Machine learning-based algorithms for dynamic patient scheduling problems with uncertainty

Tu-San Pham, Antoine Legrain, Patrick De Causmaecker, Louis-Martin Rousseau

November 4th 2022 - Focus Period Linköping
OPTIMIZATION IN THE HEALTHCARE DOMAIN

Time to tackle health-care wait times in Canada
— Mackenzie Moir, Bacchus Barua

https://nationalpost.com › news › canada › why-canada...
Why Canada's hospital capacity was so easily overwhelmed ...
Jan 17, 2022 — The latest numbers from the OECD show Canada with just one hospital bed for every 400 citizens, a ratio that puts us in the bottom tier of OECD ...

Canada Research Chair in Analytics and Logistics in Healthcare

SeniorS and HouSing: THE Challenge ahead
As Canada's aging population grows, and as Canadians live longer, there is an overwhelming desire among seniors to "age in place." But how can this be achieved when the country is facing a severe shortage of housing for seniors?

Overwhelming evidence—it’s time to fix Canadian health care
— Appeared in the Montreal Gazette, September 9, 2022

Overwhelming evidence—Canada's health care system is failing the elderly, and it's time to fix it.

Optimizing treatment planning and delivery in healthcare

Mental health among healthcare workers in Canada during the COVID-19 pandemic
Release date: 2021-02-02

How can Canada improve worsening wait times? - CMAJ News
Aug 28, 2020 — Simpson: Canadians have accepted that some delay is reasonable in order to have an equitable health care system, and that's not necessarily a ...

Canada ranks last on number of hospital beds, wait times
For example, Canada ranks 26th (out of 28 countries) for the number of doctors (2.8 per 1,000 people) and 26th (out of 27) for the number of hospital beds ...
UNCERTAINTY IN HEALTHCARE SCHEDULING

- Dynamic problems
- Online decisions

- Stochastic optimization
- MDP - (approximate) dynamic programming
- Simulation-based

Machine-learning based approaches

- Deferrals
- Resources
- Staff availability
- No-shows Cancellations
- Stochastic patient arrivals
- Stochastic service time

Scheduling
Radiotherapy scheduling for cancer treatments
Cancer incidence (2020)
- 10,000,000 deaths
- 1 out of 6 deaths

Challenges
- Growing and aging population

How to reduce death rates?
- Early detection
- Early treatment

Optimize treatment scheduling

https://www.who.int/news-room/fact-sheets/detail/cancer
Optimizing radiotherapy treatment schedules to reduce patients’ waiting times
RADIOTHERAPY TREATMENT

- Linear accelerators (linac)
- **Fraction**: a small dose of radiation
- Treatment plan
  - Multiple consecutive fractions
  - Fraction duration

Patient scheduling with multi-appointments, multi-resources
LITERATURE

- **Markov Decision Process & Approximate Dynamic Programming (ADP)**
  - Patrick et al. 2008
  - Saure et al. 2012, 2020 (3 linacs)
  - Gocgun 2018
- **Stochastic Programming**
  - Legrain et al. 2015 (2 linacs)
CHUM - CENTRE HOSPITALIER DE L'UNIVERSITÉ DE MONTRÉAL

10 linacs
5 generics
4 specialized
1 cyberknife

4400 consultations
3500 new patients
40,000 fractions
(2019)

https://www.graysuite.com/
2021 INNOVE-ACTION BEST STARTUP AWARD
Objective: minimizing overdue treatment and waiting time

<table>
<thead>
<tr>
<th>Category</th>
<th>Percentages (%)</th>
<th>Treatment deadline (days)</th>
<th>Percentage of overdue treatment (%)</th>
<th>Average waiting time (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.4</td>
<td>1</td>
<td>14.29</td>
<td>1.09</td>
</tr>
<tr>
<td>P2</td>
<td>27.2</td>
<td>3</td>
<td>79.89</td>
<td>6.91</td>
</tr>
<tr>
<td>P3</td>
<td>41.4</td>
<td>14</td>
<td>74.55</td>
<td>18.11</td>
</tr>
<tr>
<td>P4</td>
<td>31.0</td>
<td>28</td>
<td>29.89</td>
<td>22.59</td>
</tr>
</tbody>
</table>

Palliative

Curative
THE MAIN CHALLENGES

- Preserving linac capacity
  - Reserved capacity vs occupancy rate?
- A prediction-based approach
  - Learn to delay low-priority patients
Scheduling strategies

- Batch scheduling
- Offline scheduling
- Online scheduling
- Prediction-based approach
BATCH SCHEDULING

number of days of simulation \( l = 10 \)

#patients admitted

\[
\begin{array}{cccccccccc}
9 & 7 & 5 & 8 & 10 & 12 & 4 & 7 & 10 & 6 \\
\end{array}
\]

day index

0 1 2 3 4 5 6 7 8 9 ...

Scheduling decision
Palliative patients: schedule at arrival
AN INTEGER PROGRAMMING MODEL FOR BATCH SCHEDULING

\[ x^i_{tl} = \begin{cases} 
1 & \text{if patient } i \text{ receives their treatment on day } t, \text{ linac } l \\
0 & \text{otherwise} 
\end{cases} \]

minimize \[ \sum_{i \in \hat{P}} \sum_{t \in T, t > a_i} \sum_{l \in \mathcal{L}} \omega_1 (t - a_i) \log(t - a_i + 1) x^i_{tl} \]

\[ + \sum_{i \in \hat{P}} \sum_{t \in T, t > d_i} \sum_{l \in \mathcal{L}} \omega_2 (t - d_i) \log(t - d_i + 1) x^i_{tl} \]

waiting time

overdue time
\[
\sum_{i \in \hat{P}} \sum_{l \in \mathcal{L}} x_{il}^i = 1 \quad \text{assignment constraint} \quad \forall i \in \hat{P}
\]

\[
x_{il}^i = 0 \quad \text{ready date} \quad \forall i \in \hat{P}, l \in \mathcal{L}, t \in \{0, \ldots, r_i - 1\}
\]

\[
\sum_{i \in \hat{P}} \sum_{t' = \max\{0, t - I_i + 1\}}^t p_i x_{t'l}^i \leq \hat{C}_l^t \quad \text{capacity constraints} \quad \forall t \in \mathcal{T}, l \in \mathcal{L}
\]

\[
\sum_{i \in \mathcal{P}^c} \sum_{t' \in \{t - I_i + 1, \ldots, t\}} p_i x_{t'l}^i \leq \max\{0, \hat{C}_l^t - \gamma C_l^t\} \quad \text{reserved capacity} \quad \forall t \in \mathcal{T}, l \in \mathcal{L}
\]

\[
x_{il}^i \in \{0, 1\} \quad \forall i \in \hat{P}, t \in \mathcal{T}, l \in \mathcal{L}
\]
OFFLINE SCHEDULING - THE PERFECT SCENARIO

number of days of simulation $l = 10$

Future arrivals are known in advance

Scheduling decision

#patients admitted

9  7  5  8  10  12  4  7  10  6

day index

0  1  2  3  4  5  6  7  8  9  ...

0
ONLINE SCHEDULING WITH A GREEDY HEURISTIC

- 1 linac, capacity 120 time slots
- A curative patient (P4) with 3 sessions, 10 time slots each

Looking for the first eligible date that can accommodate the whole treatment

Remaining linac capacity

Patient admitted

Start looking at one/two weeks after admission

The first eligible date
How do we predict a “good” starting date for a patient?

- 1 linac, capacity 120 time slots
- a curative patient with 3 sessions, 10 time slots each

Looking for the first eligible date that can accommodate the whole treatment

Remaining linac capacity

Patient admitted

Predicted starting date given by a regression model

The first eligible date
TRAINING THE REGRESSION MODEL

Start

Problem instances

Offline scheduling

Generating training data

Offline solutions

Train regression model

Training the regression model

Training data

Datapoints:
\[ X_i = \{ r_i, I_i, d_i, p_i, \hat{C}_{\phi,\phi=\alpha_i} \} \Rightarrow \mathcal{X} \]

Labels:
\[ y_i = w_i \Rightarrow \mathcal{Y} \]

Estimate:
\[ \xi : \mathcal{X} \rightarrow \mathcal{Y} \]

Objective:
\[ \min \mathcal{L} \]
Numerical results

- Data generation
- Model selection
- Results on simulated data
- Results on real data
- Explainability with SHAP values
DATA GENERATION

- **Patient arrivals**: Poisson distribution
- **Treatment plans**: based on historical data
- **Instance setting**
  - Number of linacs
  - Arrival rate (average daily number of patients)
- **For each instance setting**: 500 instances
  - 400 for training the regression model
  - 100 for testing
## PREDICTION MODELS

<table>
<thead>
<tr>
<th>Model</th>
<th>Training time</th>
<th>Training MSE</th>
<th>Training MAE</th>
<th>Testing MSE</th>
<th>Testing MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLP</td>
<td>116.19</td>
<td>3.45</td>
<td>1.32</td>
<td>3.33</td>
<td>1.29</td>
</tr>
<tr>
<td>SGD</td>
<td>0.35</td>
<td>6.06</td>
<td>1.84</td>
<td>5.61</td>
<td>1.77</td>
</tr>
<tr>
<td>Lasso</td>
<td>0.44</td>
<td>5.97</td>
<td>1.81</td>
<td>5.52</td>
<td>1.74</td>
</tr>
<tr>
<td>ElasticNet</td>
<td>0.25</td>
<td>6.26</td>
<td>1.85</td>
<td>5.83</td>
<td>1.8</td>
</tr>
<tr>
<td>SVR</td>
<td>43.16</td>
<td>3.19</td>
<td>1.07</td>
<td>3.12</td>
<td>1.07</td>
</tr>
<tr>
<td>Decision Tree</td>
<td>0.84</td>
<td>2.41</td>
<td>0.48</td>
<td>6.59</td>
<td>1.4</td>
</tr>
<tr>
<td>Random forest</td>
<td>51</td>
<td><strong>0.38</strong></td>
<td><strong>0.39</strong></td>
<td>2.64</td>
<td>1.03</td>
</tr>
<tr>
<td>XGBoost</td>
<td>7.71</td>
<td>0.96</td>
<td>0.66</td>
<td><strong>2.44</strong></td>
<td><strong>0.97</strong></td>
</tr>
</tbody>
</table>
## Scheduling Strategies

<table>
<thead>
<tr>
<th>Scheduling strategy</th>
<th>Scheduling palliative patients</th>
<th>Scheduling curative patients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offline</td>
<td>Scheduling once with all future arrivals known in advance</td>
<td></td>
</tr>
<tr>
<td>Daily</td>
<td>Every day</td>
<td>Every day</td>
</tr>
<tr>
<td>Weekly</td>
<td>Every day</td>
<td>Every Friday</td>
</tr>
<tr>
<td>Daily greedy</td>
<td>Every day</td>
<td>Every day</td>
</tr>
<tr>
<td>Greedy</td>
<td>At admission</td>
<td>At admission</td>
</tr>
<tr>
<td>Prediction-based</td>
<td>At admission</td>
<td>At admission</td>
</tr>
</tbody>
</table>
4 LINACS

Arrival rate of 5.0

Arrival rate of 6.0

[Box plots showing average waiting time per patient and average overdue time per patient for different scenarios labeled P1 to P4.]
8 LINACS

Arrival rate of 10.0

Arrival rate of 12.0
EXPERIMENT ON A REAL PATIENT FLOW

- 7 linacs operating 8 hours/day
- High fluctuation in arrival rate
  - Instance setting for training: arrival rate of 10.1
  - Simulation horizon: 30 days
## RESULTS ON THE REAL INSTANCE

<table>
<thead>
<tr>
<th>Scheduling strategy</th>
<th>Avg. occupancy (%)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Overall</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>Overall</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>online-greedy</td>
<td>97.45</td>
<td>33.02</td>
<td>5.14</td>
<td>6.13</td>
<td>43.67</td>
<td>44.02</td>
<td>44.02</td>
<td>5.14</td>
<td>3.91</td>
<td>29.74</td>
<td>16.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>daily-greedy</td>
<td>97.51</td>
<td>32.91</td>
<td>6.00</td>
<td>6.23</td>
<td>43.48</td>
<td>43.80</td>
<td>17.71</td>
<td>6.00</td>
<td>3.99</td>
<td>29.58</td>
<td>16.00</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>daily</td>
<td>97.72</td>
<td>33.53</td>
<td>9.79</td>
<td>9.63</td>
<td>42.87</td>
<td>43.44</td>
<td>18.25</td>
<td>9.79</td>
<td>7.15</td>
<td>28.93</td>
<td><strong>15.65</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weekly</td>
<td>97.61</td>
<td>33.04</td>
<td>7.86</td>
<td>7.72</td>
<td><strong>42.42</strong></td>
<td>44.10</td>
<td>17.76</td>
<td>7.86</td>
<td>5.37</td>
<td><strong>28.51</strong></td>
<td>16.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prediction-based</td>
<td>97.14</td>
<td>32.93</td>
<td><strong>3.29</strong></td>
<td><strong>4.05</strong></td>
<td>44.21</td>
<td>44.94</td>
<td>17.69</td>
<td><strong>3.29</strong></td>
<td><strong>1.99</strong></td>
<td>30.22</td>
<td>16.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
EXPLAINABILITY
WITH SHAPLEY VALUES

○ **SHAP** (SHapley Additive exPlanations)
  □ represent the relative strength of a variable on the outcome
○ Widely use for explainability in machine learning
○ Highly appreciated in the healthcare domain
GLOBAL INTERPRETATION

BEESWARM PLOT

SHAP value (impact on model output)

noSections
dueday
duration
c49
c12
c48
c11
c10
c13
c35
c26
c37
c36
readyday
c14
c23
c28
c24
c5
Sum of 35 other features

High
Low

6 linacs - arrival rate 9.0
CONCLUSIONS AND CHALLENGES?

- A **machine learning-based** approach for **online dynamic patient scheduling**
- Empirical results
  - Improve overdue times of palliative patients, especially on large and crowded hospitals
  - Not too sensitive with the fluctuation on the arrival rate
- Explainability with Shapley values
- Challenges: generating offline solutions is expensive
A reinforcement learning approach for the dynamic home health care scheduling and routing problem

Ta Dinh Quy, Tu-San Pham,
Minh Hoang Ha, Louis-Martin Rousseau

November 3rd 2022 - focus period Linköping
HOME HEALTHCARE (HHC)

○ Lack of medical resources and expensive health care service costs
○ People want to stay at home as long as possible
○ Cost-effective and flexible
  □ costs 32% less than hospital care
○ HHC services is one of the fastest growing market in the US and Canada
  □ In Canada, 2.2 million people relied on home care services. (2012)

Mix between an assignment problem and a VRP

<table>
<thead>
<tr>
<th>Hard constraints</th>
<th>Soft constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nurse skills</td>
<td>Continuity of care</td>
</tr>
<tr>
<td>Type of care</td>
<td>Optional requirements</td>
</tr>
<tr>
<td>Forbidden nurses</td>
<td>Travel time</td>
</tr>
<tr>
<td>Time windows</td>
<td>Min/Max worktime week</td>
</tr>
<tr>
<td>Available days</td>
<td>Min/Max worktime workday</td>
</tr>
<tr>
<td>Workdays</td>
<td>Number of visits over the week</td>
</tr>
<tr>
<td>Time-dependent travel time</td>
<td></td>
</tr>
</tbody>
</table>
HOME HEALTHCARE (HHC)

- Delivery of professional care by workers in a client’s home.
- Objective: increase service quality and decrease costs
CHALLENGES

3.500,000 visits/year in an avg agency

- Staff availability
- Client needs & preferences
- Travel routes
- Continuity of care
- Union rules
- Stochastic travelling time
- Stochastic request arrival
- Cancellations
- Sick leaves
## Problem Description

- **Patient requests**
  - Episode of cares (nb of weeks)
  - Frequency (nb of visits per week)
  - Visit pattern

- **Nurses**
  - Skill level
  - Regulation

- **Constraints**
  - Continuity of care
  - Time window

<table>
<thead>
<tr>
<th></th>
<th>Time slot 1</th>
<th>Time slot 2</th>
<th>Time slot 3</th>
<th>Time slot 4</th>
<th>Time slot 5</th>
<th>Time slot 6</th>
<th>Time slot 7</th>
<th>Time slot 8</th>
<th>Time slot 9</th>
<th>Time slot 10</th>
<th>Time slot 11</th>
<th>Time slot 12</th>
<th>Time slot 13</th>
<th>Time slot 14</th>
<th>Time slot 15</th>
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</thead>
<tbody>
<tr>
<td>Monday</td>
<td>Patient 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Patient 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tuesday</td>
<td></td>
<td>Patient 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Patient 3</td>
<td></td>
</tr>
<tr>
<td>Wednesday</td>
<td>Patient 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Patient 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thursday</td>
<td>Patient 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Patient 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Patient 3</td>
<td></td>
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<tr>
<td>Friday</td>
<td>Patient 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Patient 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Travel time*
THE DYNAMIC HHCSP (DHHCSP)

- Patient’s request arrives dynamically.
- Decision:
  - Accept or reject?
  - Online decision
- Obj: maximize the number of patients served.
SOLUTION APPROACH

- Greedy approach
- Scenario-based
- Reinforcement learning
**DISTANCE-BASED INSERTION HEURISTIC**

Bennett & Erera (2011)

**Greedy heuristic:**
- Each nurse
- Each visit pattern
- Find the cheapest insertion
- Accept if there’s an eligible slot
- Reject otherwise

<table>
<thead>
<tr>
<th>Nurse</th>
<th>Visit pattern</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>(M, W)</td>
<td>600</td>
</tr>
<tr>
<td>N2</td>
<td>(M, Th)</td>
<td>400</td>
</tr>
<tr>
<td>N3</td>
<td>(Tu, Th)</td>
<td>300</td>
</tr>
</tbody>
</table>

**Visit pattern**
- (M, W) -> 100
- (M, Th) -> 60
- (Tu, Th) -> 70

**Route**

Cost: total travelling time

8:00 - 8:30 | 27
8:30-9:00 | 37
10:30-11:00 | 25
9:30-10:00 | 17
1. **Generate** a set of future scenarios
2. For each scenario
3. Run **greedy heuristic**
4. If the patient is not accepted in any scenario
5. **Reject**
6. Else
7. **Schedule** the patient to the most frequently assigned

---

**Condition | Time**
---
S1 | Accepted | 9:00
S2 | Rejected |
S3 | Accepted | 10:00
... |
S75 | Accepted | 9:00

---

**Random request**

**Already scheduled visit**

**Current request**
REINFORCEMENT LEARNING

Agent

Environment

Interacts in a Markov Decision Process

State

Reward

Action
REINFORCEMENT LEARNING
FOR DYNAMIC REQUEST ACCEPTANCE

Decision: when a new request arrives

<table>
<thead>
<tr>
<th>STATE</th>
<th>ACTION</th>
<th>REWARD</th>
<th>POST-DECISION STATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Request’s information and current schedules</td>
<td>Accept, schedule patient using a heuristic</td>
<td>1</td>
<td>New schedule</td>
</tr>
<tr>
<td>Reject</td>
<td></td>
<td>0</td>
<td>Schedule doesn’t change</td>
</tr>
</tbody>
</table>

**Diagram**

- **Accept**
  - new patient
- **Reject**
  - new patient
STATE PRESENTATION

<table>
<thead>
<tr>
<th>Patient’s characteristic</th>
<th>Nurse j’s information</th>
<th>...</th>
<th>Cheapest insertion cost</th>
</tr>
</thead>
</table>

- **Episode of care**
- **Frequency**
- **Duration of visits**

- **Number of assigned visits**
- **Total idle time**
- **Total traveling time**

![Diagram showing episode of care, frequency, and duration of visits with corresponding times and costs.]
DOUBLE DEEP Q-LEARNING WITH EXPERIENCE REPLAY

1: Initialize replay memory $D$
2: Initialize network $\theta$, target network $\hat{\theta}$
3: for episode = 1, $M$ do
4:   $s_t \leftarrow s_0$
5:   for $t = 0, T$ do
6:     With probability $\epsilon$ select a random action $a_t$
7:     Execute $a_t$, observe $r_t$, $s_{t+1}$
8:     Store $(s_t, a_t, r_t, s_{t+1})$ in $D$
9:     $s_t \leftarrow s_{t+1}$
10:    Sample random minibatch of $N$ transitions from $D$
11:   for every transition $(s_j, a_j, r_j, s_{j+1})$ in minibatch do
12:      $y_j \leftarrow r_j + \gamma Q_\theta(s_{j+1}, \arg\max_{a'} Q_\hat{\theta}(s_{j+1}, a'))$
13:   end for
14:   Calculate the loss $\mathcal{L} = 1/N \sum_{j=0}^{N-1} (y_j - Q(s_j, a_j; \theta))^2$
15:   Update $\theta$ by gradient descent by minimizing the loss $\mathcal{L}$
16:   Update target network $\hat{\theta} \leftarrow \theta$ every $K$ steps
17:   end for
18: end for
Experimental results
EXPERIMENTAL DESIGN

● Nurses,
  ○ Working time 8:00 to 16:30
  ○ 3 skill levels

● Patients
  ○ Location: uniform distribution in a square of 80*80
  ○ Travelling time: 1’ per unit
  ○ Time between requests: exponential distribution with expected value
    ■ 150 (~10 requests/day)
    ■ 240 (~6 requests/day)
    ■ 360 (4 requests/day)

● Baselines
  ○ Greedy
  ○ SBA
1 nurse

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Avg. decision time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBA</td>
<td>10.17</td>
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<tr>
<td>RL</td>
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</tbody>
</table>
CONCLUSION AND FUTURE WORK

- RL approach for a dynamic HHC scheduling problem
- Future work
  - Multiple objectives (soft constraints, travelling time...)
  - More sources of uncertainty
THANKS!

Any questions?

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