheduling once with all future arrivals known in advance | |
| | Daily | Every day | Every day |
| | Weekly | Every day | Every Friday |
| | Daily greedy | Every day | Every day |
| Online scheduling | Greedy | At admission | At admission |
| | Prediction-based | At admission | At admission |

4 LINACS

Arrival rate of 5.0

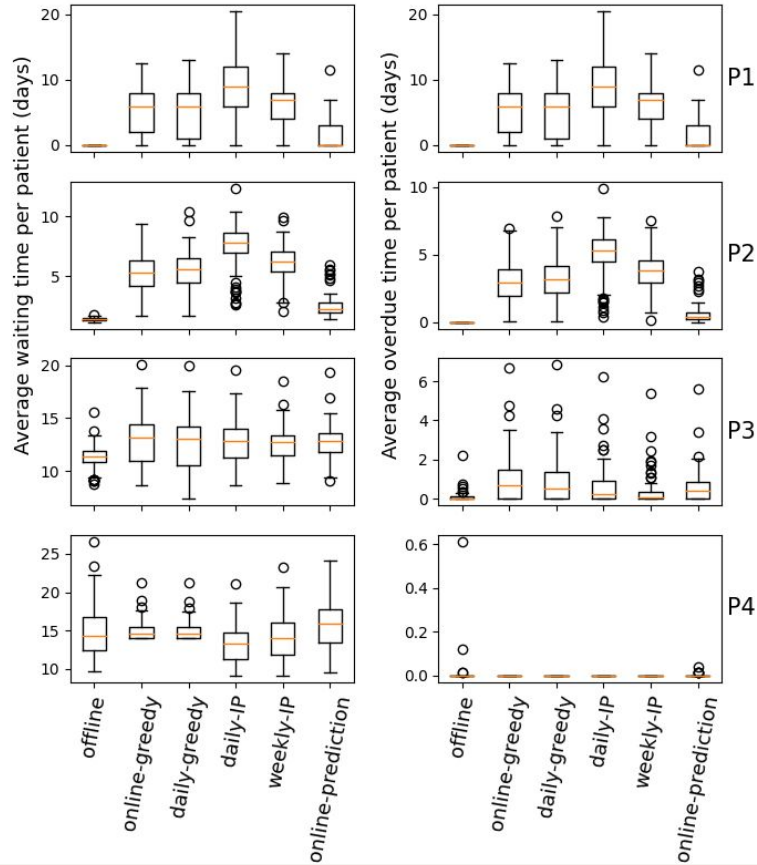


Arrival rate of 6.0

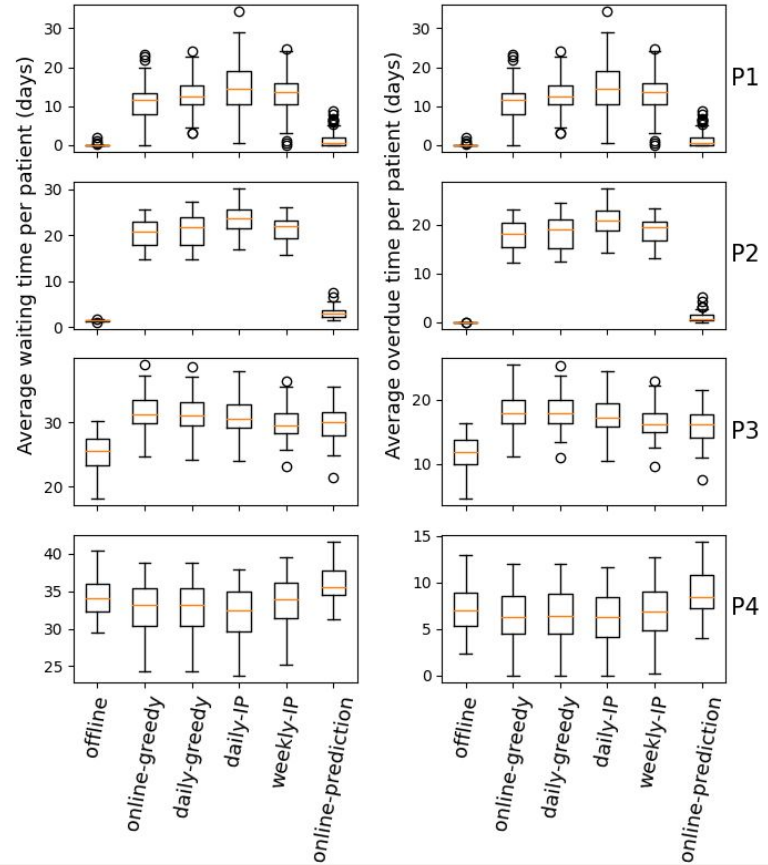


8 LINACS

Arrival rate of 10.0

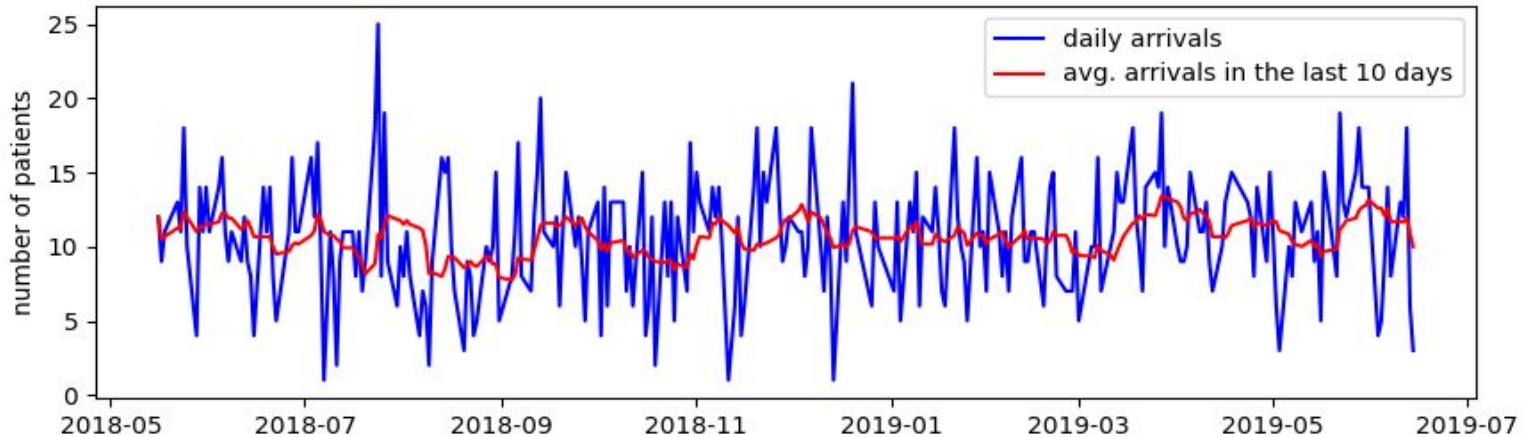


Arrival rate of 12.0

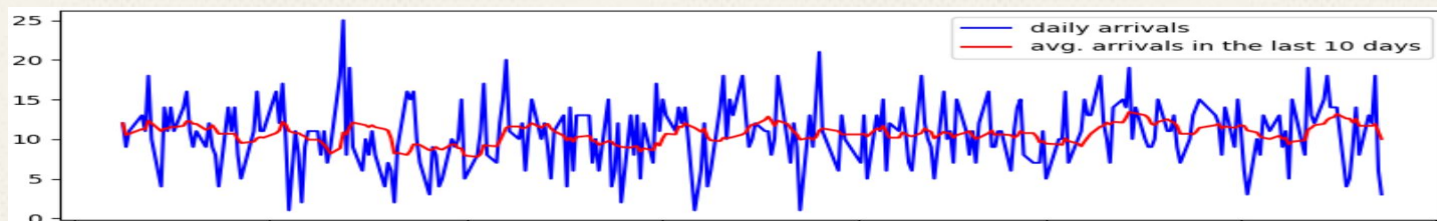


EXPERIMENT ON A REAL PATIENT FLOW

- 7 linacs operating 8 hours/day
- High fluctuation in arrival rate
 - Instance setting for training: arrival rate of 10.1
 - Simulation horizon: 30 days



RESULTS ON THE REAL INSTANCE

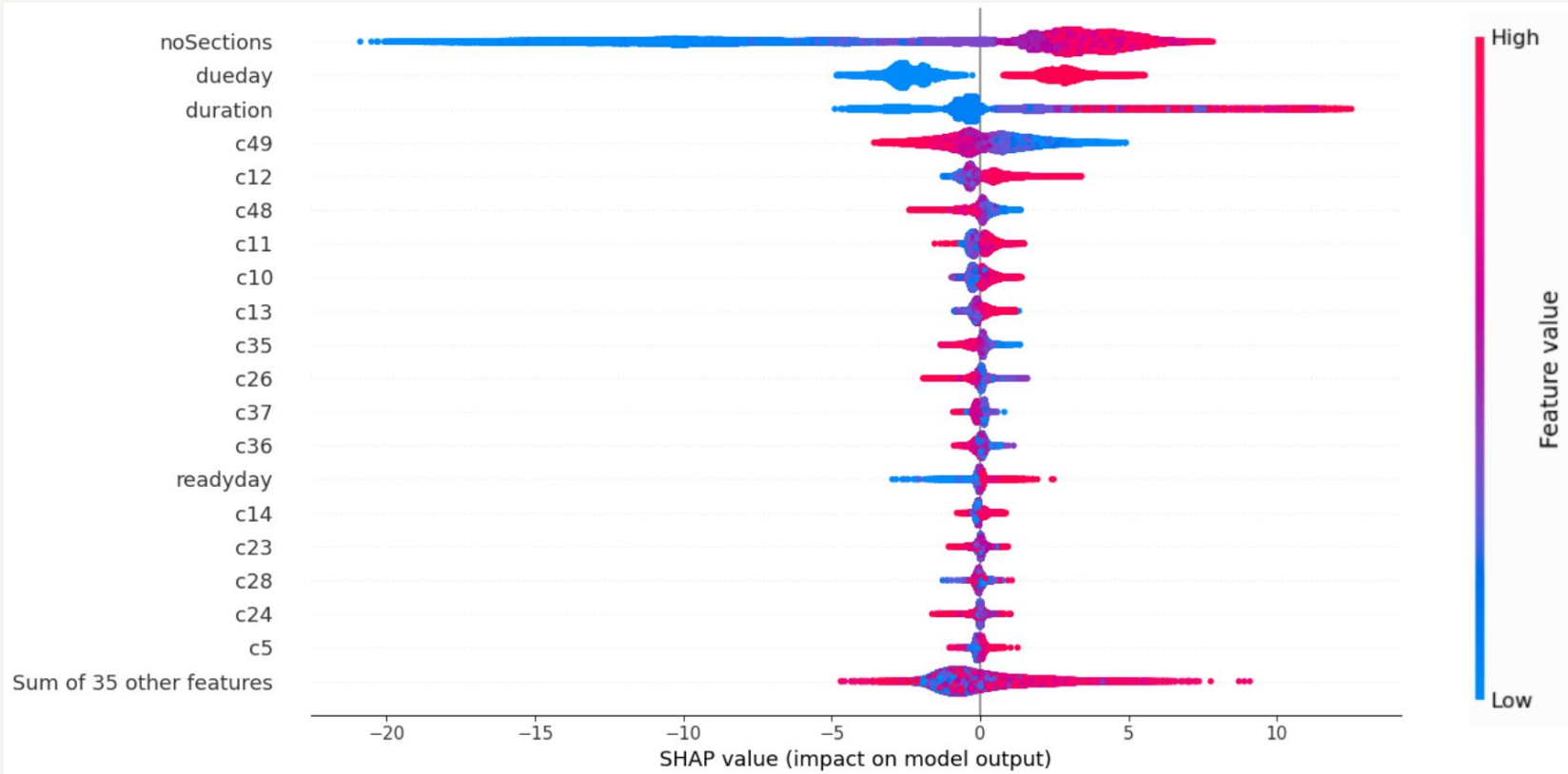


| Scheduling strategy | Avg. occupancy (%) | Waiting time (days) | | | | | Overdue time (days) | | | | |
|---------------------|--------------------|---------------------|-------------|-------------|--------------|--------------|---------------------|-------------|-------------|--------------|--------------|
| | | overall | P1 | P2 | P3 | P4 | overall | P1 | P2 | P3 | P4 |
| online-greedy | 97.45 | 33.02 | 5.14 | 6.13 | 43.67 | 44.02 | 44.02 | 5.14 | 3.91 | 29.74 | 16.18 |
| daily-greedy | 97.51 | 32.91 | 6.00 | 6.23 | 43.48 | 43.80 | 17.71 | 6.00 | 3.99 | 29.58 | 16.00 |
| daily | 97.72 | 33.53 | 9.79 | 9.63 | 42.87 | 43.44 | 18.25 | 9.79 | 7.15 | 28.93 | 15.65 |
| weekly | 97.61 | 33.04 | 7.86 | 7.72 | 42.42 | 44.10 | 17.76 | 7.86 | 5.37 | 28.51 | 16.19 |
| prediction-based | 97.14 | 32.93 | 3.29 | 4.05 | 44.21 | 44.94 | 17.69 | 3.29 | 1.99 | 30.22 | 16.96 |

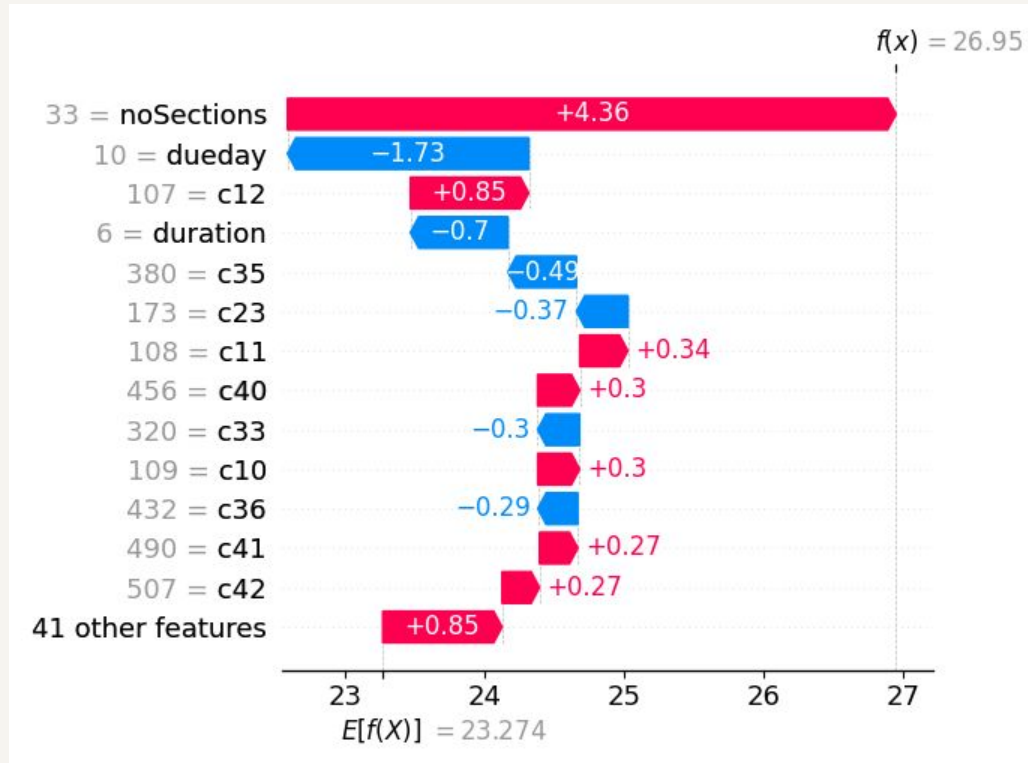
EXPLAINABILITY WITH SHAPLEY VALUES

- **SHAP** (SHapley Additive exPlanations)
 - represent the relative strength of a variable on the outcome
- Widely use for explainability in machine learning
- Highly appreciated in the healthcare domain

GLOBAL INTERPRETATION BEESWARM PLOT



LOCAL INTERPRETATION WATERFALL PLOT



CONCLUSIONS AND CHALLENGES?

- A **machine learning-based** approach for **online dynamic** patient scheduling
- Empirical results
 - Improve overdue times of palliative patients, especially on large and crowded hospitals
 - Not too sensitive with the fluctuation on the arrival rate
- Explainability with Shapley values
- Challenges: generating offline solutions is expensive

A reinforcement learning approach for the dynamic home health care scheduling and routing problem

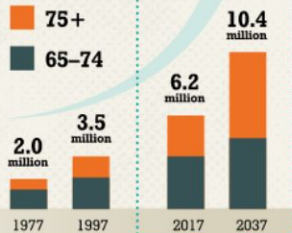
*Ta Dinh Quy, Tu-San Pham,
Minh Hoang Ha, Louis-Martin Rousseau*

November 3rd 2022 - focus period Linköping

HOME HEALTHCARE (HHC)

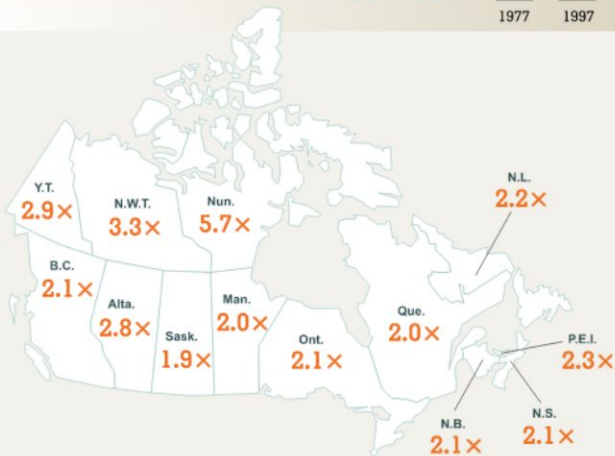
Canada's seniors population outlook: Uncharted territory

Over the next 20 years,
Canada's seniors population
is expected to
grow by **68%**



The 75+
age group
will
double

Canada 
2.1x



- Lack of medical resources and expensive health care service costs
- People want to stay at home as long as possible
- Cost-effective and flexible
 - costs **32%** less than hospital care
- HHC services is one of the fastest growing market in the US and Canada
 - In Canada, **2.2 million** people relied on home care services. (2012)

HHC SCHEDULING

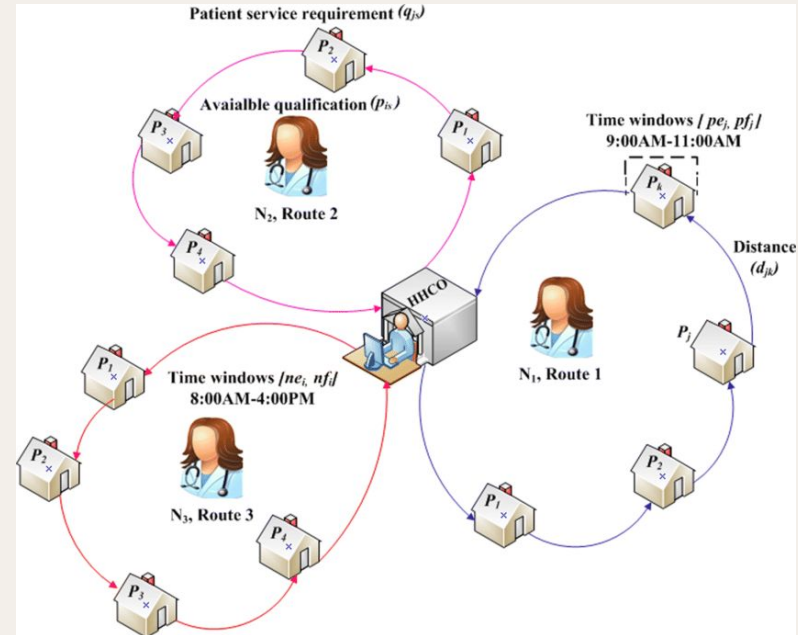
Mix between an **assignment problem** and a **VRP**

Hard constraints

- Nurse skills
- Type of care
- Forbidden nurses
- Time windows
- Available days
- Workdays
- Time-dependent travel time

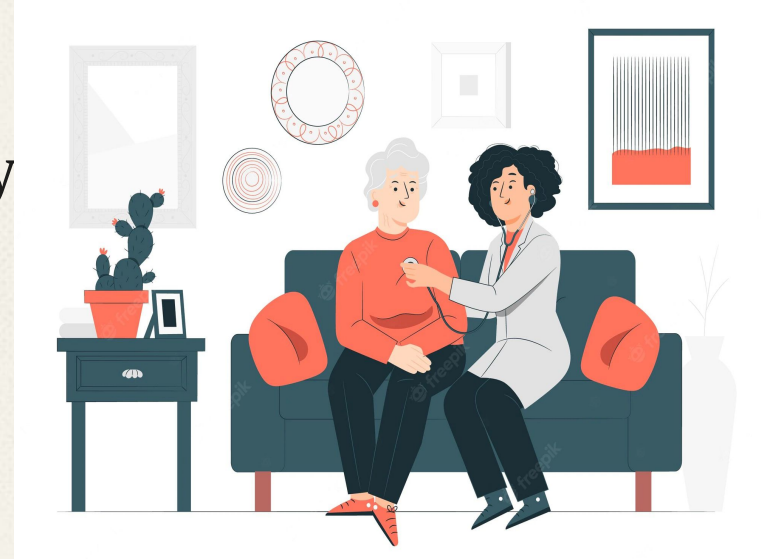
Soft constraints

- Continuity of care
- Optional requirements
- Travel time
- **Min/Max** worktime week
- **Min/Max** worktime workday
- Number of visits over the week



HOME HEALTHCARE (HHC)

- Delivery of professional care by workers in a client's home.
- Objective: increase service quality and decrease costs



CHALLENGES



PROBLEM DESCRIPTION

- Patient requests

- Episode of cares (nb of weeks)
- Frequency (nb of visits per week)
- Visit pattern

- Nurses

- Skill level
- Regulation

- Constraints

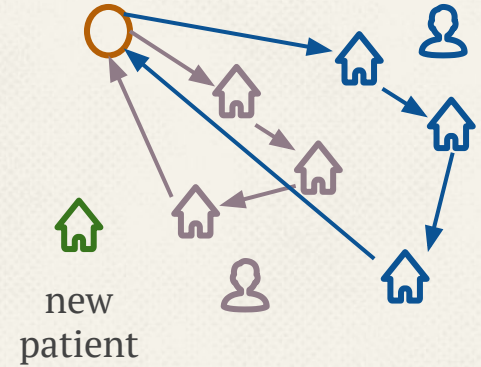
- Continuity of care
- Time window..

| | Time slot 1 | Time slot 2 | Time slot 3 | Time slot 4 | Time slot 5 | Time slot 6 | Time slot 7 | Time slot 8 | Time slot 9 | Time slot 10 | Time slot 11 | Time slot 12 | Time slot 13 | Time slot 14 | Time slot 15 |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Monday | | Patient 1 | | → | Patient 2 | | | | | | | | | | |
| Tuesday | | | | | | Patient 2 | | | → | → | Patient 3 | | | | |
| Wednesday | | Patient 1 | | → | Patient 2 | | | | | | | | | | |
| Thursday | | Patient 1 | | → | Patient 2 | | | → | → | Patient 3 | | | | | |
| Friday | | | | | | Patient 2 | | | | | | | | | |

→
Travel time

THE DYNAMIC HHCSP (DHHCSP)

- Patient's request arrives dynamically.
- Decision:
 - Accept or reject?
 - Online decision
- Obj: maximize the number of patients served.



| | Time slot 1 | Time slot 2 | Time slot 3 | Time slot 4 | Time slot 5 | Time slot 6 | Time slot 7 | Time slot 8 | Time slot 9 | Time slot 10 | Time slot 11 | Time slot 12 | Time slot 13 | Time slot 14 | Time slot 15 |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Monday | | Yellow bar | | | → | Green bar | | | | | | | | | |
| Tuesday | | | | | | Green bar | | | → | → | Blue bar | | | | |
| Wednesday | | Yellow bar | | | → | Green bar | | | | | | | | | |
| Thursday | | Yellow bar | | | → | Green bar | | | → | → | Blue bar | | | | |
| Friday | | | | | Green bar | | | | | | | | | | |

Nurse 1



| | Time slot 1 | Time slot 2 | Time slot 3 | Time slot 4 | Time slot 5 | Time slot 6 | Time slot 7 | Time slot 8 | Time slot 9 | Time slot 10 | Time slot 11 | Time slot 12 | Time slot 13 | Time slot 14 | Time slot 15 |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|---------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Monday | | | Blue bar | | | | | | | | | | | | |
| Tuesday | | | Blue bar | | | | | | | | | | | | |
| Wednesday | | | | Orange bar | | | → | Dark grey bar | | | | | | | |
| Thursday | | | Blue bar | | | | | | | | | | | | |
| Friday | | | | Orange bar | | | → | Dark grey bar | | | | | | | |

Nurse 2

SOLUTION APPROACH

- *Greedy approach*
- *Scenario-based*
- *Reinforcement learning*

DISTANCE-BASED INSERTION HEURISTIC

Bennett & Erera (2011)

Greedy heuristic:

- Each nurse
- Each visit pattern
- Find the cheapest insertion
- Accept if there's an eligible slot
- Reject otherwise

Nurse

N1 -> 600

N2 -> 400

N3 -> 300

Visit pattern

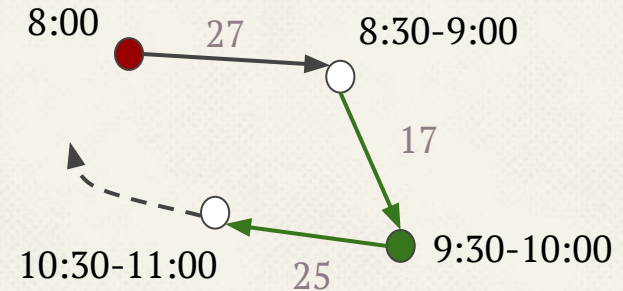
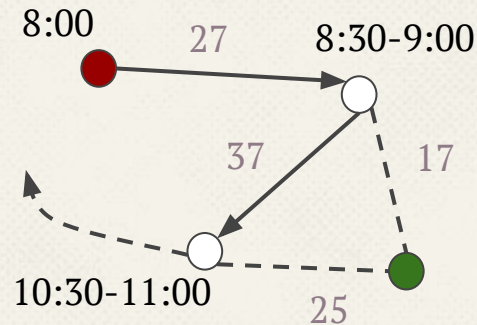
(M, W) -> 100

(M, Th) -> 60

(Tu, Th) -> 70

Route

Cost: total travelling time



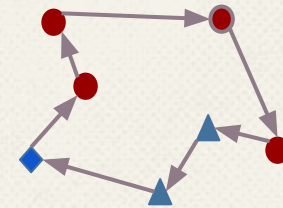
SCENARIO BASED APPROACH (SBA)

Demirbilek et al. (2019a)

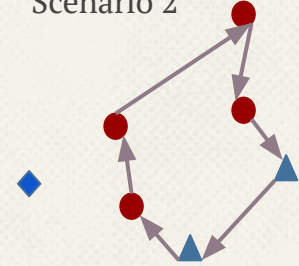
1. **Generate** a set of future scenarios
2. For each scenario
3. Run **greedy heuristic**
4. If the patient is not accepted in any scenario
5. **Reject**
6. Else
7. **Schedule** the patient to the most frequently assigned

- Random request
- ▲ Already scheduled visit
- ◆ Current request

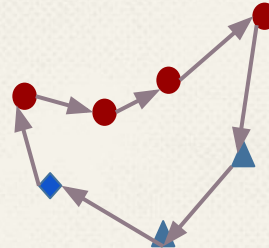
Scenario 1



Scenario 2

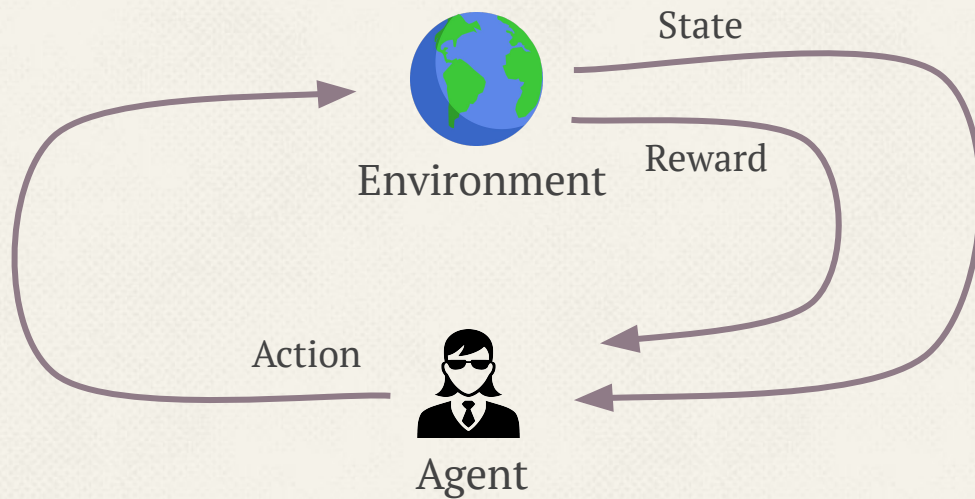


Scenario 75



| | Condition | Time |
|-----|------------------|-------------|
| S1 | Accepted | 9:00 |
| S2 | Rejected | |
| S3 | Accepted | 10:00 |
| ... | | |
| S75 | Accepted | 9:00 |
| | | <hr/> |
| | Accepted | 9:00 |

REINFORCEMENT LEARNING

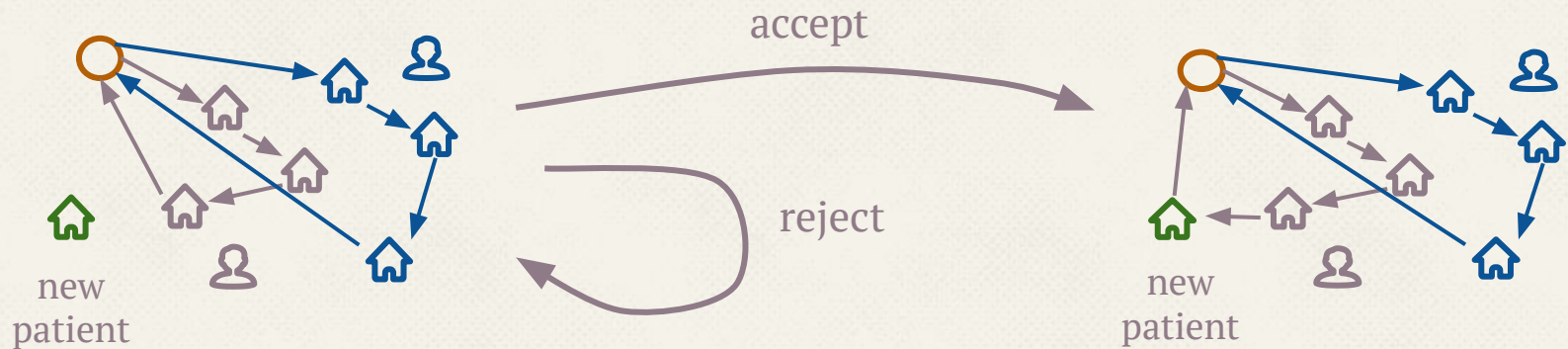


Interacts in a
Markov Decision Process

REINFORCEMENT LEARNING FOR DYNAMIC REQUEST ACCEPTANCE

Decision: when a new request arrives

| STATE | ACTION | REWARD | POST-DECISION STATE |
|---|--|--------|-------------------------|
| Request's information and current schedules | Accept, schedule patient using a heuristic | 1 | New schedule |
| | Reject | 0 | Schedule doesn't change |

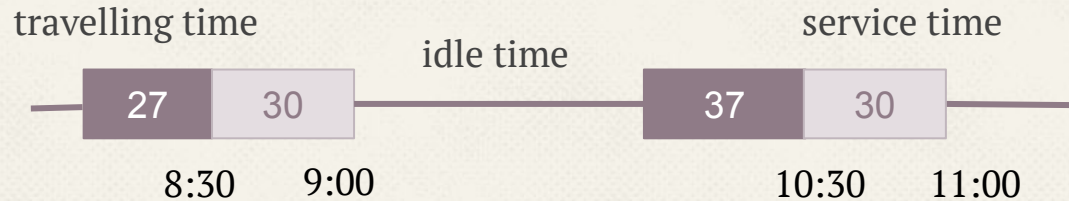
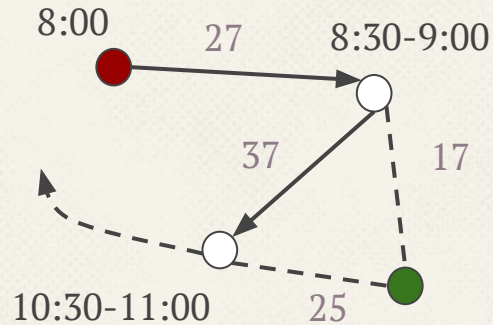


STATE PRESENTATION

| | | | |
|--------------------------|-----------------------|-----|-------------------------|
| Patient's characteristic | Nurse j's information | ... | Cheapest insertion cost |
|--------------------------|-----------------------|-----|-------------------------|

- *Episode of care*
- *Frequency*
- *Duration of visits*

- *Number of assigned visits*
- *Total idle time*
- *Total traveling time*



DOUBLE DEEP Q-LEARNING WITH EXPERIENCE REPLAY

```
1: Initialize replay memory  $\mathcal{D}$ 
2: Initialize network  $\theta$ , target network  $\hat{\theta}$ 
3: for  $episode = 1, M$  do
4:    $s_t \leftarrow s_0$ 
5:   for  $t = 0, T$  do
6:     With probability  $\epsilon$  select a random action  $a_t$ 
7:     Execute  $a_t$ , observe  $r_t, s_{t+1}$ 
8:     Store  $(s_t, a_t, r_t, s_{t+1})$  in  $\mathcal{D}$ 
9:      $s_t \leftarrow s_{t+1}$ 
10:    Sample random minibatch of  $\mathcal{N}$  transitions from  $\mathcal{D}$ 
11:    for every transition  $(s_j, a_j, r_j, s_{j+1})$  in minibatch do
12:       $y_j \leftarrow r_j + \gamma Q_{\theta}(s_{j+1}, \operatorname{argmax}_{a'} Q_{\hat{\theta}}(s_{j+1}, a'))$ 
13:    end for
14:    Calculate the loss  $\mathcal{L} = 1/N \sum_{j=0}^{N-1} (y_j - Q(s_j, a_j; \theta))^2$ 
15:    Update  $\theta$  by gradient descent by minimizing the loss  $\mathcal{L}$ 
16:    Update target network  $\hat{\theta} \leftarrow \theta$  every  $K$  steps
17:  end for
18: end for
```

Calculate target

Back-tracking

Update target network every K steps

Experimental results

EXPERIMENTAL DESIGN

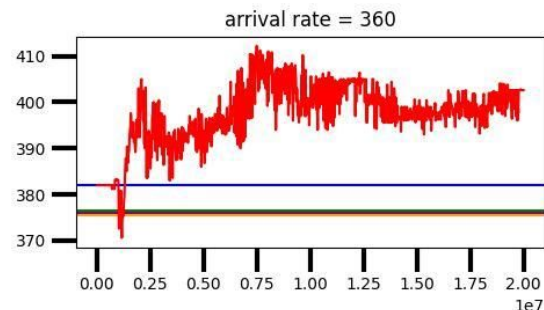
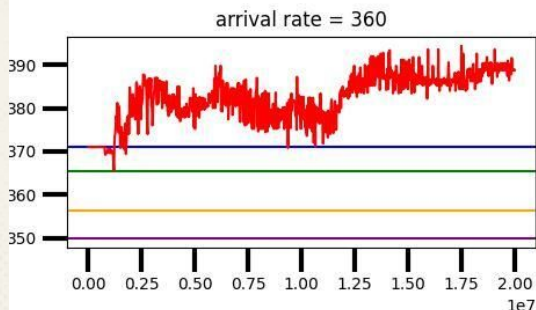
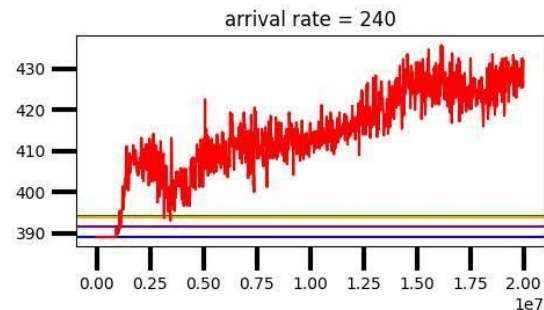
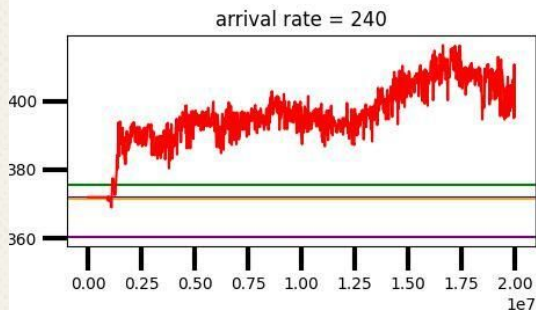
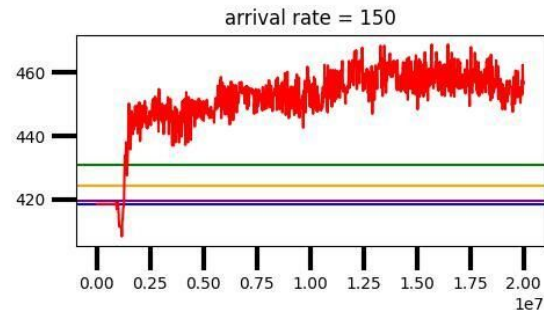
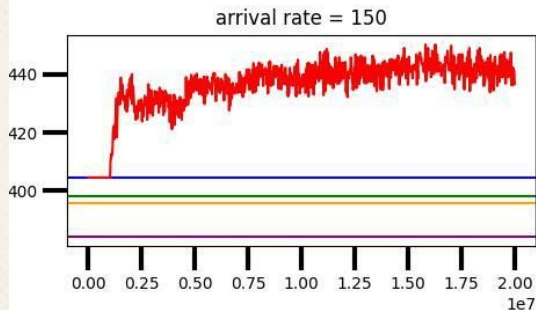
- Nurses,
 - Working time 8:00 to 16:30
 - 3 skill levels
- Patients
 - Location: uniform distribution in a square of 80*80
 - Travelling time: 1' per unit
 - Time between requests: exponential distribution with expected value
 - 150 (~10 requests/day)
 - 240 (~6 requests/day)
 - 360 (4 requests/day)
- Baselines
 - Greedy
 - SBA

1 nurse

Avg. decision time (s)

SBA 10.17

RL 0.033

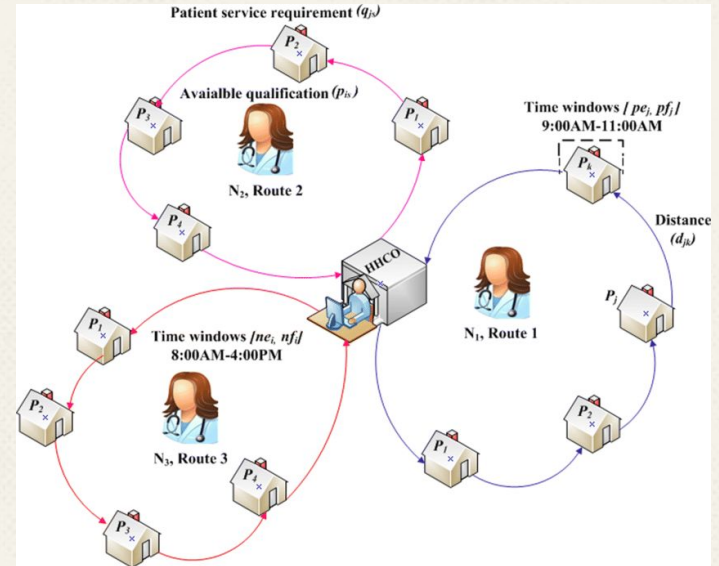


Uniform

Cluster

CONCLUSION AND FUTURE WORK

- RL approach for a dynamic HHC scheduling problem
- Future work
 - Multiple objectives (soft constraints, travelling time...)
 - More sources of uncertainty



THANKS!

Any questions?

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