# Robot learning on the edge

Online learning in hardware





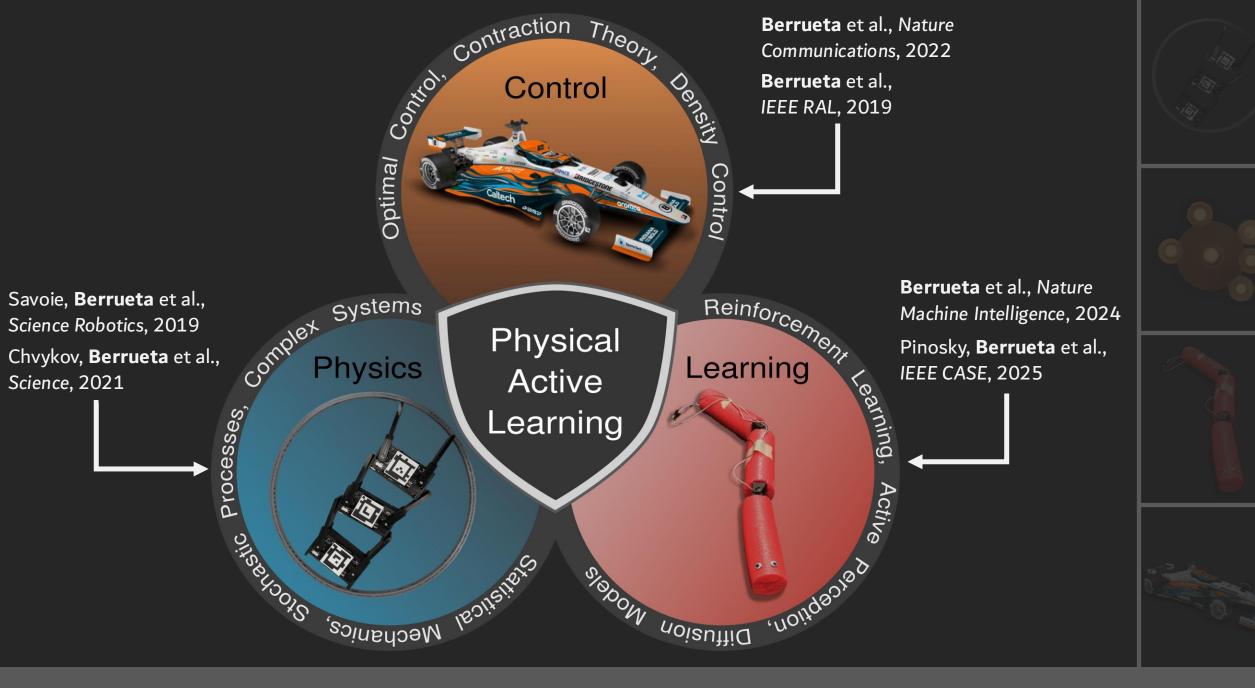




#### Thomas A. Berrueta

Postdoctoral Scholar, Computing + Mathematical Sciences ELLIIT Robot Learning Symposium (2025/11/19)

# Caltech



#### INDY CAST

LAP 1 89.556

LAP 2 100.547

LAP 3 110.962

LAP 4 111.436

LAP 5 121.915

LAP 6 122.360

LAP 7 133.129

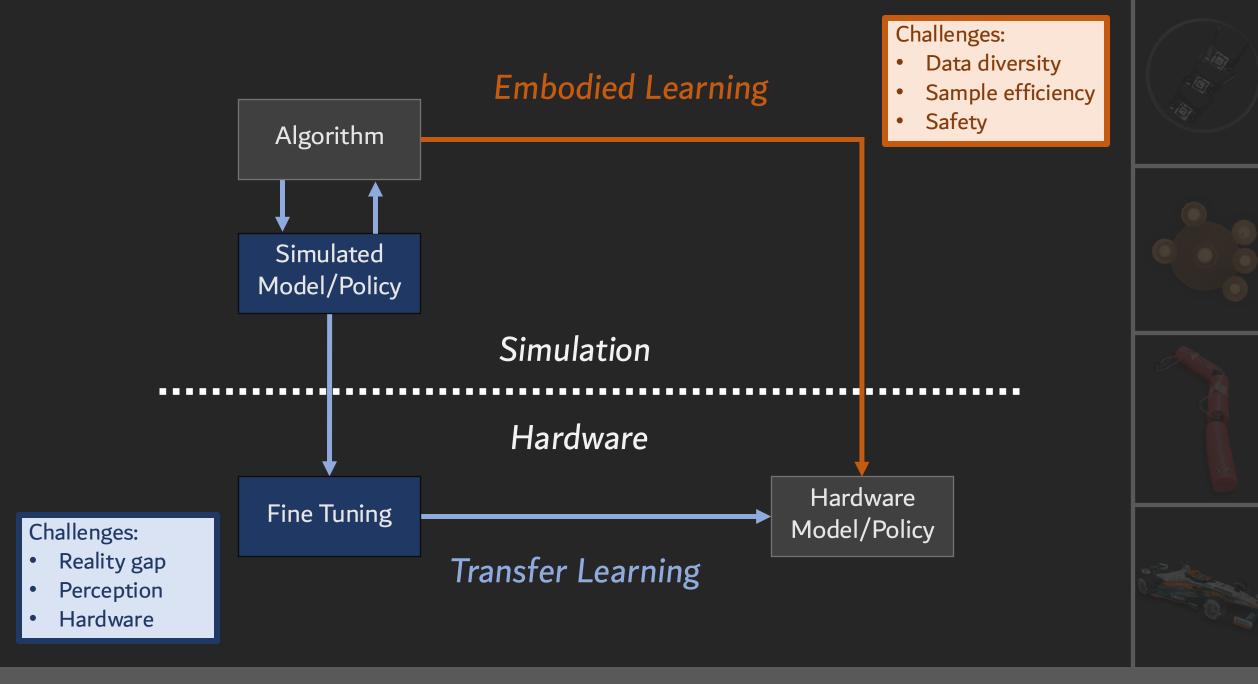
LAP 8 133.514

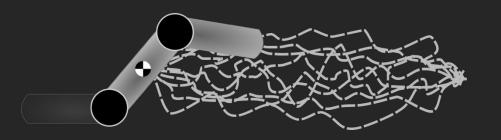
LAP 9 144.130



#### **CAST Racer**

THROTTLE BRAKE
SPEED 146.4 mph GEAR 5 RPM 5704





#### Path decorrelation for efficient online learning



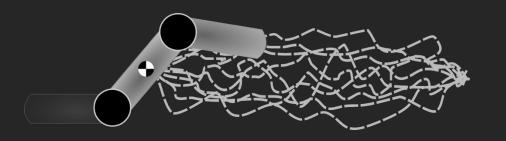
Operational safety through modular design











#### Path decorrelation for efficient online learning



Operational safety through modular design

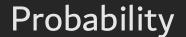


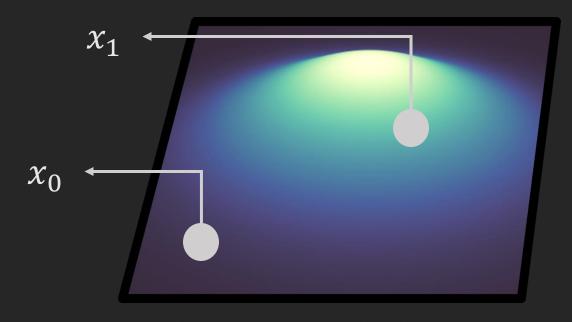






#### Physics gets in the way of data diversity





$$p(x_0, x_1) = p(x_1)p(x_0)$$

$$S[p(x_0, x_1)] = S[p(x_1)] + S[p(x_0)]$$



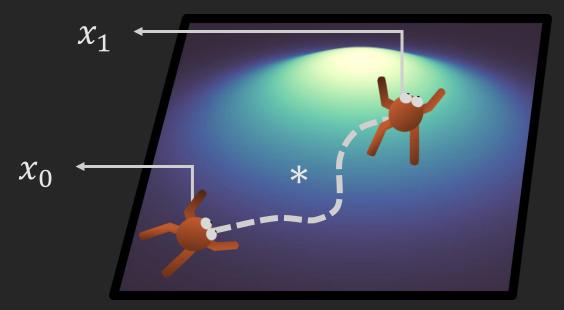






#### Physics gets in the way of data diversity

#### Probability



\* Transitions along path distribution (i.e., dynamics)

$$p(x_0, x_1) = p(x_1|x_0)p(x_0)$$

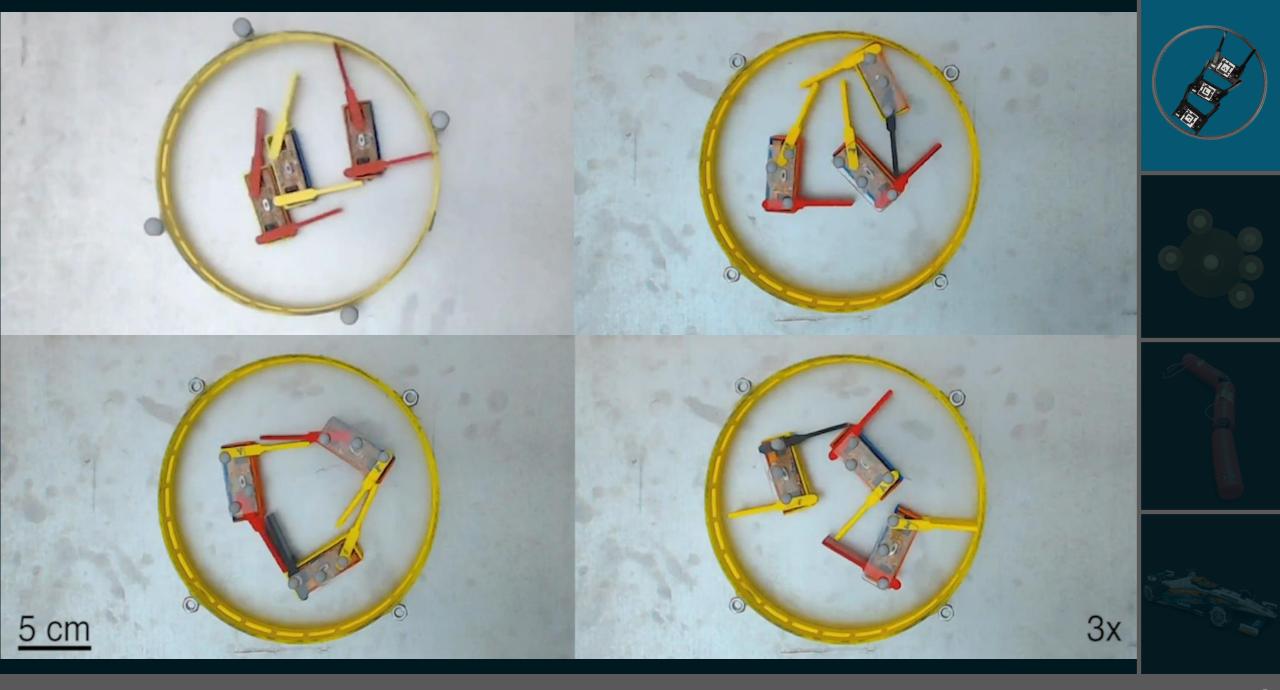
$$S[p(x_0, x_1)] \le S[p(x_1)] + S[p(x_0)]$$

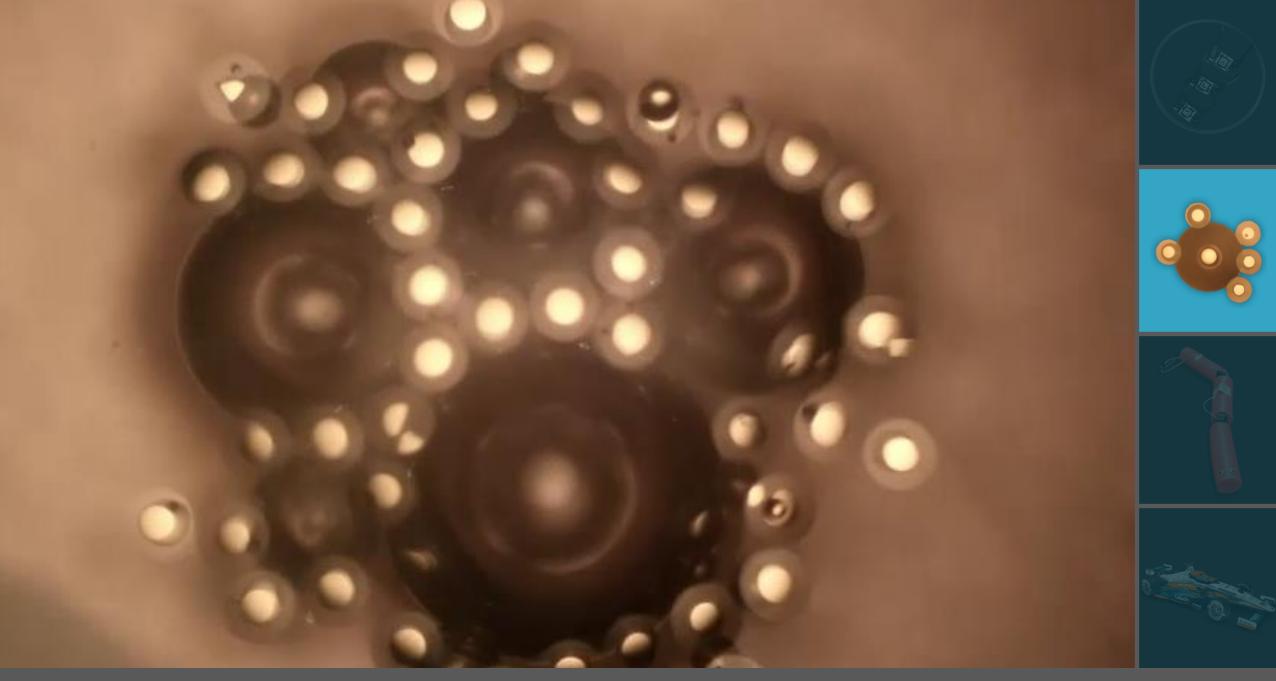








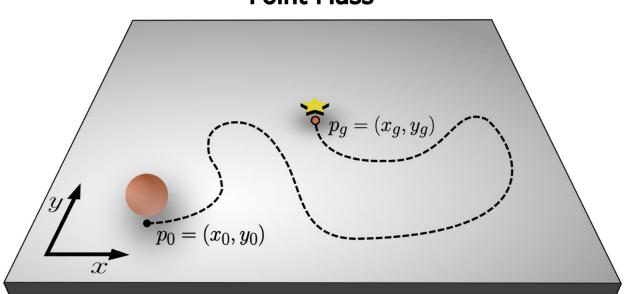




Berrueta, et al., "Emergent microrobotic oscillators via asymmetry-induced order," Nature Communications, 13 (1), 1-11 (2022)

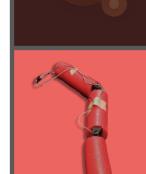


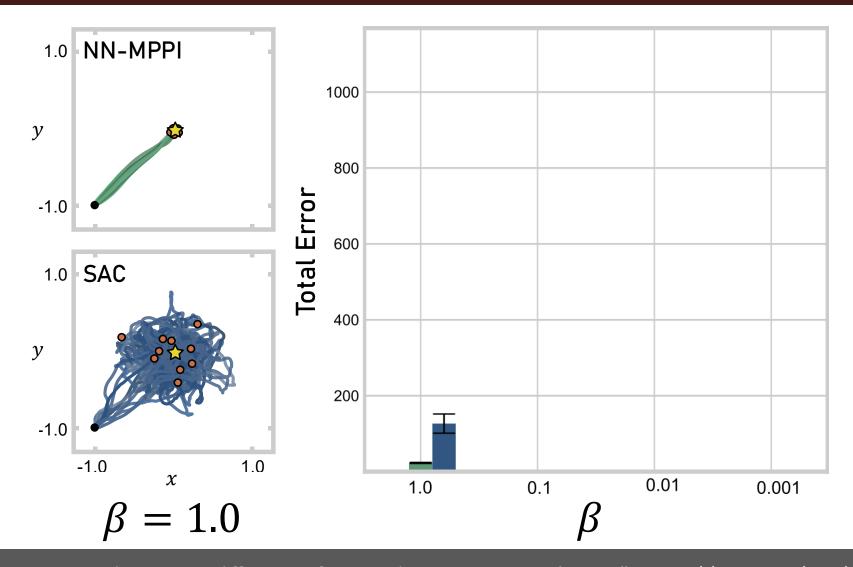




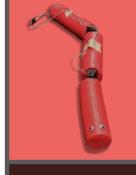
Dynamics: 
$$\vec{x}_{t+1} = A\vec{x}_t + B\vec{u}_t$$

$$m{A} = egin{bmatrix} 1 & 0 & m{eta} & 0 \ 0 & 1 & 0 & 1 \ 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 1 \end{bmatrix} \quad m{B} = egin{bmatrix} 0 & 0 \ 0 & 0 \ 1 & 0 \ 0 & 1 \end{bmatrix}$$

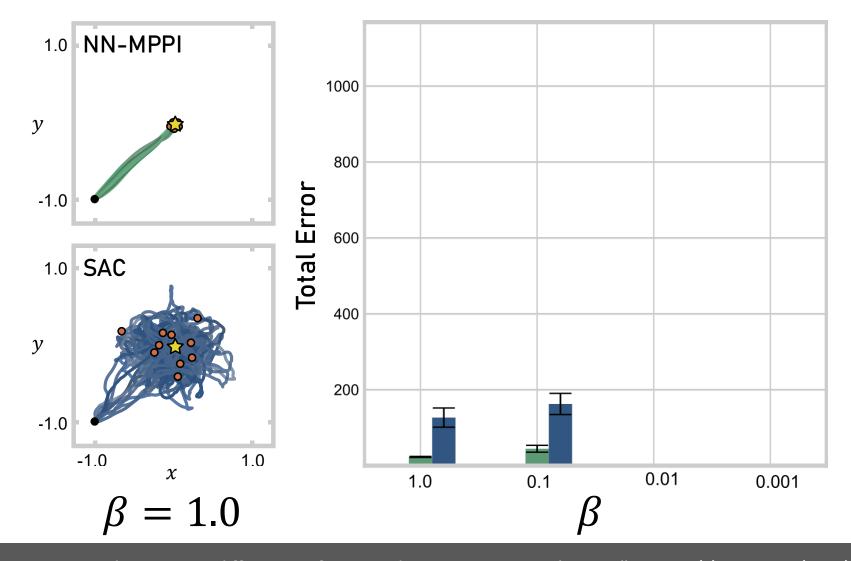




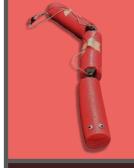




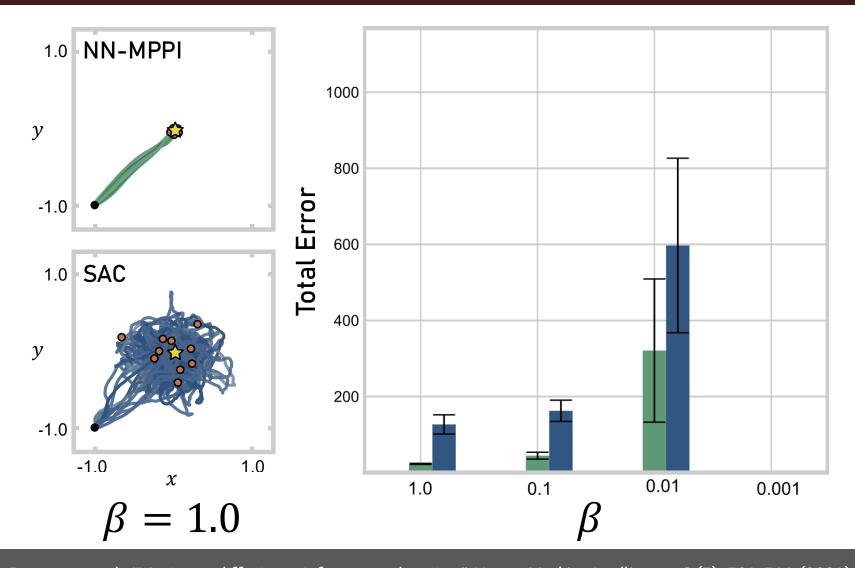


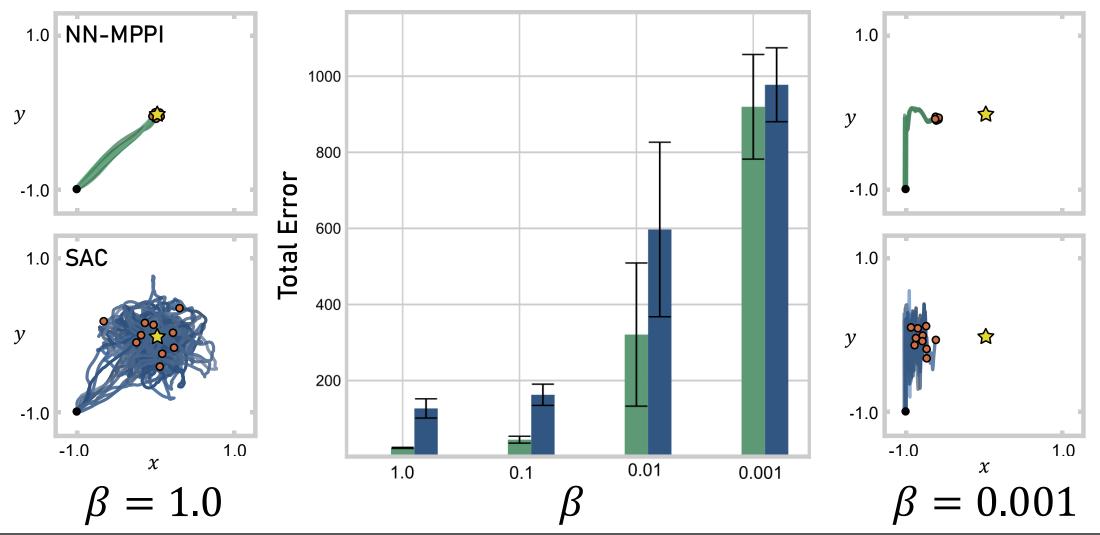






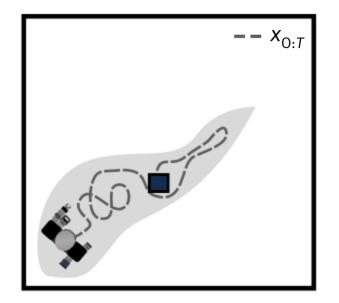






- Entropy maximization as a means of sample path decorrelation.
- However, this is intractable for general unknown dynamics.

$$P[x_{0:T}] = p(x_0) \prod_{t=0}^{T} p(x_{t+1}|x_t)$$



 $\underset{P[x_{0:T}]}{\operatorname{argmax}} S[P[x_{0:T}]]$ 

 $P_{\max}[x_{0:T}]$ 



$$\underset{P[x_{0:T}]}{\operatorname{argmax}} - \int P[x_{0:T}] \log P[x_{0:T}] \mathcal{D} x_{0:T} - \lambda_0 \left( \int P[x_{0:T}] \mathcal{D} x_{0:T} - 1 \right)$$

$$-\int Tr\Big(\Lambda(x^*)^{\intercal}\big(\langle\Delta x_{0:T}^2\rangle_{x^*}-\mathbf{C}[x^*]\big)\Big)dx^*$$

where:

$$\langle \Delta x_{0:T}^2 \rangle_{x^*} = \int P[x_{0:T}] \left[ \sum_{i=0}^T (x_{i+1} - x_i)^{\intercal} (x_{i+1} - x_i) \delta(x_i - x^*) \right] \mathcal{D}x_{0:T}$$

$$\mathbf{C}[x^*] = \sum_{ au=t_i}^{t_i+\Delta t} K_{XX}(t_i, au), \;\; (w/x_{t_i}=x^*)$$









For systems with continuous sample paths, we can prove that:

$$\max_{P} S[P[x_{0:T}]] \leq \sum_{t=0}^{T} \frac{1}{2} \log \det \mathbf{C}[x_t] \propto \frac{\log \text{-Volume of locally reachable states}}{1 + \log t}$$

which is a concave and easily computable quantity.

Optimizing this expression leads to <u>diffusive exploration</u>, because its optimum describes the sample paths of a class of diffusion processes.









### Maximum diffusion reinforcement learning

• To illustrate these results, we developed an RL pipeline based on our derivations:

$$\operatorname*{argmax}_{\pi} E_{p,\pi} \left[ \sum_{t=0}^{T} \hat{r}(x_t, u_t) \right]$$

• The rewards are augmented with a term that decorrelates sample paths:

$$\hat{r}(x_t, u_t) = r(x_t, u_t) + \frac{\alpha}{2} \log \det \mathbf{C}[x_t]$$

• Agents that optimize MaxDiff objectives are <u>ergodic</u> and asymptotically inherit robustness and online learning guarantees.









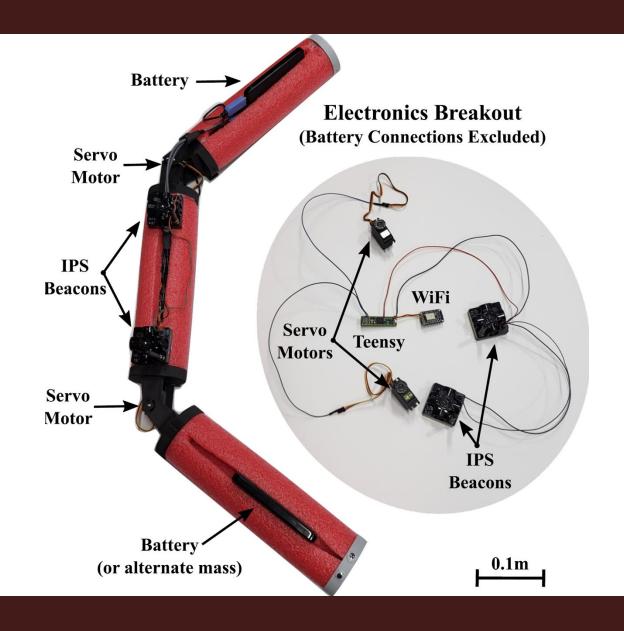
#### Maximum diffusion reinforcement learning

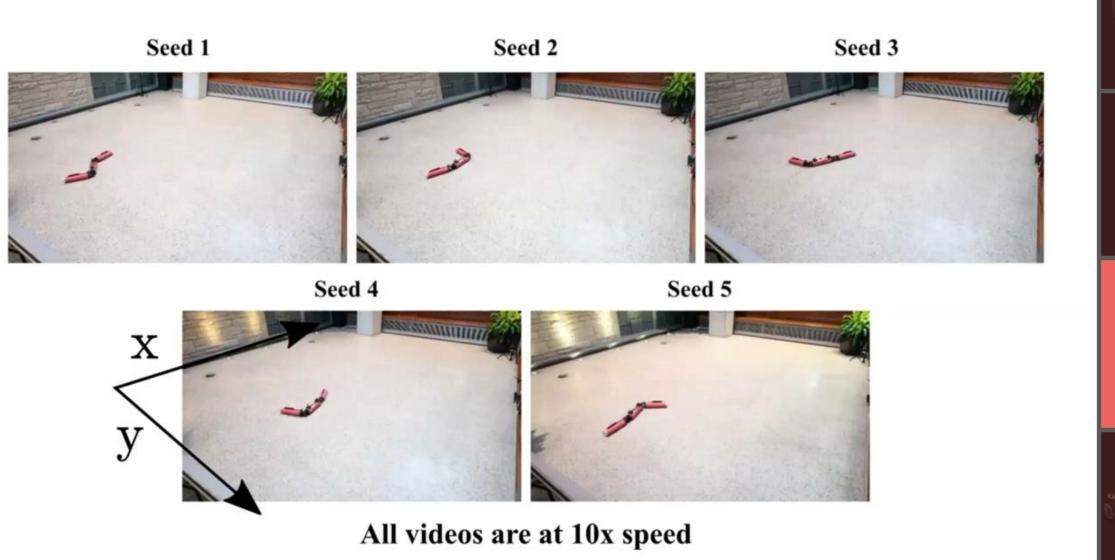
**Theorem 2.** (MaxDiff RL Agents are Reliable) If there exists a PAC-MDP algorithm with policy  $\pi^{\max}$ , then the Markov chain induced by  $\pi^{\max}$  is ergodic and  $\pi^{\max}$  will be  $\epsilon$ -optimal regardless of initialization.

**Theorem 3.** (MaxDiff RL Agents can Learn in Single-Shot) If there exists a PAC-MDP algorithm with policy  $\pi^{\max}$ , then the Markov chain induced by  $\pi^{\max}$  is ergodic and any realization of  $\pi^{\max}$  will asymptotically achieve the same  $\epsilon$ -optimality as an ensemble.

• To be <u>PAC-MDP</u> is  $\epsilon$ -optimal  $(1 - \delta)$ -percent of the time:

$$\Pr(\mathcal{V}_{\pi^*}(x_0) - \mathcal{V}_{\pi^{\max}}(x_0) \le \epsilon) \ge 1 - \delta$$



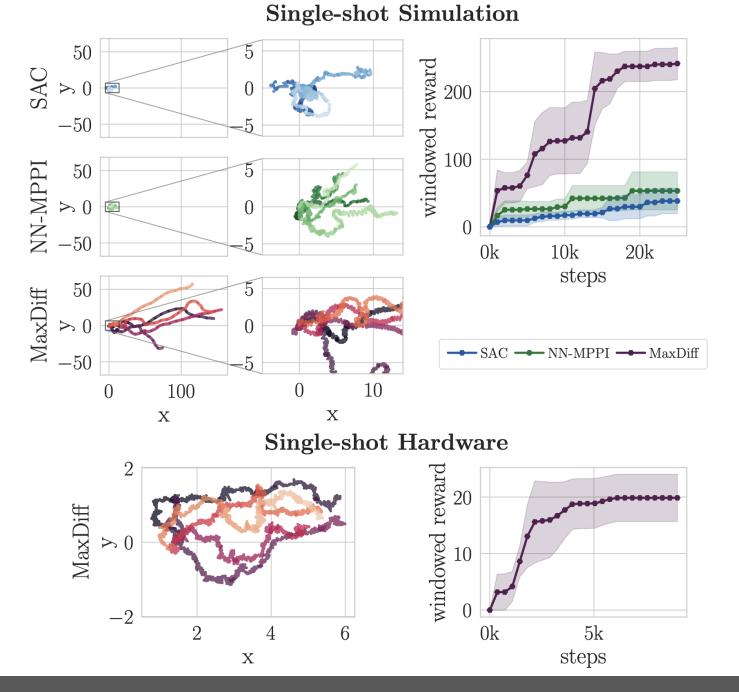














Path decorrelation for efficient online learning



Operational safety through modular design

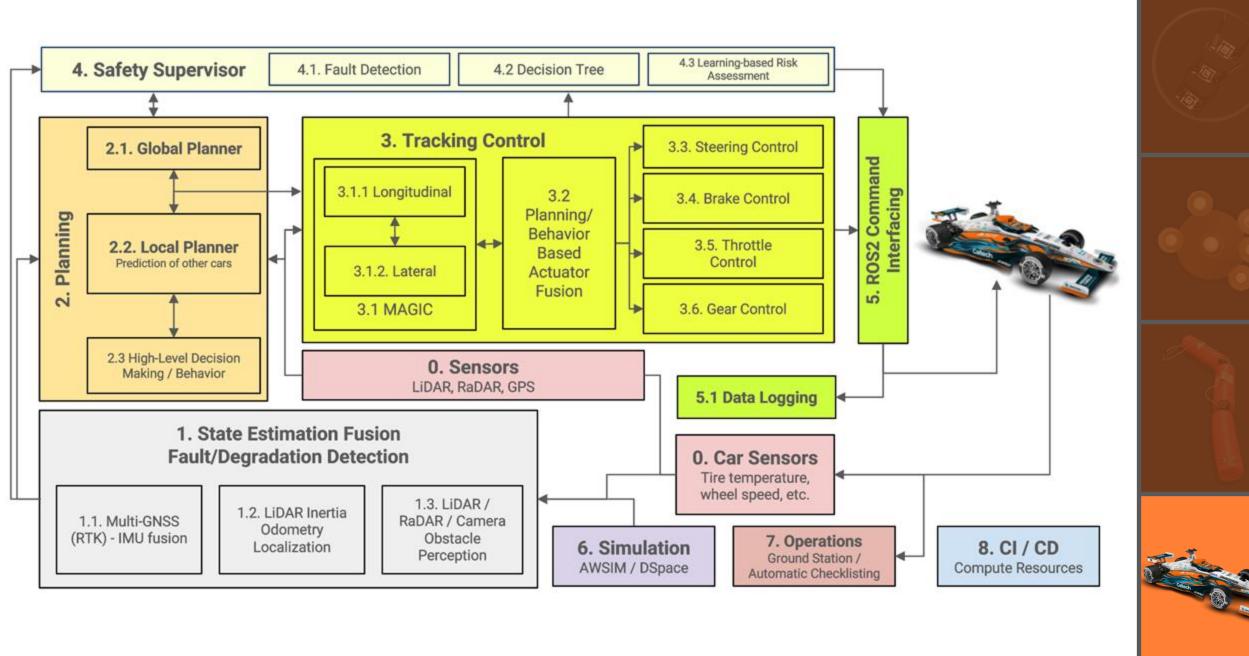








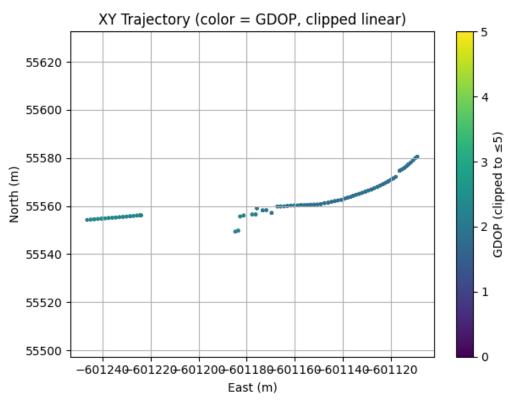


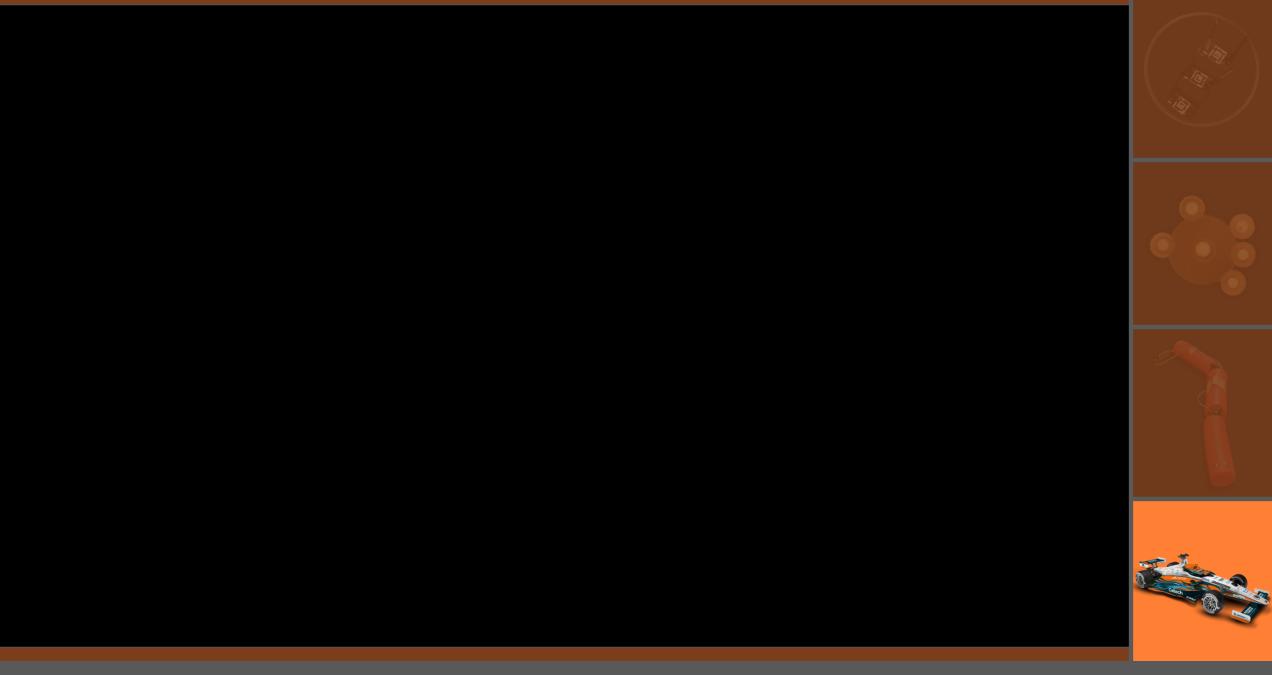


#### Modeling the dynamics of uncertainty

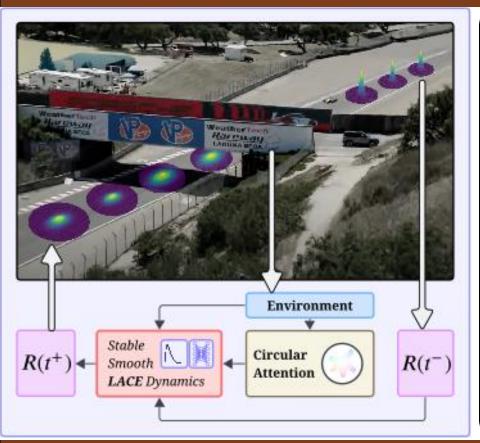


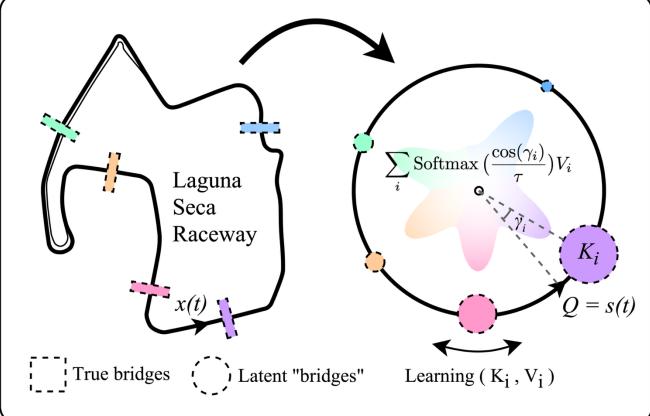




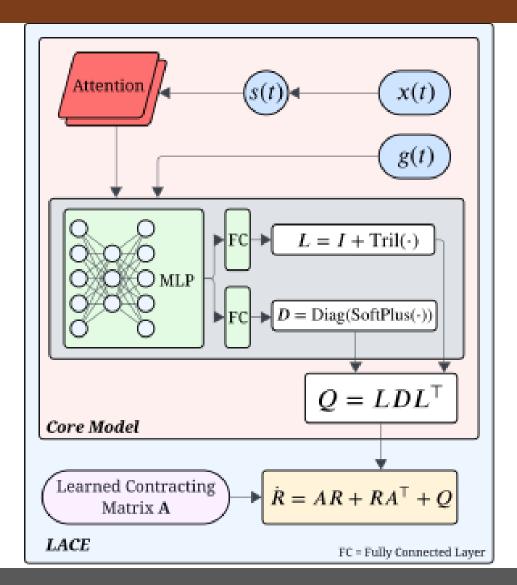


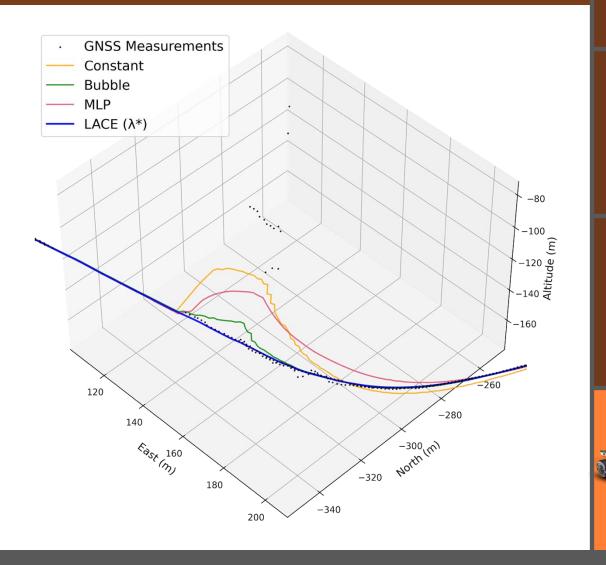
### Modeling the dynamics of uncertainty



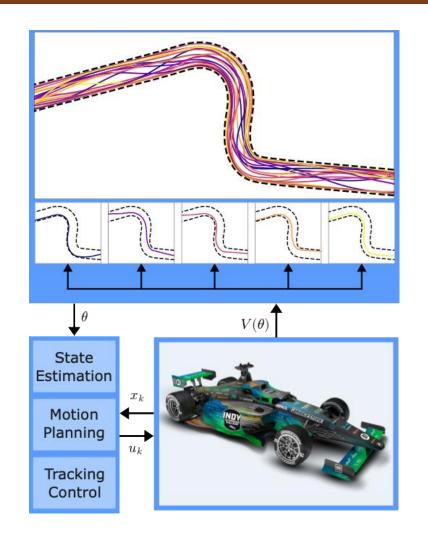


### Modeling the dynamics of uncertainty





#### Online learning of closed-loop racelines



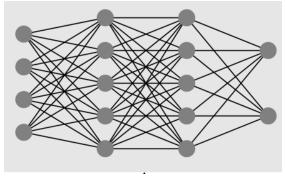
Engine parameters, brake bias, margin to track bounds, etc. Deep Kernel Learning Evaluate  $V(\theta)$ Propose  $\theta$ **Thompson Sampling** Lap time





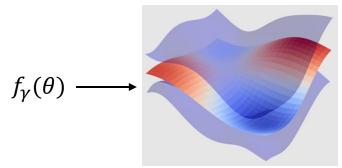
TRACK TITAN

Representation learning



γ

Deep GP with compound kernel



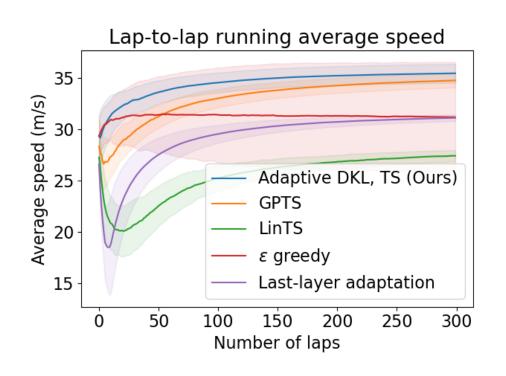
$$\hat{V}(\theta') \sim N(\mu(\theta'), k^*(f_{\gamma}(\theta), f_{\gamma}(\theta')))$$

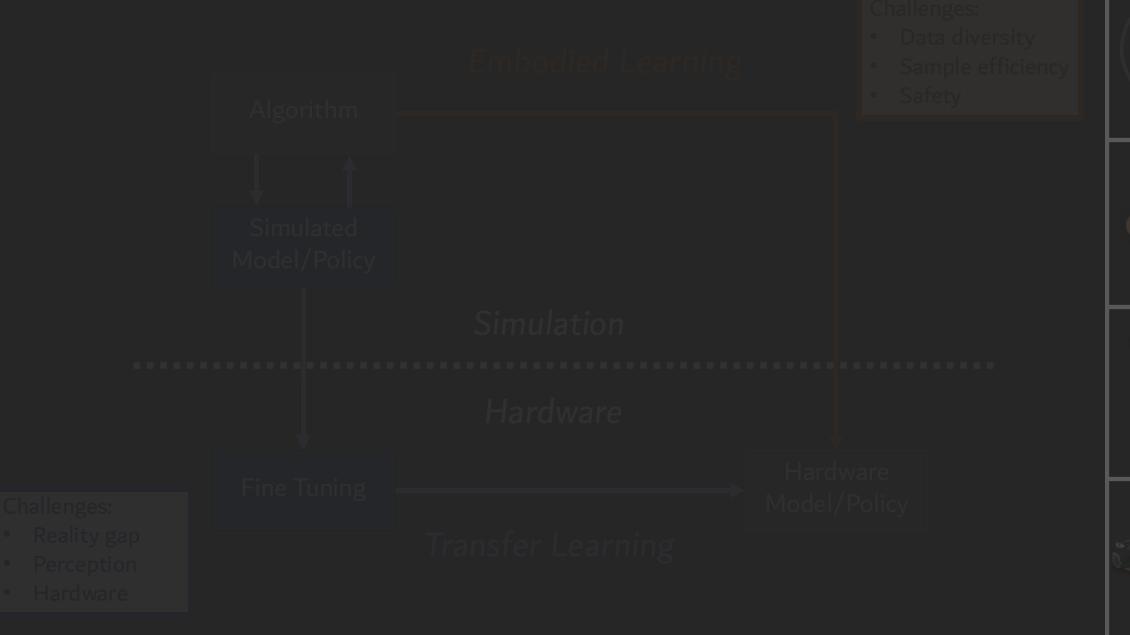
$$k^* = k(f_{\gamma}(\theta), f_{\gamma}(\theta')) + k(\theta, \theta')$$
Offline Online



#### Online learning of closed-loop racelines

- Our approach can leverage strong priors to learn faster.
- Representation learning is key to finding better lap times.
- Since the MPC is the ultimate determinant of safety, this work is directly transferable to hardware.



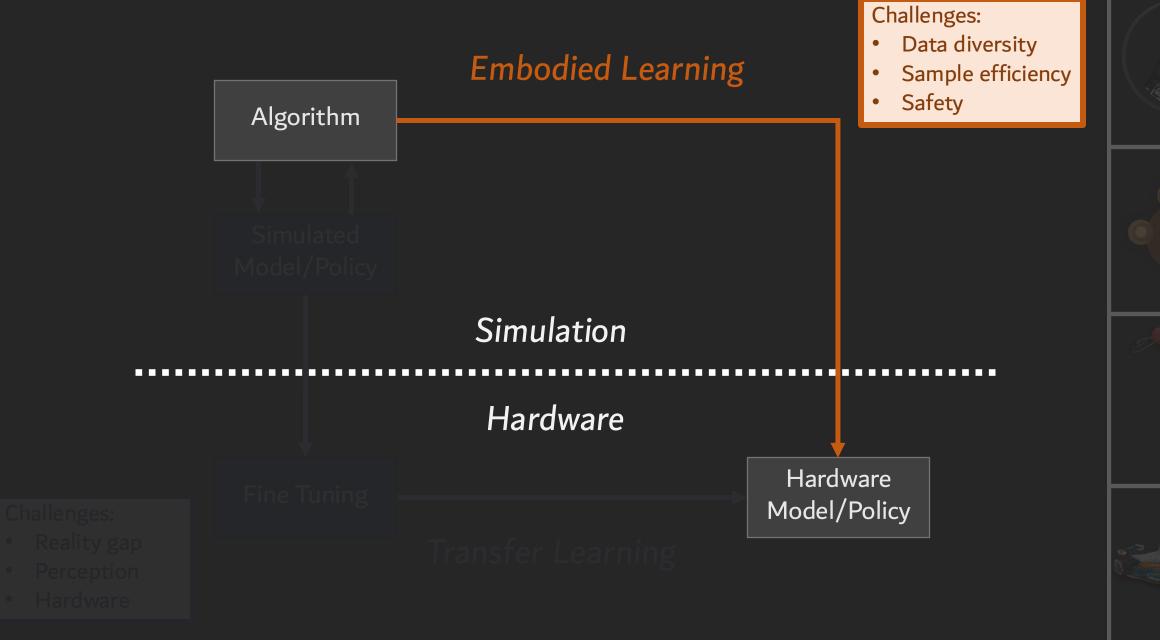




















#### Questions?



