Decomposition approaches for a large-scale scheduling problem

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What?

A challenging scheduling problem

- Electronic systems in aircraft
- Multiprocessor scheduling with precedence relations between tasks
  (lb and ub on time lag)
  & communication network
- Feasibility: find a schedule or prove that none exists

Of importance for development of future aircraft

Why?

- Make sure software functions are assigned the hardware resources they need: run applications & pass data
- Configurable system—each configuration needs a schedule
- Part of design process:
  - iterative development and changes
  - changes → new schedule
  - updates during whole life-time (decades)

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- Scheduling fails → costly design changes
- Heuristic approaches not suitable to determine feasibility
- Large-scale problem: Out of reach for generic DO solvers

Who?

Close collaboration between Linköping University and Saab

- LiU side: Optimisation perspective
- Saab side: Technical perspective
- Joint team
  - Elina Rönnberg (research leader, LiU & Saab)
  - Robert Petersson (technical leader at Saab)
  - Emil Karlsson (PhD student, LiU & Saab)
  - Andreas Stenberg (software developer, Saab)
  - Hannes Uppman (algorithm developer, Saab)
  - some more software developers / system engineers at Saab
  - steering group with managers and technical fellows

How?

Pre-runtime scheduling tool = method development

- Modelling + decomposition approaches that exploit
  - problem structure
  - power of generic solvers
- Exploring data ⇒
  - preprocessing
  - adapt decomposition

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Today's talk: Overview of two decomposition approaches, the use of MIP vs. CP, and the importance of understanding the data

Outline

Introduction

Technical background

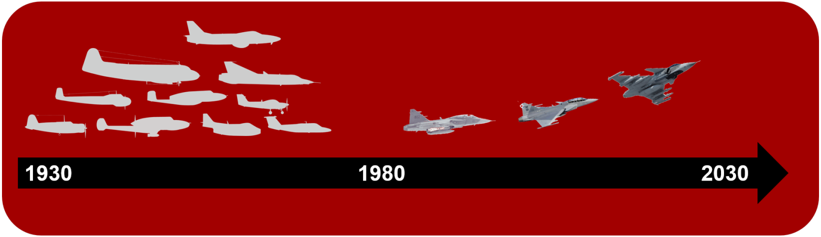
Problem formulation

Decomposition approaches

Concluding comments

Aircraft development

From conquering the laws of physics—
to becoming computers with wings



Avionics

Electronics in an aircraft

- sensors that collect information
- units where the information is processed
- actuators that control the aircraft
- equipment that presents information to the pilot

Prescribe—down to the nanosecond—what the electronics does

Avionics design

Making sure that the system can be trusted is key

Avionics design

Making sure that the system can be trusted is key

Examples of aspects:

- Create subsystems that can be validated independently
- Prevent faults from propagating between functions
- All possible scenarios are covered and evaluated
- Information is correct and protected from unauthorized access

Extensive documentation, testing and certification processes

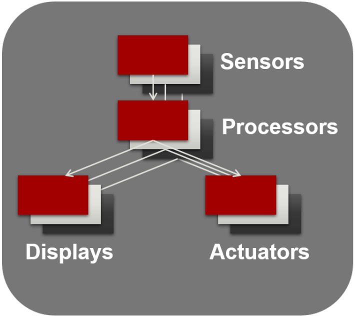
Classic design

Federated system

- Each function has a separate hardware
- A hard-wired system

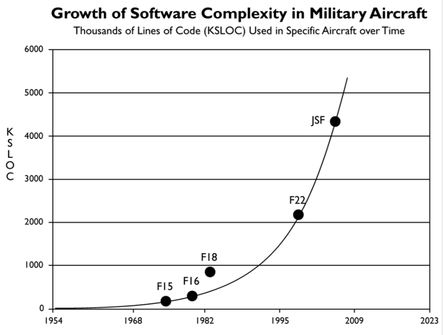
+ Simple integration and verification

– Limited synergy and system integration



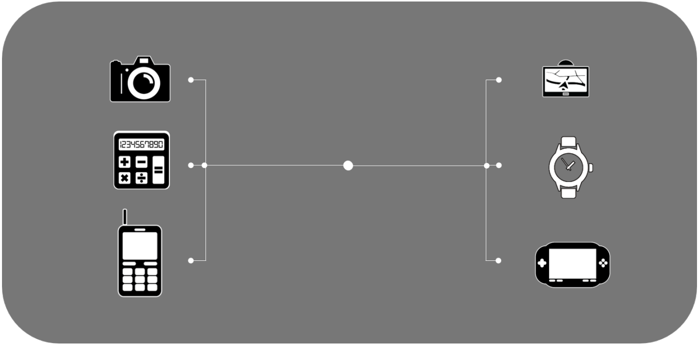
Times are changing

- Digital systems and computers introduced new possibilities
- System complexity increases over time
- New needs:
  - upgradable
  - adaptable
  - reconfigurable



https://savi.avsi.aero/about-savi/savi-motivation/exponential-system-complexity/

Change of design philosophy—an analogy



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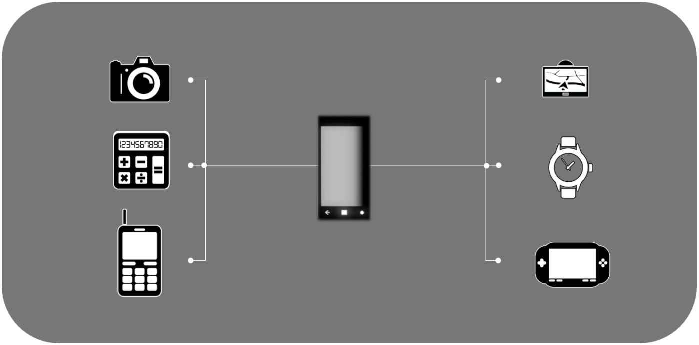

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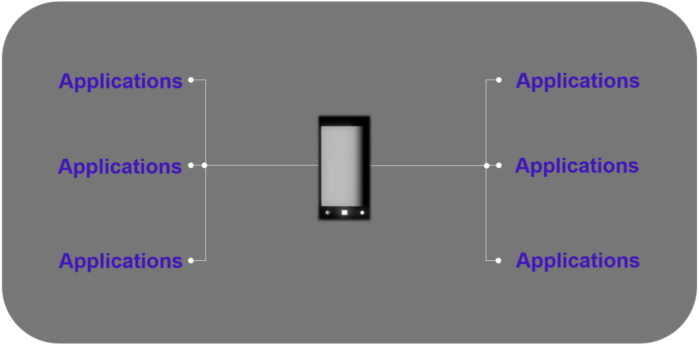
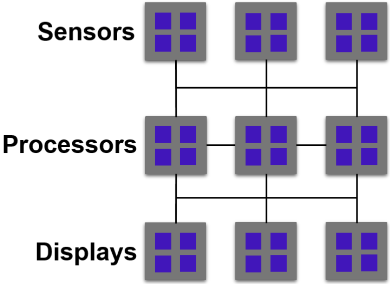


Modern design

IMA: Integrated modular avionics

- Shared hardware



Modern design

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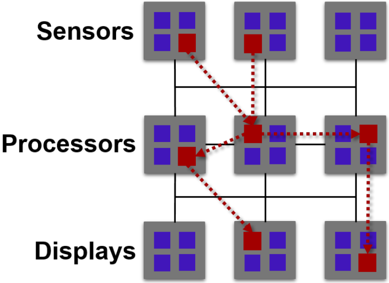
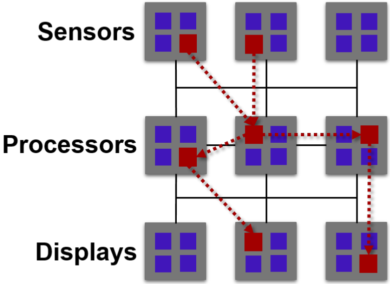
- Software defines the functionality



Modern design

IMA: Integrated modular avionics

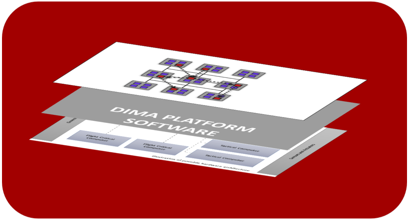
- ▶ Shared hardware
- ▶ Software defines the functionality
- + Facilitates synergies and integration
- – Complex integration and verification



Separation of software and hardware

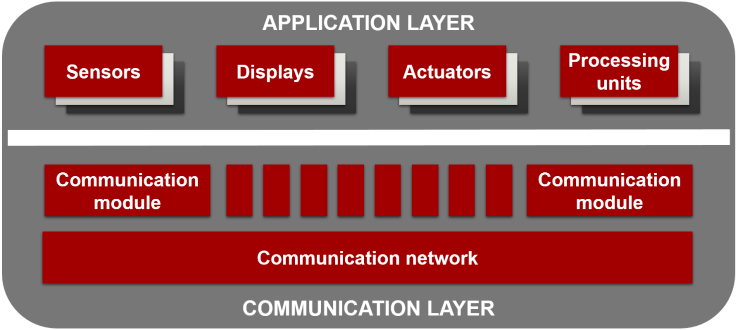
Three independent layers

- Software
- "The glue"
  Code, tools, tests
- Hardware



One responsibility of "the glue" is to allocate hardware resources to the software processes—and make sure the system can be trusted

SAAB avionics design case



APPLICATION LAYER

Sensors

Displays

Actuators

Processing units

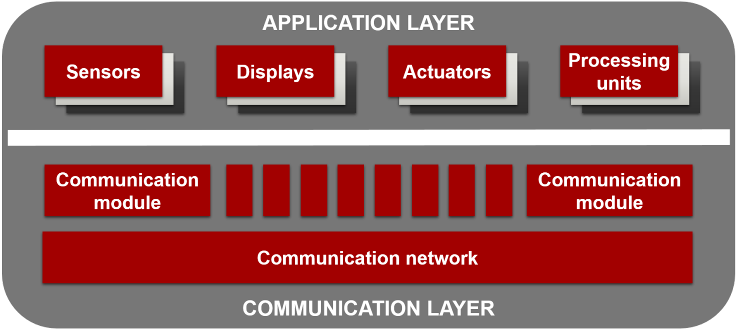
Communication module

Communication module

Communication network

COMMUNICATION LAYER

SAAB avionics design case



- Application layer: Application development view
- Communication layer:
  Infrastructure to provide communication

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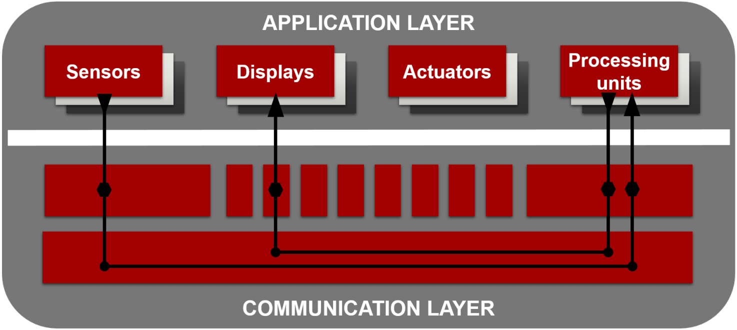

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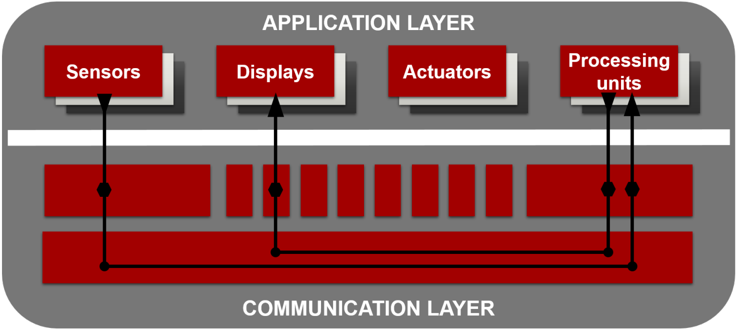
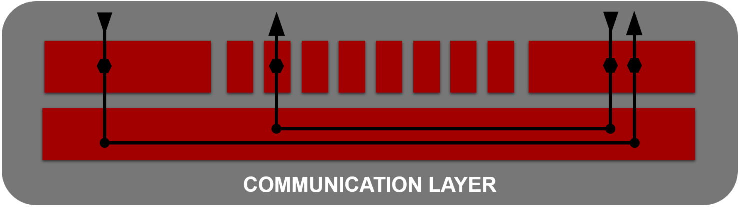


- Completely synchronous system
- Each communication activity is explicitly scheduled and data is available at a determined point in time

SAAB avionics design case: communication details



- Communication network:
  Switched Ethernet with messages sent in discrete time slots
  - Access to the full bandwidth at that instant
  - Can guarantee fast communication
  - Schedule determine when data will arrive
  - Mulitcast

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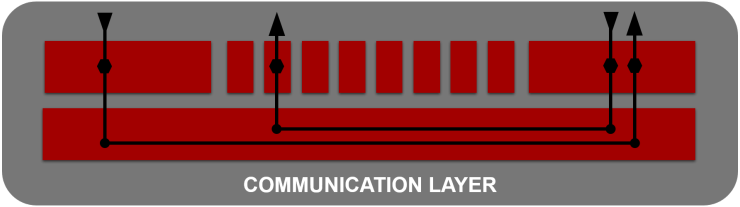

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  - Mulitcast
- Communication modules: Chains of tasks to be scheduled

SAAB avionics design case: characteristics

- Independence between different applications by
  - a pre-runtime schedule with start times for all activities
  - known worst-case execution times for all activities
  - spatial partitioning (not part of scheduling)

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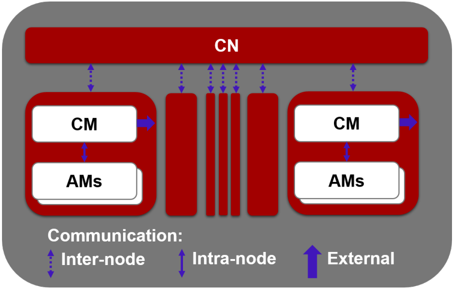
- Highly advanced ”glue” between software and hardware
- Complex scheduling
  - All activities at once
  - Communication scheduling intricate and detailed

From system design to scheduling



- Communication network (CN)
  Communication between nodes
- Application module (AM)
  Run software processes
- Communication module (CM)
  Handle communication

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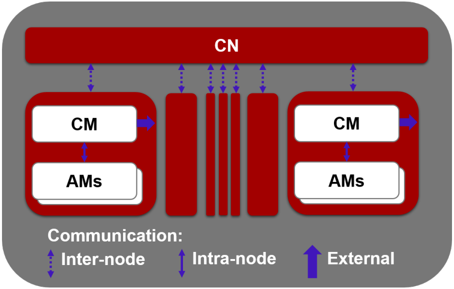

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- One cycle = a major frame (system period) ∼1s long

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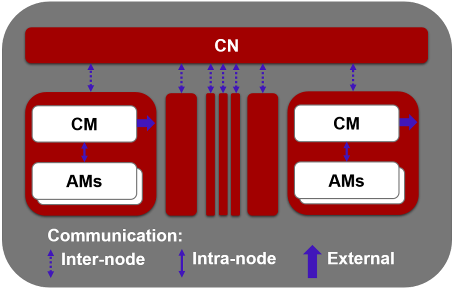

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- Cyclic schedule
- One cycle = a major frame (system period) $\sim$1s long
- Time resolution in nanoseconds
- Find a **feasible** solution or deduce that none exists

Multi-processor scheduling with ...

Scheduling of periodic tasks

- AMs: Few tasks, several instances per major frame
- CMs: Huge number of tasks, one instance per major frame

Multi-processor scheduling with ...

Scheduling of periodic tasks

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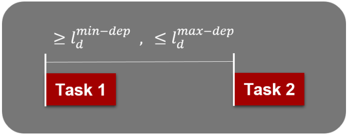
- Dependencies

Dependency:

- Precedence relation with time lag
- On the same or on different modules



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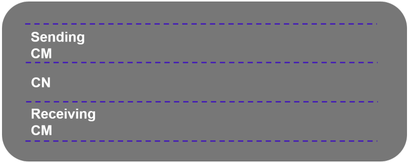
Sequence and assign start time

Multi-processor scheduling with ...

Scheduling of communication



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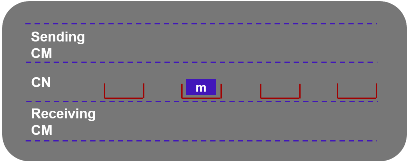

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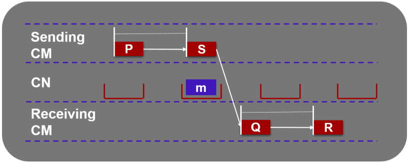

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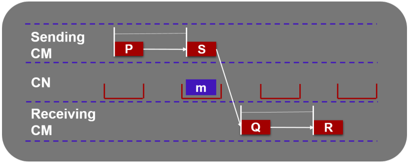

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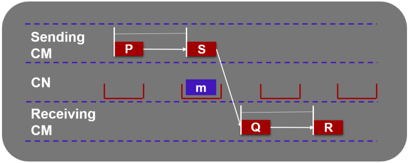
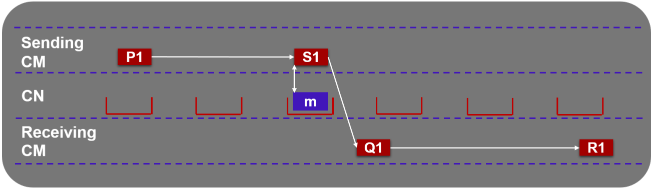


Choose a time slot for each message

BUT choice of time slot ⇒
additional restrictions on the involved tasks …

Interaction between task and communication scheduling

- The choice of time slot impacts the release times and deadlines of some tasks



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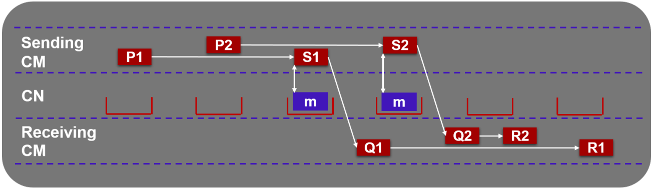
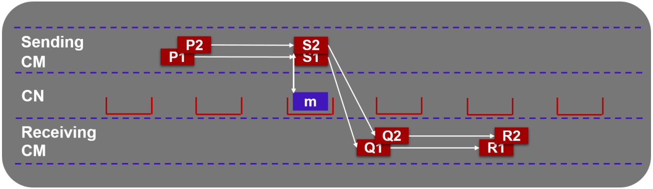
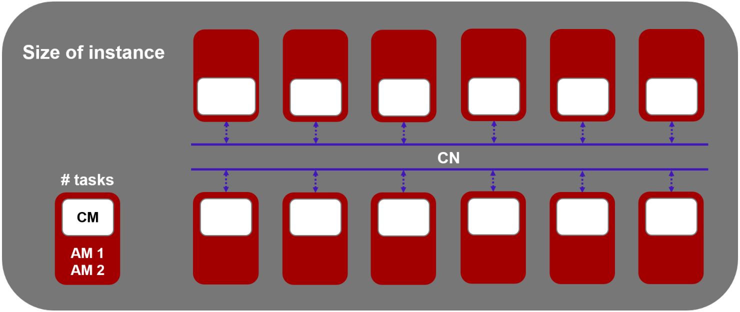


Interaction between task and communication scheduling

- The choice of time slot impacts the release times and deadlines of some tasks
- Same relative order between messages and some of the tasks
- Co-allocation of messages ⇒ merging of tasks



Goal: instance of practical relevance for future design



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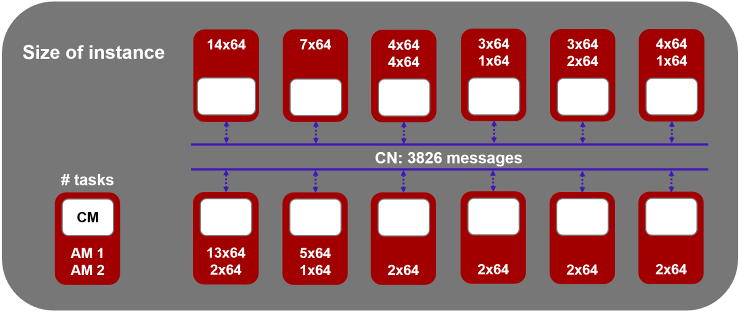

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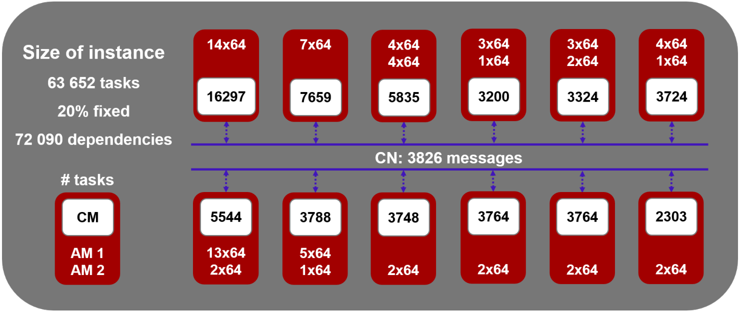
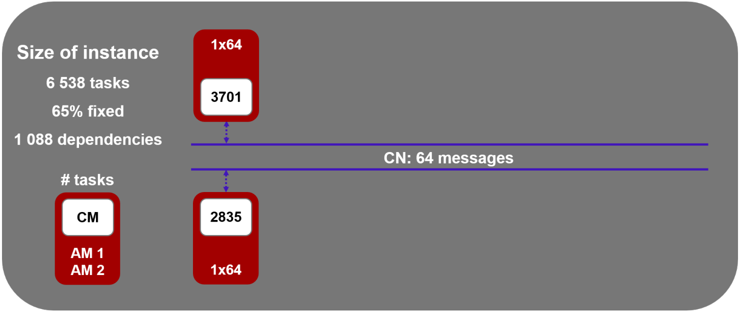


Point of departure



Full MIP model: not solved within a week

Tried different MIP and CP approaches for small scale problems

Problem analysis

Main computational challenges

- Interaction between task and communication scheduling
  - ”Messy” part of the model
  - Impact on the task sequencing

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    A magnitude of 100 million "Task *i* before *j*" decisions

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  - Impact on the task sequencing
- Sequence tasks on the CMs:
  - Huge number of tasks: $> 15,000$ on a single module
    A magnitude of 100 million ''Task *i* before *j*'' decisions
  - Long scheduling horizon: 10^9 time points
    A magnitude of 10^{12} ''Task *i* start at time *t*'' decisions

Problem analysis

Important design considerations

- Find a feasible schedule or prove that none exists

Problem analysis

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- Find a feasible schedule or prove that none exists
- Interaction between communication & tasks

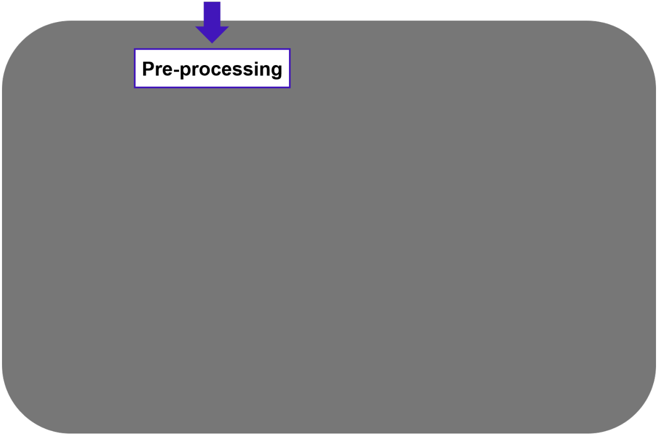
Problem analysis

Important design considerations

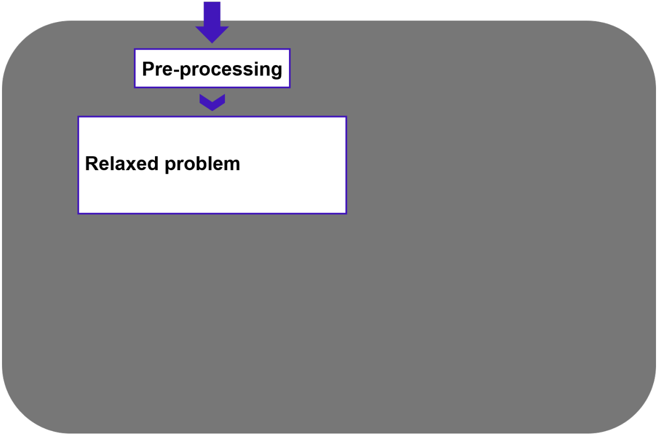
- Find a feasible schedule or prove that none exists
- Interaction between communication & tasks
- Need to handle the sequencing
  MIP: Time-indexed formulations are not viable,
  does order-based formulations stand a chance?
  CP: Better equipped for such sequencing?

Good news: Problem structure to exploit!

Promising type of decomposition scheme


Pre-processing

Promising type of decomposition scheme



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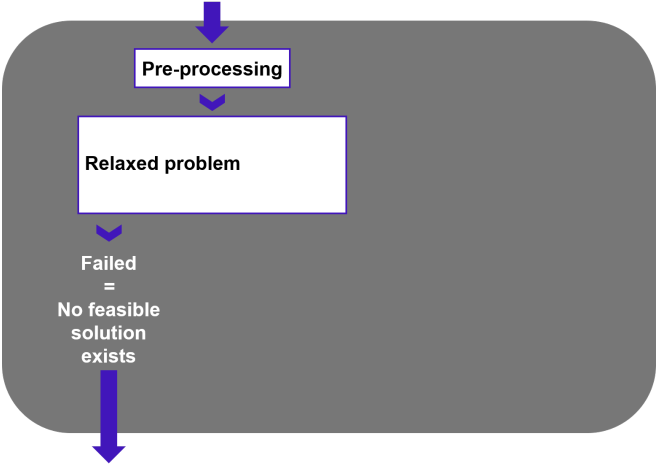

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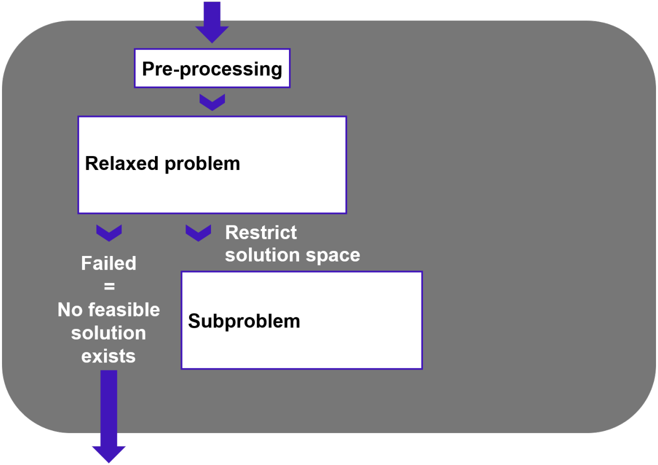

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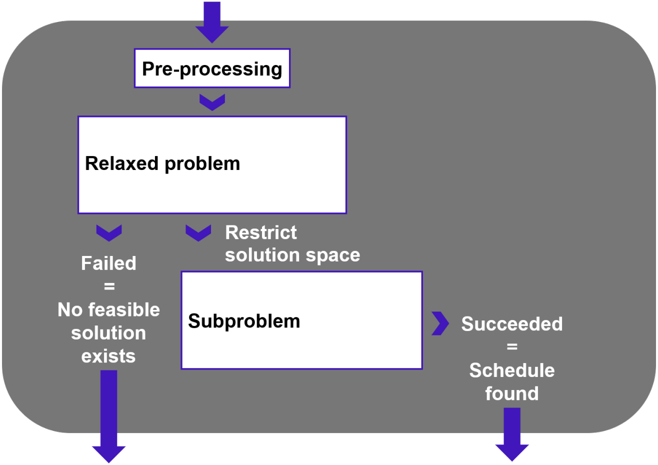
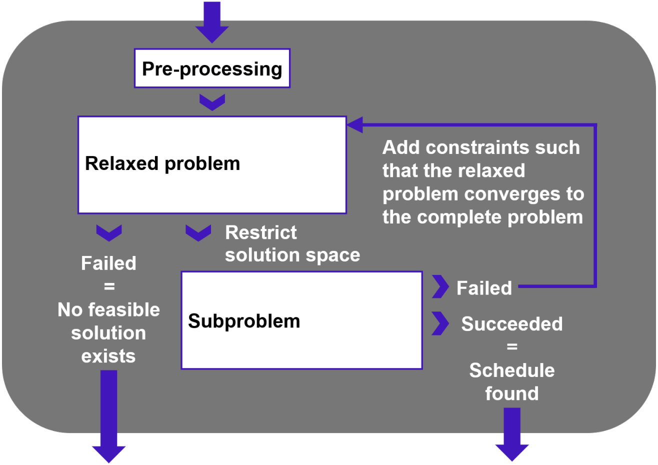

Pre-processing
Relaxed problem
Restrict solution space
Failed = No feasible solution exists
Subproblem
Succeeded = Schedule found

Promising type of decomposition scheme



Pre-processing

Relaxed problem

Add constraints such that the relaxed problem converges to the complete problem

Restrict solution space

Failed = No feasible solution exists

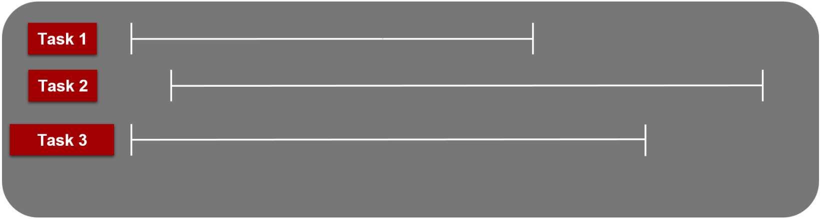
Subproblem

Failed

Succeeded = Schedule found

Problem structure: sequencing of tasks on CMs

Original data: Each task has a release time and deadline



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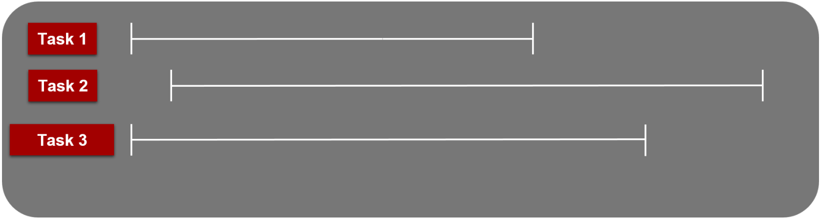

$$\left.\begin{array}{c}\text{Technical restrictions: Discrete times, fixed tasks, ...}\\ +\\ \text{Pre-processing: Propagate precedence relations and time lags}\end{array}\right\}$$

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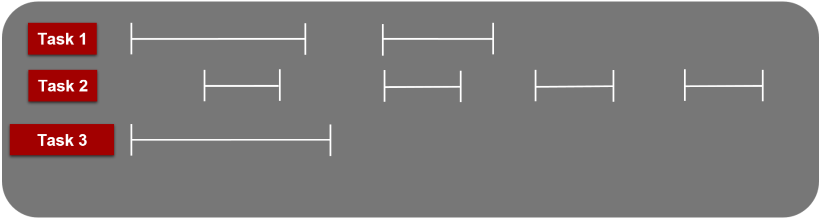


Technical restrictions: Discrete times, fixed tasks, ...

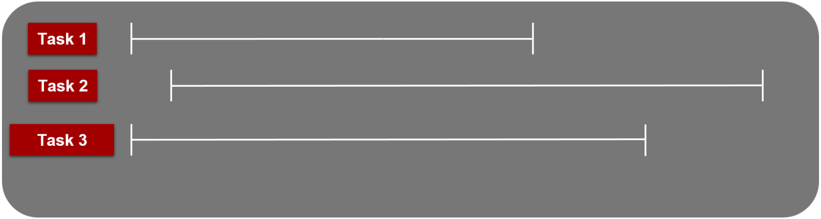
+

Pre-processing: Propagate precedence relations and time lags }

⇒ Only some sub-intervals within the interval between release time and deadline are feasible

Problem structure: sequencing of tasks on CMs

Impact of these sub-intervals?



"Task *i* before *j*" decisions

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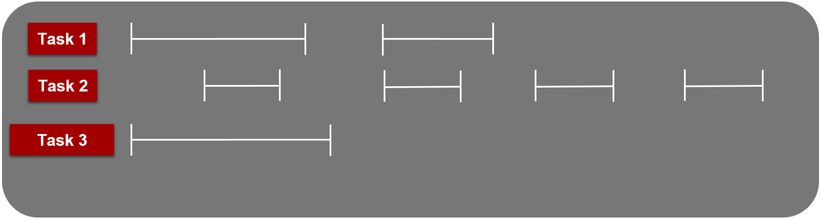
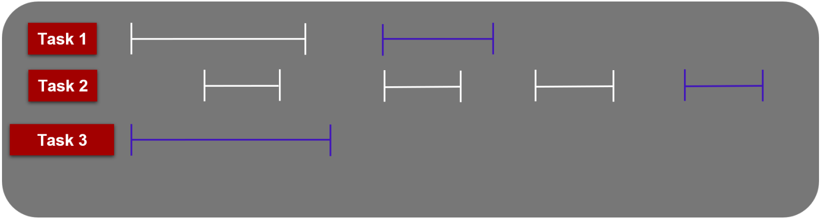


"Task i before j" decisions

- Reduces to $\sim 1/10$ when considering the sub-intervals

Problem structure: sequencing of tasks on CMs

Impact of these sub-intervals?

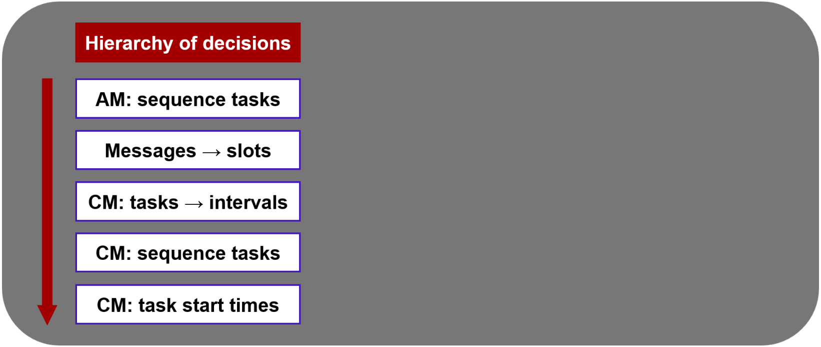


"Task *i* before *j*" decisions

- Reduces to $\sim 1/10$ when considering the sub-intervals
- Reduces to $\sim 1/10$ for a fixed assignment of sub-intervals

Model structure

Including the knowledge about the task sub-intervals— the model can be seen as:



Hierarchy of decisions

- **AM: sequence tasks**
- **Messages → slots**
- **CM: tasks → intervals**
- **CM: sequence tasks**
- **CM: task start times**

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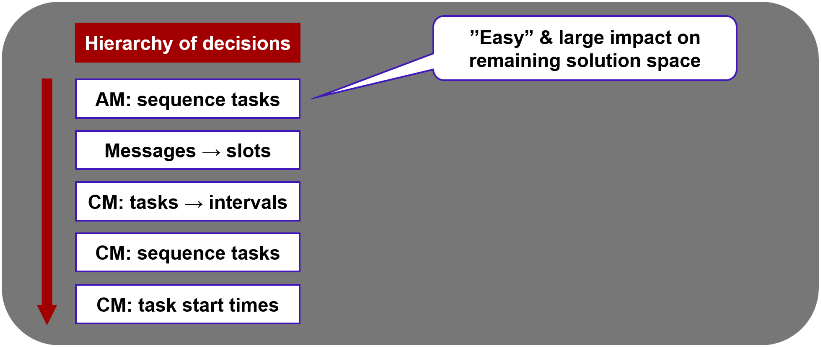

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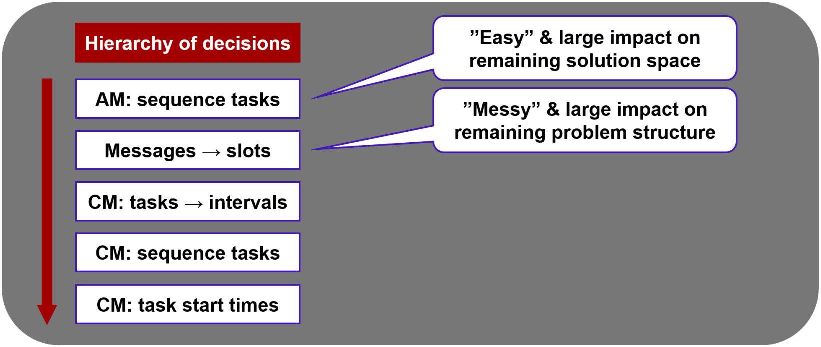

Hierarchy of decisions

AM: sequence tasks

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”Easy” & large impact on remaining solution space

”Messy” & large impact on remaining problem structure

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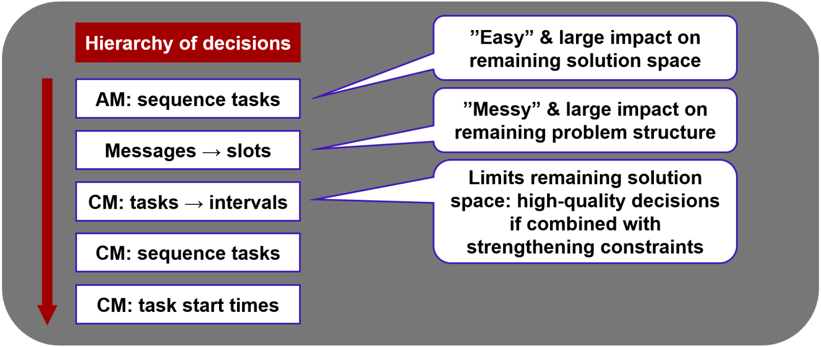
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Limits remaining solution space: high-quality decisions if combined with strengthening constraints

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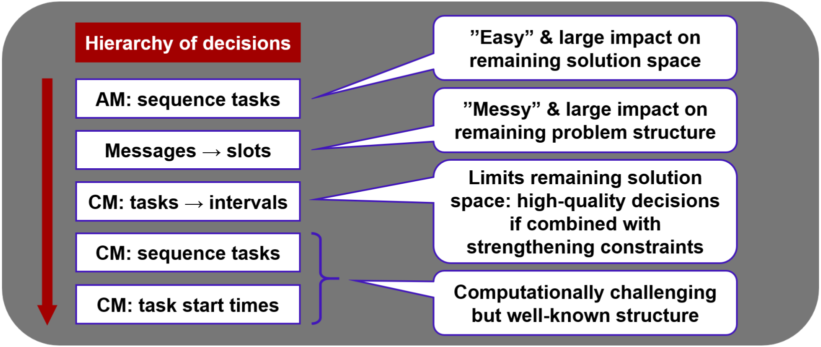

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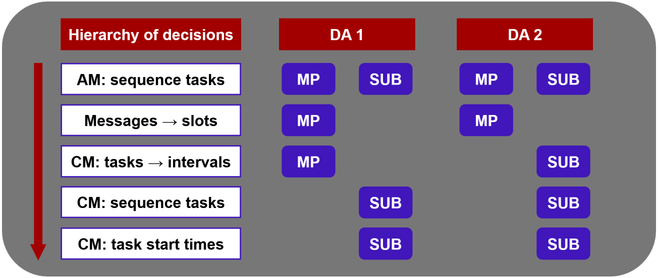
Computationally challenging but well-known structure

Two decomposition approaches

- Differs in decisions made by master problem and subproblem
- Different type of feedback information to the master problem



Hierarchy of decisions	DA 1		DA 2	
AM: sequence tasks	MP	SUB	MP	SUB
Messages → slots	MP		MP	
CM: tasks → intervals	MP			SUB
CM: sequence tasks		SUB		SUB
CM: task start times		SUB		SUB

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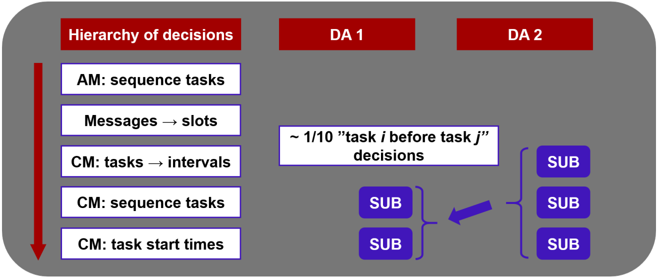

Hierarchy of decisions

DA 1

DA 2

AM: sequence tasks

Messages → slots

CM: tasks → intervals

CM: sequence tasks

CM: task start times

~ 1/10 "task *i* before task *j*" decisions

SUB

SUB

SUB

SUB

SUB

DA1 characteristics

- Master problem:
  - Place tasks in sub-intervals and messages in slots
  - Messy and challenging MIP-model (Gurobi)

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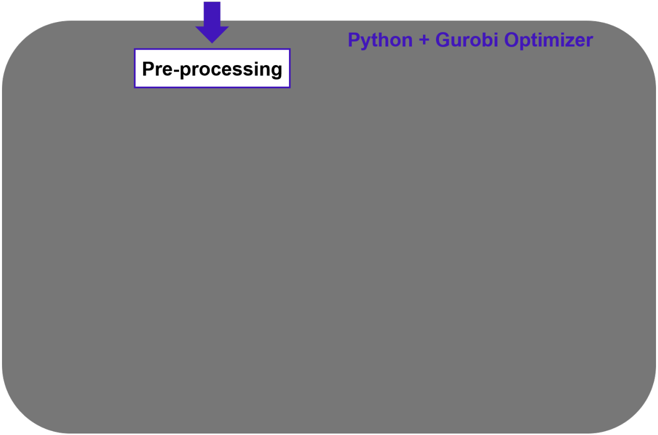
Mindset: Strong relaxation that can discover infeasibility ...
... and if the problem seems feasible—find a schedule

DA1: A constraint generation approach



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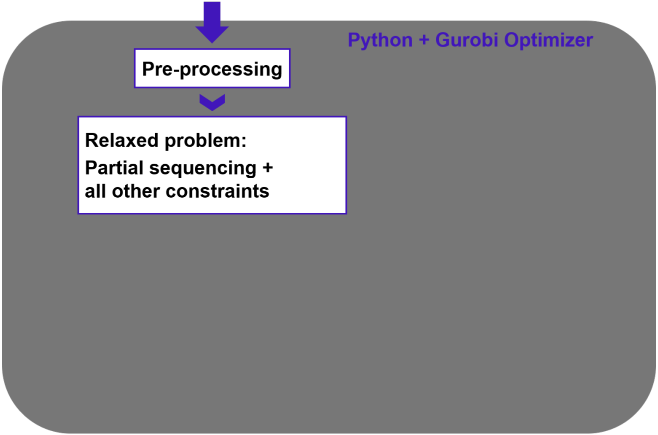

Python + Gurobi Optimizer

Pre-processing

Relaxed problem:
Partial sequencing +
all other constraints

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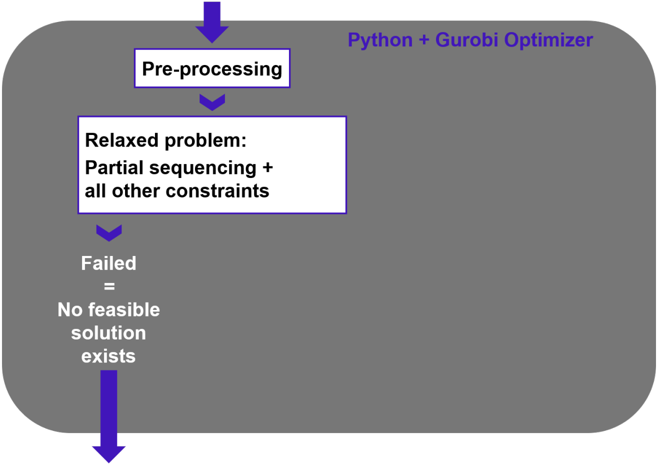

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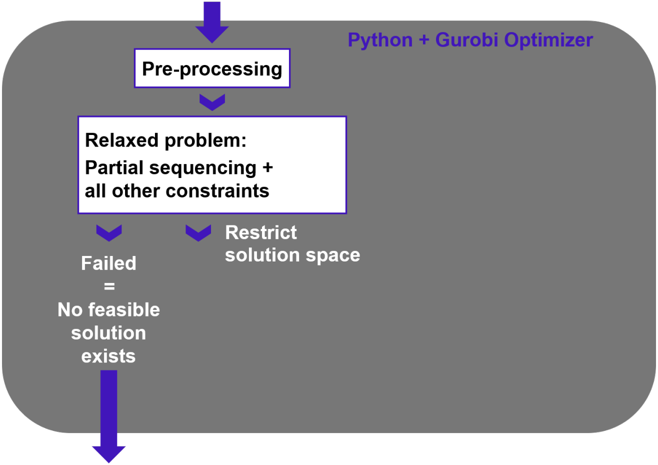

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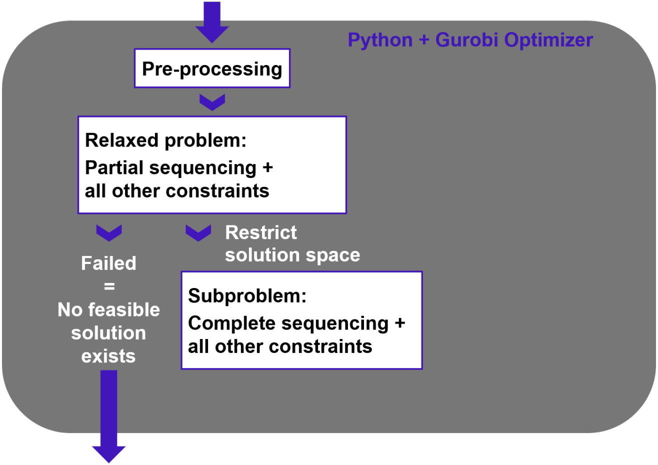

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Failed
=
No feasible
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exists

Restrict
solution space

Subproblem:
Complete sequencing +
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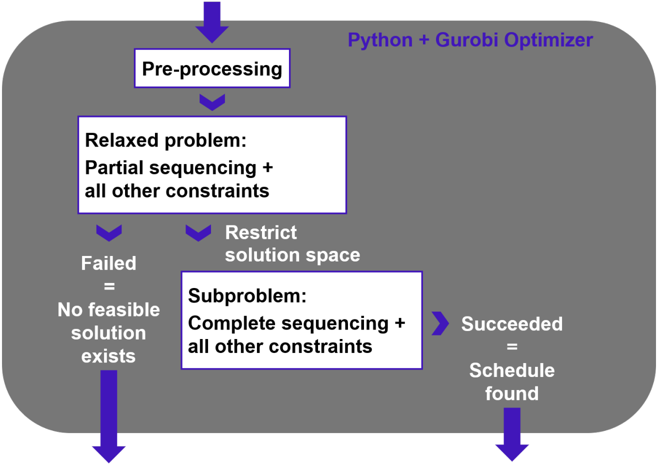

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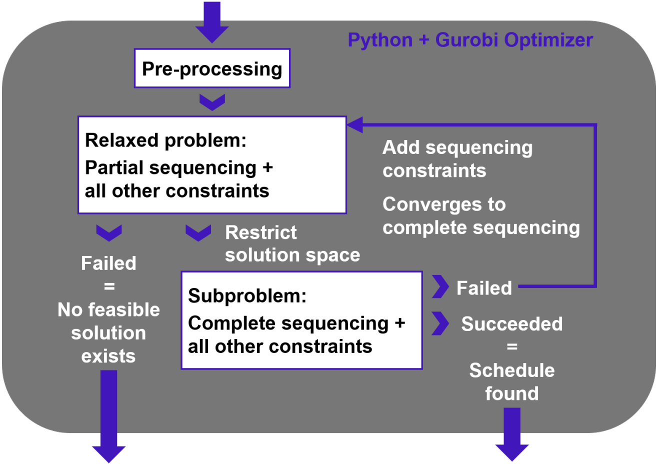


Python + Gurobi Optimizer

Pre-processing

Relaxed problem:
Partial sequencing +
all other constraints

Add sequencing constraints

Converges to complete sequencing

Restrict solution space

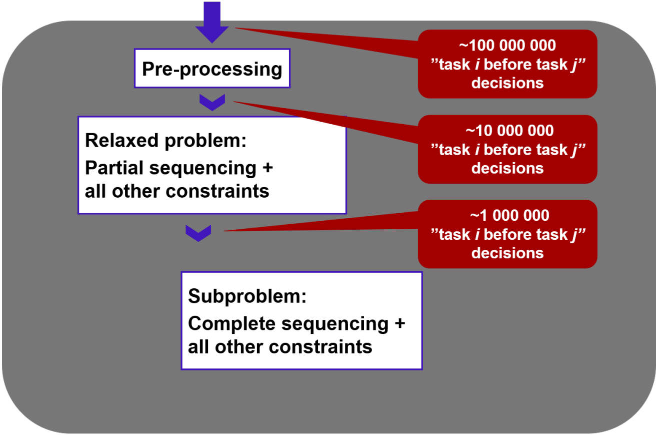
Failed = No feasible solution exists

Subproblem:
Complete sequencing +
all other constraints

Failed

Succeeded = Schedule found

DA1: A constraint generation approach



Pre-processing

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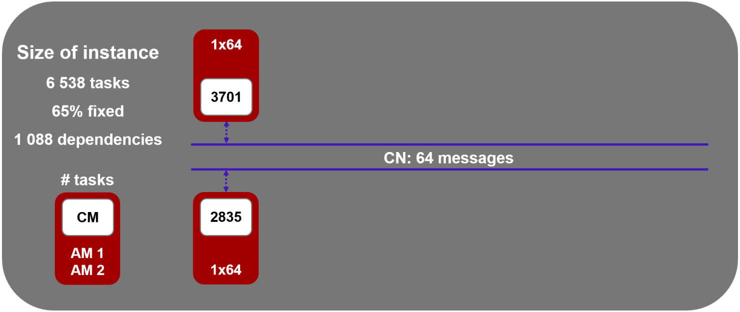
~100 000 000
"task *i* before task *j*"
decisions

~10 000 000
"task *i* before task *j*"
decisions

~1 000 000
"task *i* before task *j*"
decisions

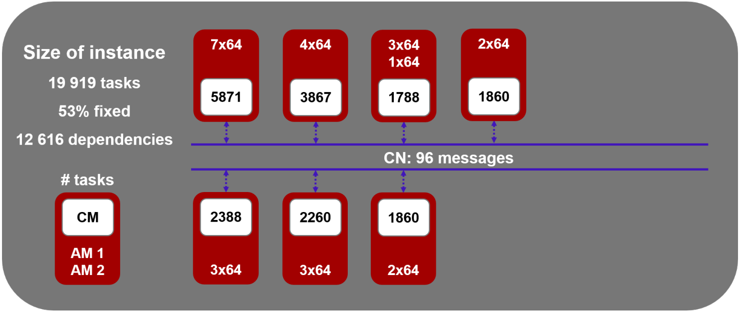
Results of DA1



Full model: not solved within a week

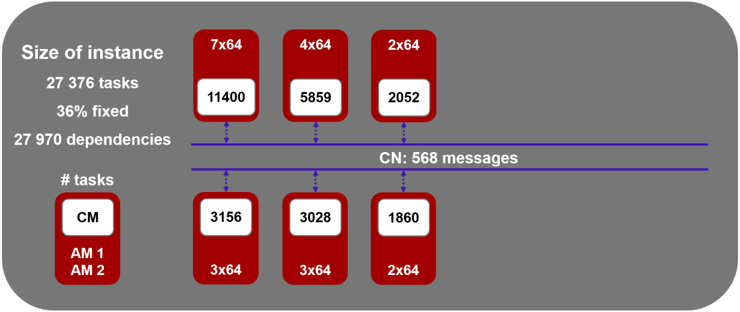
DA1: solved within < 1 minute

Results of DA1



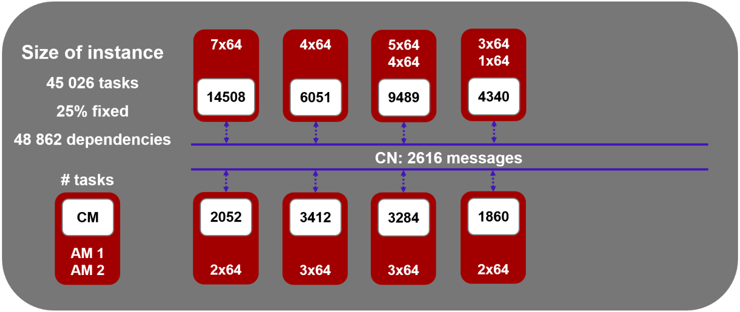
DA1: solved within 2 minutes

Did DA1 take us all the way?



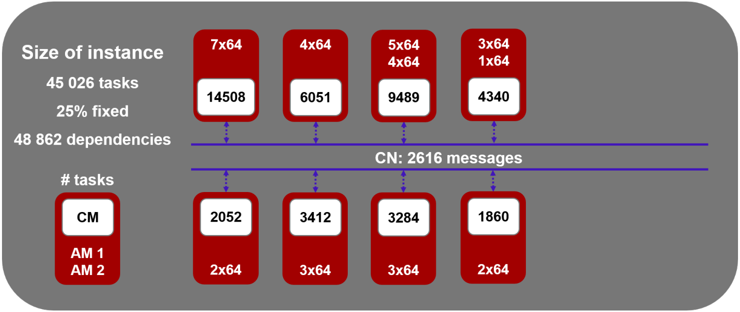
Relaxed model: solved in $11\frac{1}{2}$h

Did DA1 take us all the way?



Relaxed model: not solved in 24h

Did DA1 take us all the way? No ...



Relaxed model: not solved in 24h

A CM with more than 10 000 tasks ⇒ Relaxed problem challenging

Conclusions & insights from DA1

- Really strong relaxation of the problem, it serves it purpose!
- *A bit* too expensive—want results for larger instances

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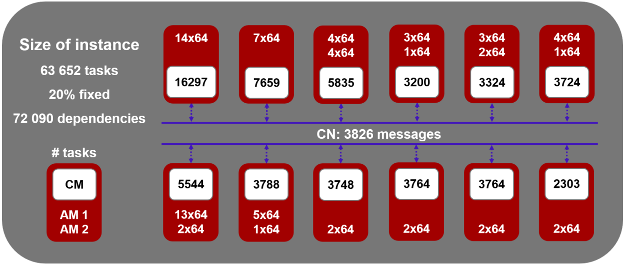
MIP-based Adaptive Large Neighbourhood Search (ALNS)
for solving the relaxed problem



Size of instance

63 652 tasks

20% fixed

72 090 dependencies

tasks

CM

AM 1
AM 2

14x64 — 16297

7x64 — 7659

4x64
4x64 — 5835

3x64
1x64 — 3200

3x64
2x64 — 3324

4x64
1x64 — 3724

CN: 3826 messages

5544 — 13x64
2x64

3788 — 5x64
1x64

3748 — 2x64

3764 — 2x64

3764 — 2x64

2303 — 2x64

Computational time: $\sim$ 3 days

DA2: Logic-Based Benders Decomposition (LBDD)

- Master problem:
  - Assign: messages to slots
  - Messy (but not very challenging) MIP-model (Gurobi)

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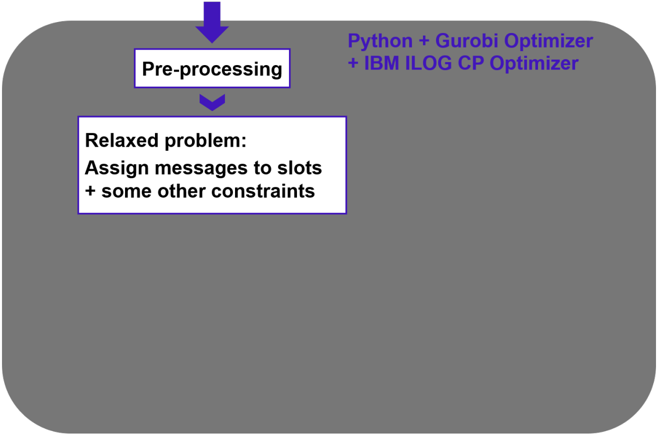
Mindset: Less focus on discover infeasibility ...
... and find a schedule faster?

DA2: A LBBD approach



Pre-processing

Relaxed problem:

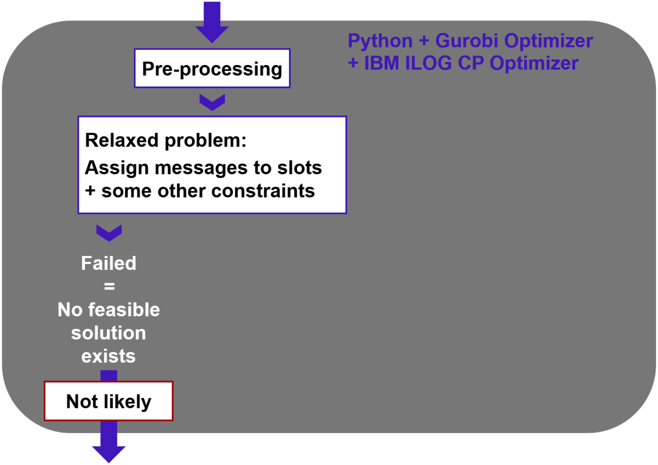
Assign messages to slots

+ some other constraints

Python + Gurobi Optimizer

+ IBM ILOG CP Optimizer

DA2: A LBBD approach



Python + Gurobi Optimizer
+ IBM ILOG CP Optimizer

Pre-processing

Relaxed problem:
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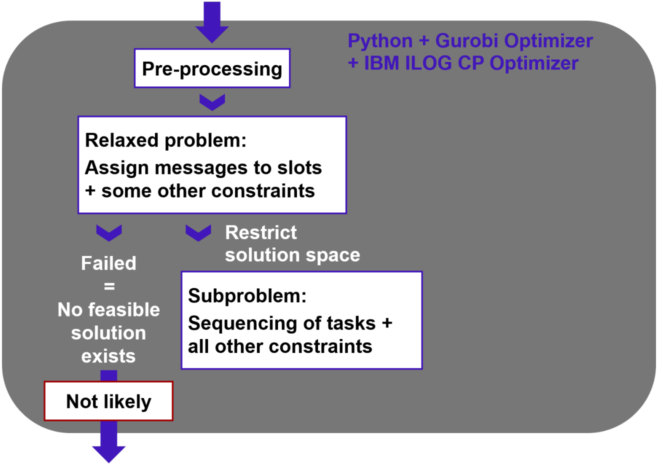
Failed
=
No feasible
solution
exists

Not likely

DA2: A LBBD approach



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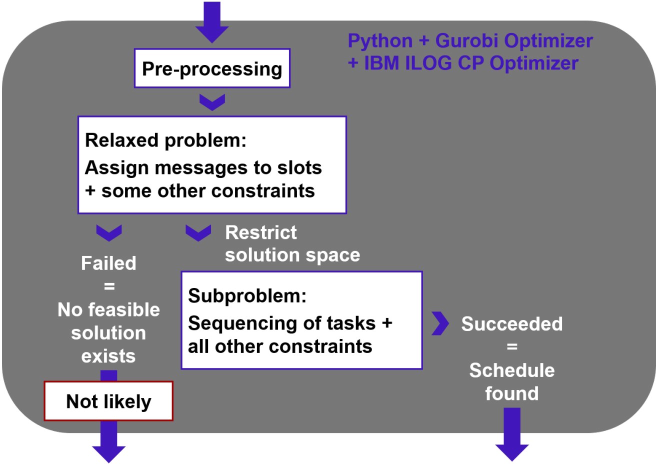

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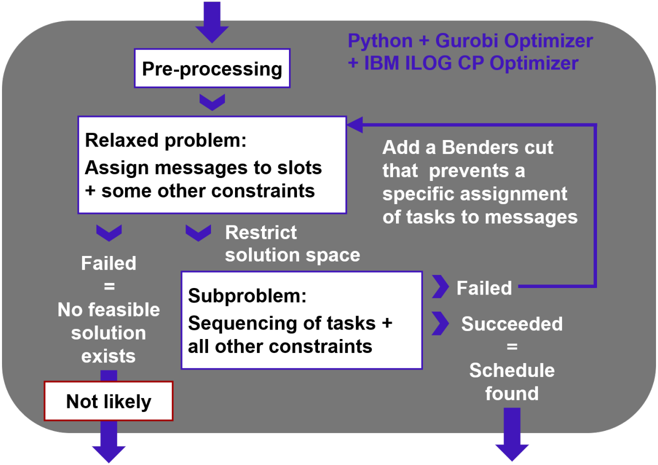

Acceleration techniques for LBBD

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  $\Rightarrow$ Apply cut-strengtheing techniques

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    Master problem variable $1 \to 0$ = relaxation of subproblem

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- New partial assignment acceleration technique
  - Based on common cut-strengthening:
    Systematic search over subsets of variables
    Master problem variable $1 \rightarrow 0$ = relaxation of subproblem
  - Our extension:
    restriction to explore in the search to find feasible solutions

Computational comparisons

Set of public instances:
https://gitlab.liu.se/eliro15/avionics_inst/tree/master

Instance category D, 30 instances with ranges:

Modules	Tasks	Messages	Fixed tasks	Dependencies
14–21	30,000–55,000	1200–2800	5000–8000	60,000–120,000

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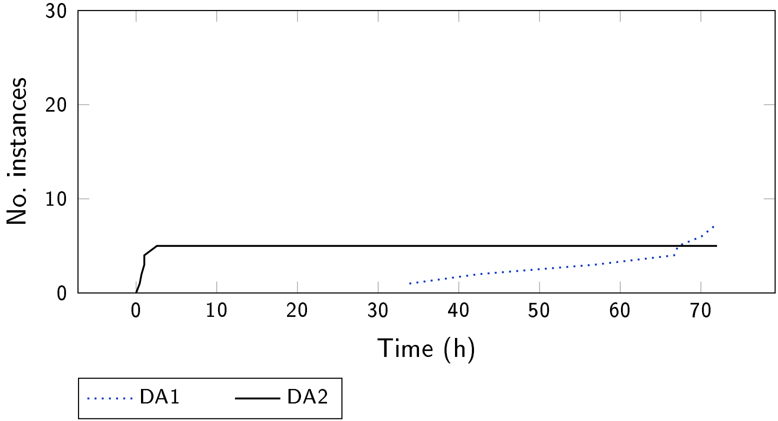
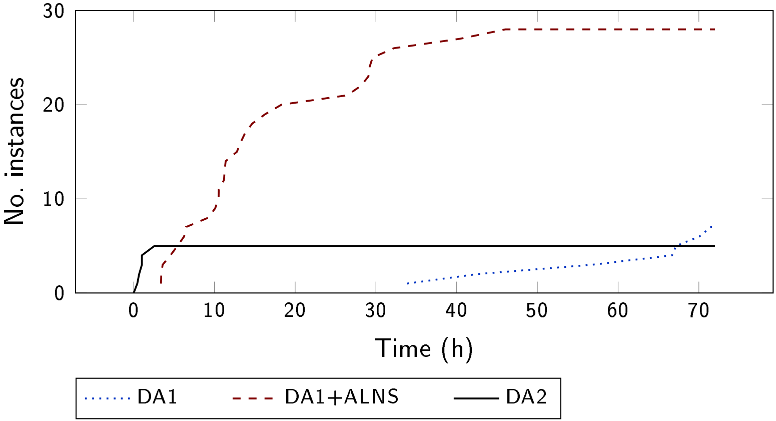
- Two Intel Xeon Gold 6130 Processors (16 cores, 2.1 GHz)
- Unfair comparison wrt memory usage:
  - DA1: 384 GB RAM
  - DA2: 96 GB RAM

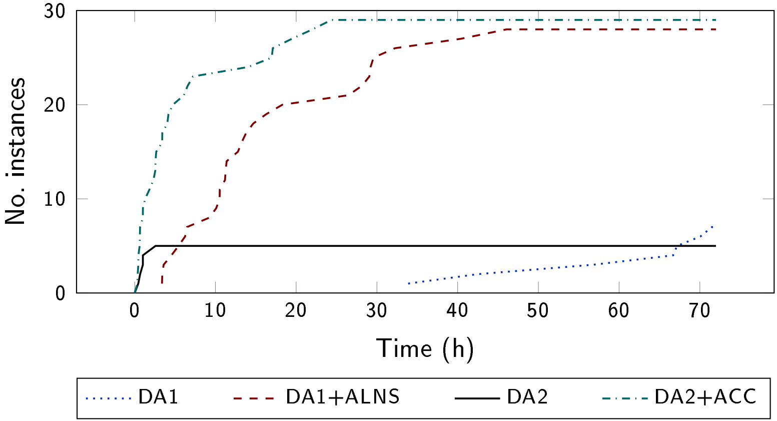
Computational results: "Pure methods"



With matheuristic component in DA1



With new acceleration technique in DA2



Mathematical programming & constraint programming

Both subproblems:

- Sequencing of tasks on modules (up to 15,000 per module)
- Release time and deadline for each task
- Precedence relations with lb and ub on time lags
- Some more ”details”

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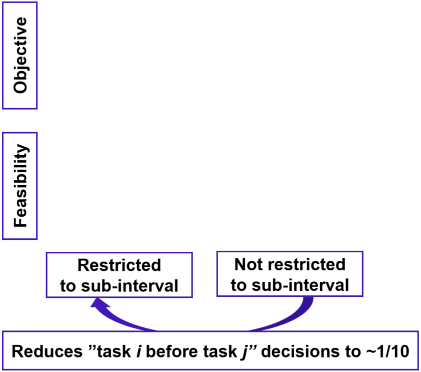

Objective

Feasibility

Restricted to sub-interval

Not restricted to sub-interval

Reduces "task *i* before task *j*" decisions to ~1/10

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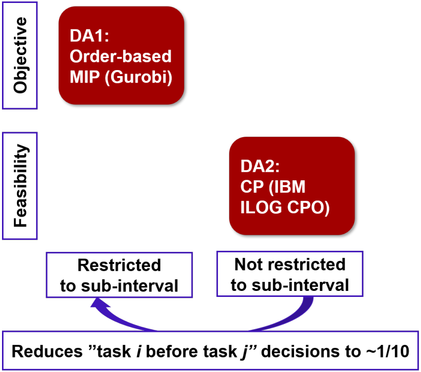

Objective

Feasibility

DA1: Order-based MIP (Gurobi)

DA2: CP (IBM ILOG CPO)

Restricted to sub-interval

Not restricted to sub-interval

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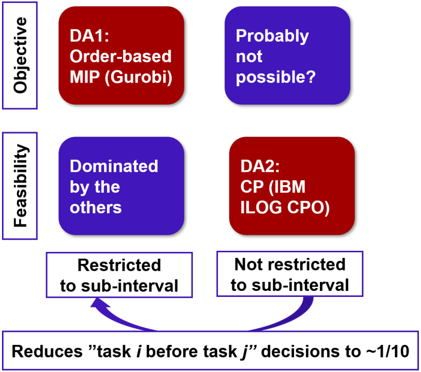
Objective
DA1: Order-based MIP (Gurobi)
Probably not possible?
Feasibility
Dominated by the others
DA2: CP (IBM ILOG CPO)
Restricted to sub-interval
Not restricted to sub-interval
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Models & data

- Technical requirements → mathematical modelling and decomposition
- Understanding data and engineering assumptions → preprocessing and decomposition

Learning on this higher level possible?

Models & data

- Technical requirements →
  mathematical modelling and decomposition
- Understanding data and engineering assumptions →
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Learning on this higher level possible?

On a lower level: very active area of research

- Select methods or decompositions, boost methods, ...
- Our current work
  - Oberweger F.F., Raidl G.R., Rönnberg E., Huber M.: *A Learning Large Neighborhood Search for the Staff Rerostering Problem.* CPAIOR 2022.
  - Ongoing: Learning in logic-based Benders decomposition

Acknowledgements & references

- Center for Industrial Information Technology (CENIIT)
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Thanks for listening!

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