A social power game for the concatenated opinion dynamics with stubborn agents

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   - An example: Paris Agreement

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   - From model to the climate talks

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   - Model analysis

4 Summary
Social opinion dynamics

- Individuals’ opinions are influenced by their neighbors over social networks, and evolve following some cognitive patterns.

Opinion dynamics: to investigate opinion evolution by system theory

- opinions - scalars, vectors...
- social networks - matrices
- cognitive pattern - dynamics

⇒ collective behaviors:
- consensus, polarization,
- oscillation...
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4 Summary
UNFCCC

Atmospheric CO₂ in last 800K years

UNFCCC: an international environmental framework to combat “dangerous human interference with the climate system”

- Parties in the UNFCCC: 195 countries + EU
- “Supreme” governing body: Conference of the parties (COP)
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Background

An example: Paris Agreement

Negotiation process of the UNFCCC

- COP meets annually and decides on climate actions
- Many constituted bodies help the COP
- COP is plenary
- Constituted bodies have restricted participation (not plenary)
- Each constituted body meets once/twice a year
What is the Paris Agreement?

- **Comprehensive accord** for coordinating the international effort to keep the effects of global warming to below 2 °C relative to the pre-industrial level
- **Many aspects**: carbon emission mitigation, adaptation to the effects of climate change, climate finance, green technology transfer, climate agreement implementation, legal and procedural matters linked to climate agreements, etc.
- **Agreement**: all parties (195 countries + EU) agree on common measures → **consensus** is needed
- **Issues at stake**:
  - Future of our planet
  - Many trillions of US $...
  → long (15 years), complex negotiation process
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Background

An example: Paris Agreement

Mathematical model for the Paris Agreement

Task

Develop a dynamical opinion model that describes the process of "achieving an agreement" like the Paris agreement

- **Ingredients:**
  1. Agents: 196 parties
  2. State variables: opinions on the agreement
  3. Interaction graph: time-varying

- **Dynamics**
  1. agents are stubborn (defend their opinions)
  2. negotiation leads to compromise
     - at each meeting final opinions must be closer than initial opinions
  3. over the long time horizon consensus must be achieved
Mathematical model for the Paris Agreement

Task

Develop a dynamical opinion model that describes the process of "achieving an agreement" like the Paris agreement

- Candidate model for each meeting: Friedkin-Johnsen (FJ) model
- Model for multiple meetings in a sequence: $\Rightarrow$ concatenated FJ model
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4 Summary
The Friedkin-Johnsen (FJ) model

- **Motivation:** people’s stubbornness will influence their opinions
- **FJ model:**
  \[ \mathbf{y}(t+1) = (I - \Theta) \mathbf{W} \mathbf{y}(t) + \Theta \mathbf{y}(0) \]
- **Opinions:** \( \mathbf{y}(t) \in \mathbb{R}^m \); weight matrix: \( \mathbf{W} \)
- **Stubbornness (‘‘memory’’ of initial opinions):**
  \[ \Theta = \text{diag}\{\theta_1, \ldots, \theta_m\}, \theta_i \in [0, 1) \]
- **Possible agents:**
  \[
  \begin{cases} 
  \theta_i > 0 & \text{stubborn ‘‘●’’} \\
  \theta_i = 0 & \text{non-stubborn ‘‘●’’} 
  \end{cases}
  \]
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Concatenated Friedkin-Johnsen (FJ) model

Model formulation

Aymptotic behavior for a single FJ model

Solution:

\[
y(\infty) = \lim_{t \to +\infty} y(t) = (I - (I - \Theta)W)^{-1}\Theta y(0)
\]

- \( V \) is a stochastic matrix

- If \( \theta_i > 0, i = 1, \ldots, u \), and \( \theta_i = 0, i = u + 1, \ldots, m \),

\[
V = \begin{bmatrix} R & 0 \\ \hline u & m-u \end{bmatrix}, \quad R \in \mathbb{R}^{m \times u}_{>0}
\]
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Concatenated Friedkin-Johnsen (FJ) model

Model formulation

Concatenated FJ model

- Agent set \( V = \{1, \ldots, n\} \)
- Opinion states \( \mathbf{y}(s, t) \in \mathbb{R}^n \) (two time scales)
- Partial participation
  - stubborn participants \( U(s) \)
  - non-stubborn participants
  - absent agents

For a single discussion \( s \), a FJ model is applied to \( \mathcal{M}(s) \)

\[
\mathbf{y}(s, t + 1) |_{\mathcal{M}(s)} = \text{FJ}(\mathbf{y}(s, t) |_{\mathcal{M}(s)})
\]

Opinions are concatenated:

\[
\mathbf{y}(s, \infty) = \mathbf{y}(s + 1, 0)
\]
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**Concatenated Friedkin-Johnsen (FJ) model**

**Model formulation**

**Concatenated FJ model (compact form)**

- Let \( \mathbf{x}(s) = \mathbf{y}(s, \infty) \)
- Update rule: \( \mathbf{x}(s) = P(s) \mathbf{x}(s - 1) \)

\[
P(s) = \Pi(s)^\top \begin{bmatrix} R(s) & 0 & 0 \\ 0 & 0 & I_{n-m(s)} \end{bmatrix} \Pi(s)
\]

- \( P(s) \) is stochastic
- \( R(s) \in \mathbb{R}^{m(s) \times u(s)} \) is positive
- **Concatenated FJ model:**

\[
\mathbf{x}(s) = P(s)P(s - 1) \ldots P(1)\mathbf{x}(0)
\]
Convergence of the CFJ model

- Consensus: \( \lim_{s \to \infty} x(s) = c1 \iff \lim_{s \to \infty} P(s) \ldots P(1) = 1c^T \)

**Consensus condition (existing result)**

Given stochastic matrices \( Q(s), s \geq 1 \)

1. \( \exists \epsilon > 0 \text{ s.t. } [Q(s)]_{ij} > \epsilon \text{ if } [Q(s)]_{ij} > 0, \forall i, j, s \)

2. \( \exists s_1 < s_2 < \ldots \text{ s.t. } Q(s_k) \text{ has a positive column} \)

\[ \implies \lim_{s \to \infty} Q(s)Q(s-1)\ldots Q(1) = 1c^T \]

- By exploiting the existing result, conditions for the CFJ model to achieve consensus can be given\(^1\)

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\(^1\) L. Wang, et., al. IEEE Trans. on Automatic Control (2022)
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Concatenated Friedkin-Johnsen (FJ) model

From model to the climate talks

Back to the UNFCCC

Data collected for 295 meetings (2001-2015)

1. Meeting participants $\rightarrow M(s)$
2. Speakers ($\leftrightarrow$ stubborn agents) $\rightarrow U(s)$
3. N. of speeches ($\leftrightarrow$ stubbornness level) $\rightarrow \theta_i(s)$
Each year of UNFCCC:

1. COP (plenary)
2. many meetings of 11 constituted bodies

Split the overall 2001– 2015 product of stochastic matrices into yearly intervals with yearly matrices $Q(k)$

$$Q(k) = P_{\text{COP}}^k P_{\text{11}}(k) P_{\text{10}}(k) \ldots P_{\text{1}}(k), \quad k = 1, \ldots, 15$$

``Yearly`` opinion dynamics:

$$x(k) = Q(k)x(k - 1), \quad k = 1, \ldots, 15$$

COP is plenary $\implies Q(k)$ has positive columns

$\implies $ ``practical convergence'' is predicted

$\implies $ Paris Agreement
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4 Summary
Strategic interactions in the UNFCCC

- The participating parties are **rational**, with many issues bargaining on table
- In the CFJ model, agents’ opinions are only passively evolving

**Task**

Develop the concatenated FJ model to reflect the **rationality** of the parties for the UNFCCC
Revisiting the concatenated FJ model

- **Observation 1**: parties can choose to speak or not
  \[ \Rightarrow \text{“speaking” is linked with stubbornness of the model} \]
  \[ \Rightarrow \text{stubbornness can be decided as an action!} \]

- **Observation 2**: \[ \mathbf{x}(s) = P(s)\mathbf{x}(s - 1) = P(s) \ldots P(1) \mathbf{x}(0) \]
  \[ Q(s) \]
  \[ \Rightarrow \lim_{s \to \infty} Q(s) = \mathbf{c}^\top, \quad \lim_{s \to \infty} \mathbf{x}(s) = \mathbf{c} \]
  \[ \Rightarrow Q(s) \text{ encodes the eigenvector centrality of each agent!} \]
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Social power game

Social power for the concatenated FJ model

- **(Cumulated) social power** = overall influence accumulated by agent \( i \) over all agents in the sequence of discussions \( 1, \ldots, M \)

\[
x(M) = Q(M)x(0) = P(M) \ldots P(1)x(0)
\]

\[
sp(M)^\top = \frac{1}{n} 1^\top Q(M) = \frac{1}{n} 1^\top \begin{bmatrix}
\vdots & Q_{1i}(M) & \cdots \\
\vdots & : & \vdots \\
\vdots & \vdots & \vdots \\
\end{bmatrix}
\]

- \( sp(M) \sim \) eigenvector centrality: \( \lim_{M \to \infty} sp(M) = c \)

- \( sp(M) = \) nonlinear function of the stubbornness parameters \( \Theta(1), \ldots, \Theta(M) \)

\[
P(s) = (I - (I - \Theta(s))W(s))^{-1}\Theta(s)
\]
Maximizing social power

- $\text{sp}(M)$ is determined by the speaking occasions $a(1), \ldots, a(M)$ through the concatenated FJ model.

Question

How should an agent take speaking opportunities to maximize its social power?
Social power game

- **Players**: agents $\mathcal{V} = \{1, \ldots, n\}$
- **Actions**: allocation of speaking occasions
  \[ a_i = (a_i(1), \ldots, a_i(M)) \iff \theta_i = (\theta_i(1), \ldots, \theta_i(M)) \]
- **Pay-off function**: social power
  \[ u_i(a_i, a_{-i}) = sp_i(M) \]
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Social power game: constraints

1. More speaking, more stubborn
   \[ \theta_i(s) = \theta a_i(s) \]

2. Limited budget of overall speaking opportunities: \( \gamma, K \)
   \[ a_i(s) \leq \gamma, \quad a_i(1) + \cdots + a_i(M) \leq K \]

3. Limited capacity of speaking occasions per meeting: \( C \)
   \[ \sum_{i \in \mathcal{V}} a_i(s) \leq C \]
Social power game: network topology

- The network is a complete graph

\[ W(s) = W = \frac{1}{n} \mathbf{1} \mathbf{1}^\top, \quad s = 1, \ldots, M \]

- Meaning: meetings are all plenary
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Problems of interest

- **P1**: given the actions of two agents, who will obtain a higher social power (social power comparison)?
- **P2**: what is the (generalized) NE of the social power game (Nash equilibrium)?
- **P3**: for a given agent, if the actions of the other agents are fixed, what is the best strategy for her (best strategy)?
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Social power game

Model analysis

Problem P1: social power comparison

\[ \theta_i(s) = \theta a_i(s) \]

\[ a_i(s) \rightarrow \theta_i(s) = \theta a_i(s) \rightarrow P(s) \rightarrow \text{CFJ model} \rightarrow sp_i(M) \]

- strategies of agents \( i \) and \( j \)

\[ a_i = (a_i(1), \ldots, a_i(M)) \quad a_j = (a_j(1), \ldots, a_j(M)) \]

Theorem (Comparison of social powers)

For small enough \( \theta \), \( a_i(s) = a_j(s), \ \forall s < s' \quad a_i(s') < a_j(s') \}

\[ \implies sp_i(M) < sp_j(M). \]

- Meaning: speaking more at early meetings gives higher social power

\[ \implies \text{early mover earns more} \]
Problem P1: binary stubbornness

Assume $\gamma = 1$, i.e., agents can choose to speak or be silent

**Theorem (Comparison of social powers)**

Let $\tau_i = \arg \min_s \{a_i(s) = 0\}$.

$\tau_i < \tau_j \implies u_i < u_j$

No constraint is made on $\theta$

Example

- $a_1 = (1, 1, 1, 0, 0, 1)$
- $a_2 = (1, 0, 1, 1, 1, 0)$

agent 1 wins!
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Problem P2: (generalized) Nash Equilibrium

\[ \sum_{i \in V} a_i(s) \leq C \]

\[ \theta_i(s) = \theta a_i(s) \]

\[ P(s) \rightarrow \text{CFJ model} \]

Nash equilibrium: \[ a_i^* = \arg \max_{a_i} u_i(a_i, a_{-i}) \]

Theorem (Generalized Nash equilibrium)

For \( \theta \) small enough, if \( \gamma \mid C \), any \( a^* \) taking the following form is a GNE

- For \( i = 1, \ldots, \frac{C}{\gamma} \):
  \[ a_i^* = (\underbrace{\gamma, \ldots, \gamma}_\text{\( \lceil \frac{K}{\gamma} \rceil \) meetings}, K - \gamma \lceil \frac{K}{\gamma} \rceil, 0, \ldots, 0) \]

- For \( i > \frac{C}{\gamma} \), \( a_i^* \) can be arbitrarily chosen such that
  \[ a_i^*(1) = \cdots = a_i^*(\lceil \frac{K}{\gamma} \rceil) = 0, \quad \sum_{j \in V} a_j^*(\lceil \frac{K}{\gamma} \rceil + 1) = C \]
Problem P2: Nash equilibrium (cont’d)

- Multiple GNEs
- On the equilibrium agents tend to speak more in early meetings
- \( \implies \) early mover strategies consist the GNE

**Theorem (Nash equilibrium: binary stubbornness)**

Assume \( \gamma = 1 \) and \( C = |\mathcal{V}| \). For small enough \( \theta \), the unique NE is

\[
\boldsymbol{a}_i^* = (1, \ldots, 1, 0, \ldots, 0)
\]

\( K \) meetings

\( \implies \) everyone takes the early mover strategy!
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Problem 3: best strategy

- Early mover strategy

\[ \tilde{a}_i = (\gamma, \ldots, \gamma, K - \gamma\left\lceil \frac{K}{\gamma} \right\rceil, 0, \ldots, 0) \]

\[ \left\lceil \frac{K}{\gamma} \right\rceil \text{ meetings} \]

Theorem (Best strategy)

For \( \theta \) small enough, it holds

\[ \tilde{a}_i = \arg \max_{a_i} u_i(a_i, a_{-i}), \quad \forall a_{-i}. \]

- Meaning: the early mover strategy is a dominant strategy
- \( \implies \) early mover advantage
Problem 3: best strategy (cont’d)

- Early mover strategy

\[
\tilde{a}_i = (\gamma, \ldots, \gamma, K - \gamma \lceil \frac{K}{\gamma} \rceil, 0, \ldots, 0) \quad \text{meetings}
\]

- The early mover strategy might not be optimal for larger \( \theta \)

Example. \( \gamma = 1 \), \( K = 6 \) and \( \theta = 0.6 \)

\[
\begin{align*}
\mathbf{a}_1' &= (1, 1, 1, 1, 1, 0, 1, 0, 0, 0) \\
\mathbf{a}_2 &= (0, 1, 1, 1, 0, 1, 0, 1, 1, 0) \\
\mathbf{a}_3 &= (1, 1, 1, 1, 0, 1, 0, 0, 1, 0) \\
\mathbf{a}_4 &= (1, 1, 0, 1, 1, 1, 1, 0, 0, 0) \\
\end{align*}
\]

\[ u_1(\tilde{a}_1, \mathbf{a}_{-1}) < u_1(\mathbf{a}_1', \mathbf{a}_{-1}) \]
Early mover advantage for general stubbornness

- Early mover strategy

\[
\tilde{a}_i = (\underbrace{\gamma, \ldots, \gamma}_K, K - \gamma \lceil \frac{K}{\gamma} \rceil, 0, \ldots, 0)
\]

\[s = \lfloor \frac{K}{\gamma} \rfloor \text{ meetings} \]

Theorem (General stubbornness)

For any \(a_{-i}\) it must be

\[
u_i(\tilde{a}_i, a_{-i}) \geq \max_{a_i \in A_i(a_{-i})} u_i(a_i, a_{-i}) - 2\left(1 - \frac{1}{n}\right) \sum_{s=\lfloor \frac{K}{\gamma} \rfloor}^{M-1} \left(\gamma \theta_s \right)^s.
\]

- Meaning: the early mover strategy is at least suboptimal
- \(\implies\) early mover advantage holds for general stubbornness
Beyond complete graph: simulation results

Graphs

Parameters: \( M = 10, K = 6, C = 24, \theta = 0.05 \)

Social power of agent 1 w.r.t \( a_1 \): ind = lexicographical order

- Social power roughly increases along the lexicographical order
- \( \Rightarrow \) early mover advantage still holds!
Why early mover advantage?

- Concatenated FJ model has contracting dynamics
- Closer to consensus, harder to impact the final outcome
- \(\Rightarrow\) early discussions are more important
- \(\Rightarrow\) diminishing return law

**Theorem (Diminishing returns)**

Let \(\Theta = (\theta_1, \ldots, \theta_n)\) be the strategy profile. It holds for \(\forall i\)

\[
\max_{\Theta} \{ sp_i(s_1 + 1) - sp_i(s_1) \} = (1 - \frac{1}{n}) \prod_{s=1}^{s_1} \max_{j \in V} \theta_j(s)
\]

- The diminishing return law does not depend on how \(a_i\) is associated with \(\theta_i\)
Back to UNFCCC: social power

- The EU has the highest social power for most of the years
- Is the EU using an early mover strategy?
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Model analysis

UNFCCC Negotiations: a few years

[Graphs showing stubbornness and social power for 2009 and 2015]
UNFCCC Negotiations: early mover strategy

Is EU taking early mover advantage?

- null model: reshuffle order in the action $a_{EU} \rightarrow \text{perm}(a_{EU})$
  recompute the social powers

$$\text{mean}(sp_{EU, \text{reshuffled}}) < sp_{EU} \implies \text{the EU is taking an early mover advantage!}$$
Validation: UNFCCC leadership

- To assess leadership in climate negotiations: use survey data from International Negotiations Survey
- → perceived leadership
- data collected in years 2008-2022
- total of 5530 responses
Validation: UNFCCC leadership

To assess leadership in climate negotiations: use survey data from International Negotiations Survey

⇒ perceived leadership
data collected in years 2008-2022
total of 5530 responses

⇒ mean(corr(leaderhship, sp))=0.6
Validation: UNFCCC leadership

- Temporal trend for the EU is captured very well.
- Less precise for other countries like China and US.

Summary: the model-based social powers seem rather close to the perceived leadership!
Summary

- **Concatenated FJ model**
  - a two time scale model representing consecutive FJ discussion events
  - opinions are contracting for each discussion

- **Social power game**
  - strategic game for the concatenated FJ model
  - allocate speaking opportunities to maximize social power

- **Results**
  - Early mover advantage: speaking more in early discussions makes an advantage
  - Diminishing return law: later discussions have lower influence on the social power

- **Application:** UNFCCC, Paris Agreement
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References


Thank You!