

Learning by Demonstration with Dynamic Movement Primitives: Adaptation, Safety and Applications

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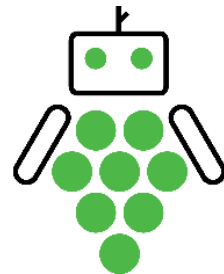
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Focus Period Lund University 2025: Robot Learning

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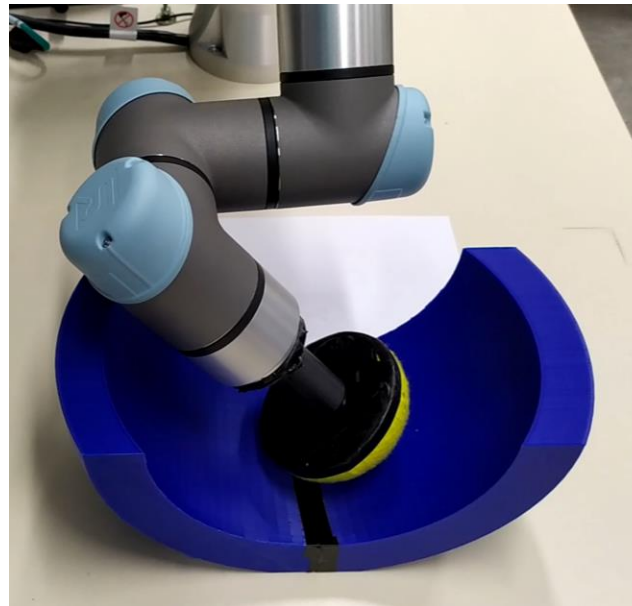
Who am I?

- Born in Thessaloniki (1996)
- Grew up in Thessaloniki (1996 – 2014)
- Integrated Master in the Aristotle University of Thessaloniki (2014 – 2019)
- PhD and working at research projects in the Aristotle University of Thessaloniki (2020-now)



Research Interests

- Bimanual Robot Control
- Non-Prehensile Object Manipulation
- Robotic Cutting
- Orientation Control
- Learning by Demonstration (DMP)



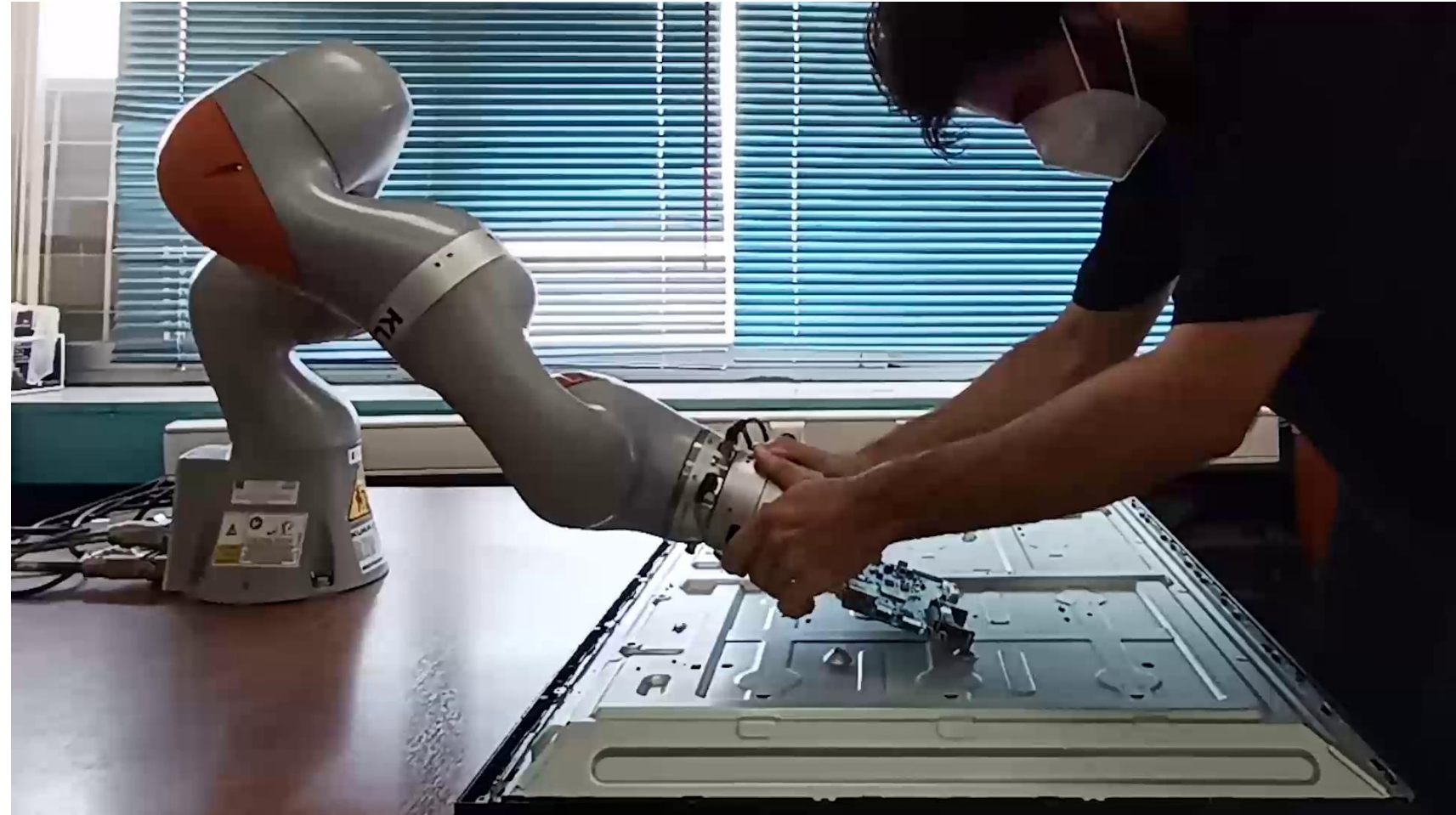
Human Skills

- Complex task execution based on years of experience.
- Dynamic adaptation on environment changes.
- Predictable behavior.
- Safe for other humans in the workspace



Learning by Demonstration

- Intuitive way to program a robotic motion without requiring tedious programming.
- Kinesthetic teaching: Physically guiding the robot to perform the desired task.



Human Robot Coexistence

- Modern robotics operate in shared workspaces with humans
- Modern industries and small businesses, domestic environments, retail
- The robot needs to be able to learn, adapt and be safe for the humans in its workspace



Dynamic Movement Primitives (DMP)

- Linear second order dynamics

$$\begin{aligned}\tau \dot{\mathbf{z}} &= a_z(\beta_z(\mathbf{g} - \mathbf{y}) - \mathbf{z}) + \mathbf{S}_g \mathbf{f}(x) \\ \tau \dot{\mathbf{y}} &= \mathbf{z}\end{aligned}$$

- Augmented by a non-linear forcing term.

$$\mathbf{S}_g = \text{diag}(\mathbf{g} - \mathbf{y}_0) \text{diag}(\mathbf{g}_d - \mathbf{y}_{0d})$$

- First order canonical system.

$$\mathbf{f}(x) = \frac{\sum_i \mathbf{w}_i \Psi_i(x)}{\sum_i \Psi_i(x)}$$

- The canonical system makes the system autonomous.

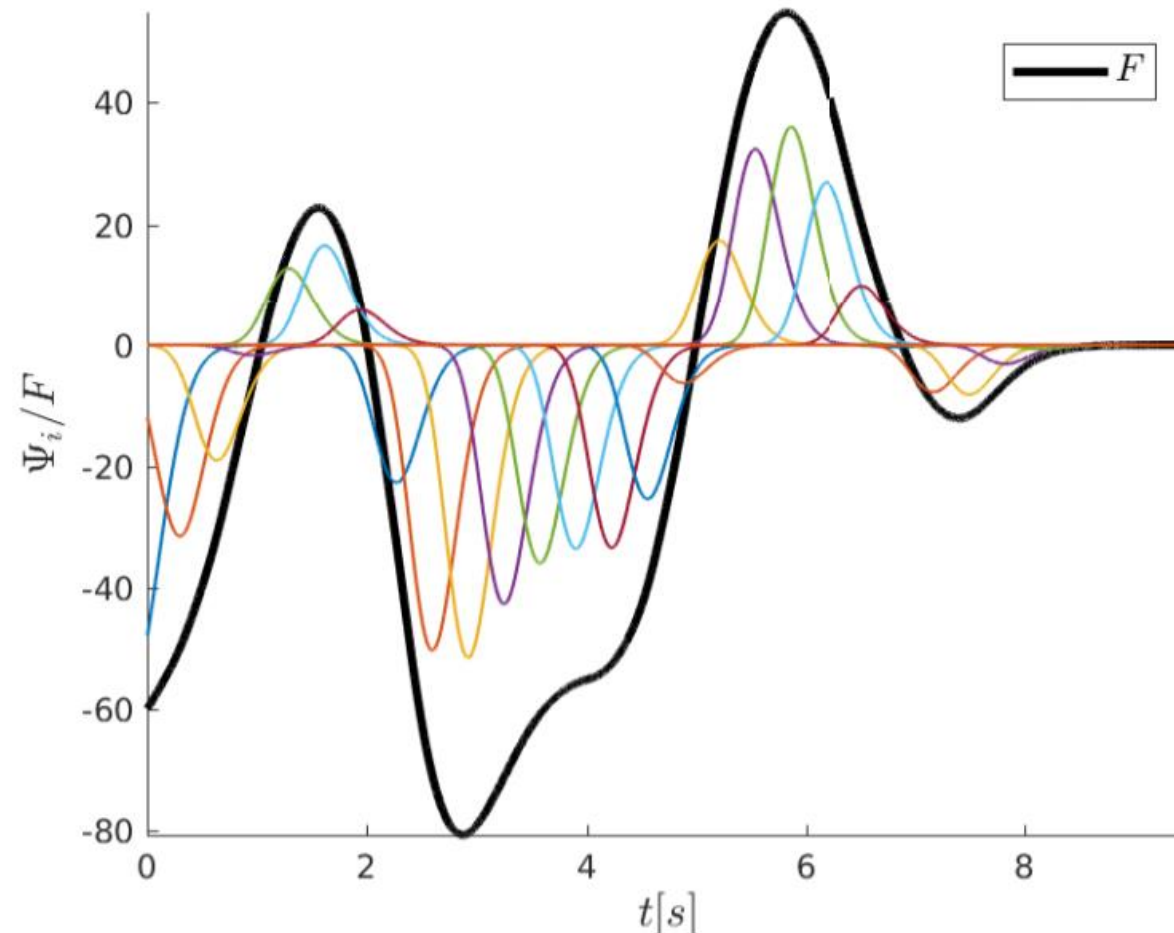
$$\Psi_i(x) = \exp(-h_i(x - c_i)^2)$$

$$\tau \dot{x} = -a_x x$$

DMP Training

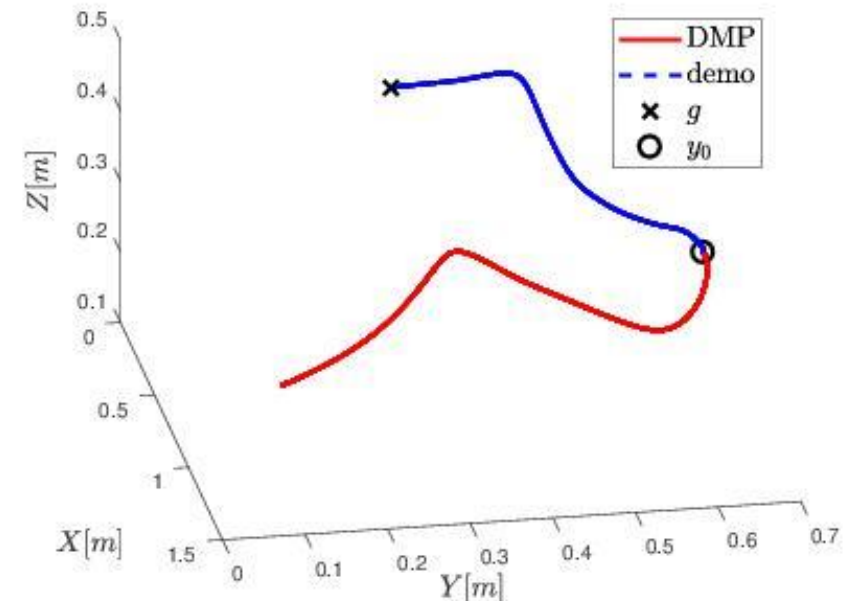
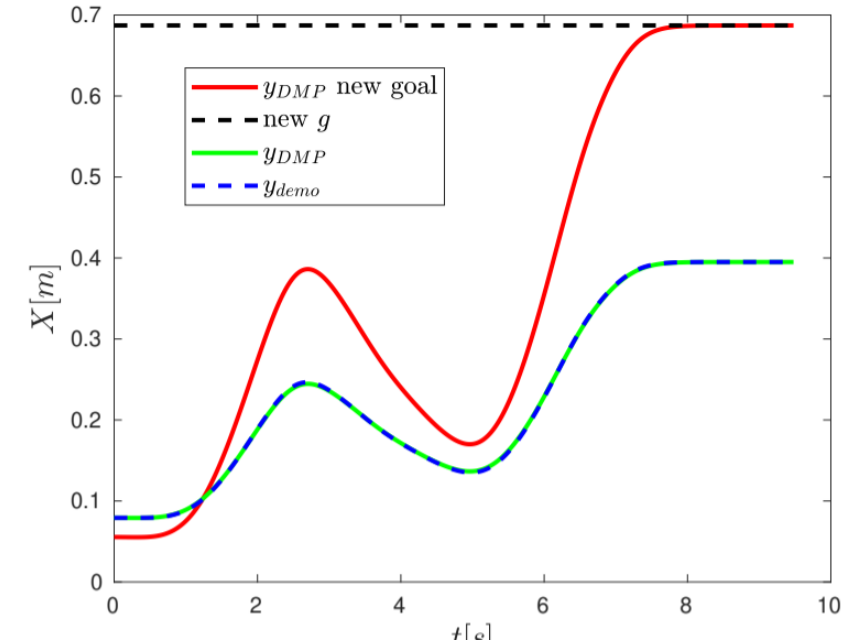
- The non linear forcing term is trained to match a desired trajectory.
- Only one demonstration is needed.

$$f_d = \tau^2 \ddot{y}_d - a_z(\beta_z(g_d - y_d) - \tau \dot{y}_d)$$



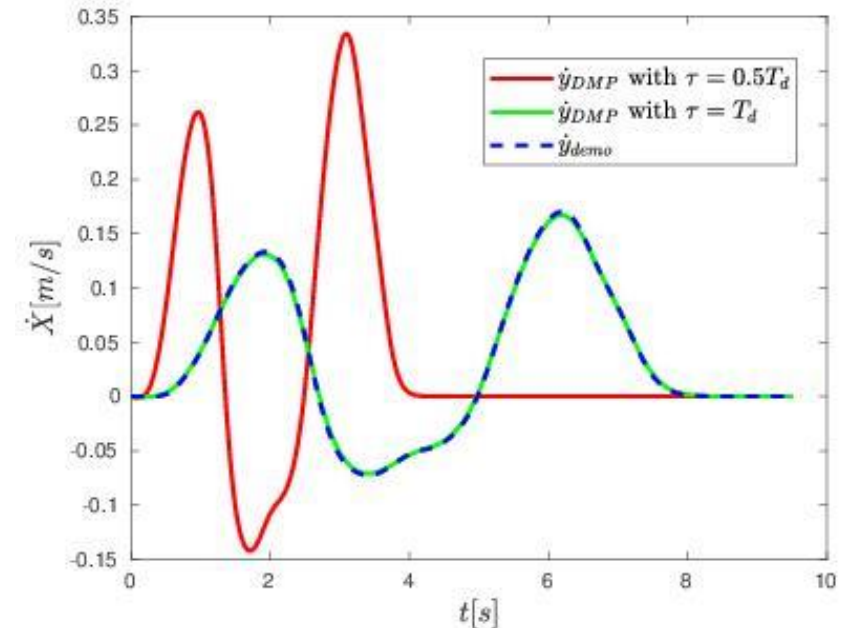
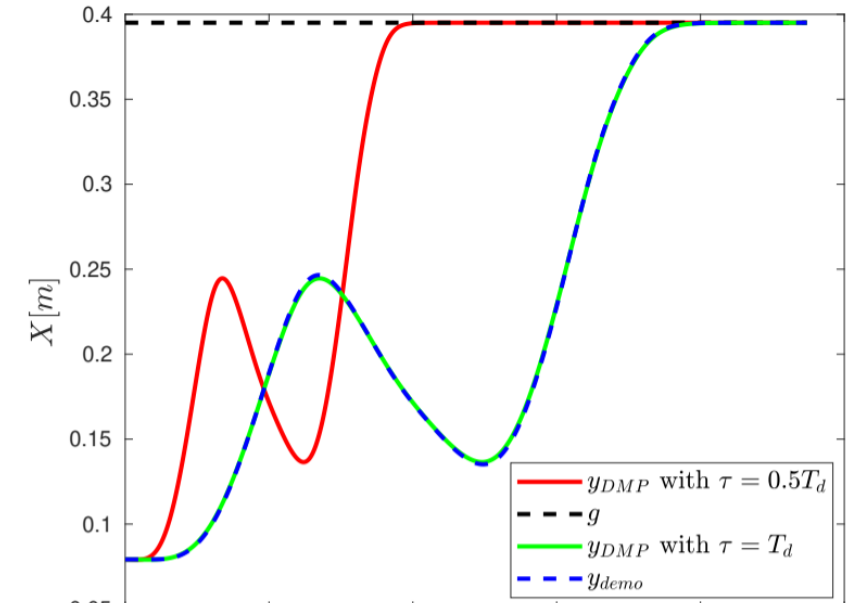
DMP Properties: Spatial Scaling

- Predictable adaptation to new goal / new initial position.
- $S_g = \text{diag}(\mathbf{g} - \mathbf{y}_0) \text{diag}(\mathbf{g}_d - \mathbf{y}_{0d})$
- Scaling according to the difference in each coordinate



DMP Properties: Temporal Scaling

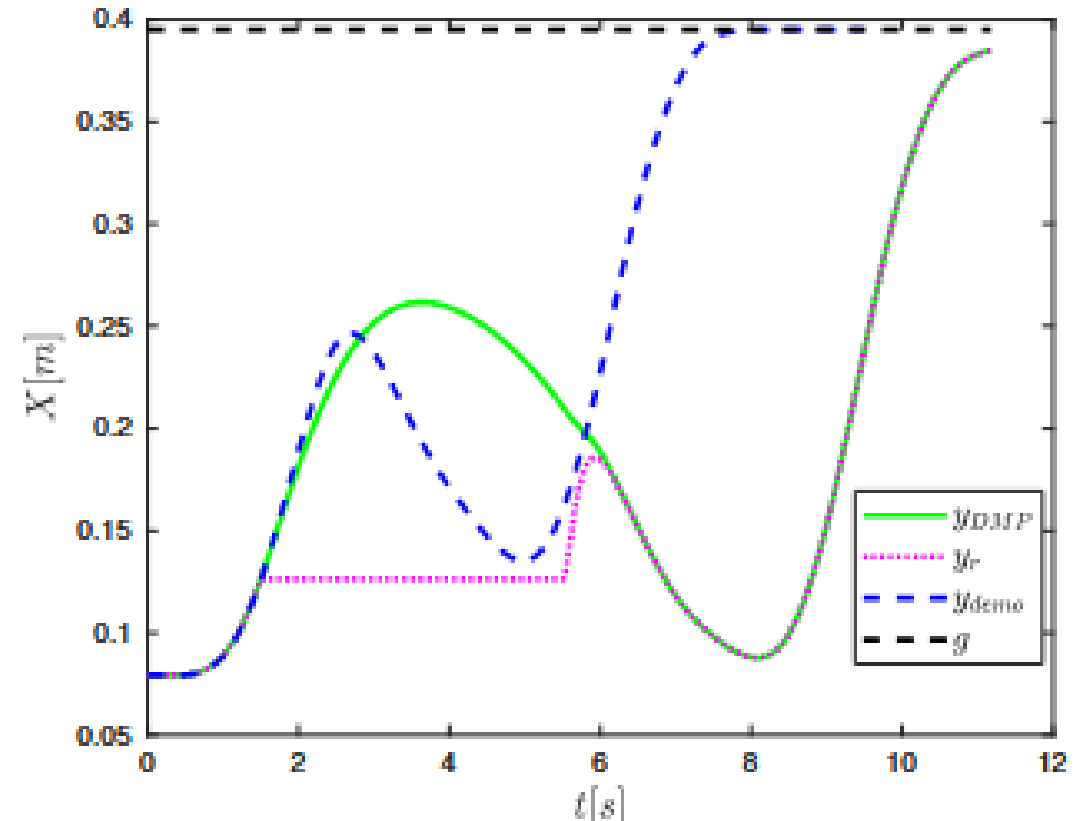
- Make the robot faster/ slower by setting a different values to τ than the demonstration.
- Demo: $\tau_d = 1$ or $\tau_d = T_d$.
- Duration: $T = \frac{\tau_d}{\tau} T_d$
- $\tau > \tau_d$ slower motion, $\tau < \tau_d$ faster motion.



DMP Properties: Phase stopping

- Stop / restart the trajectory execution.
- $\tau \dot{x} = \frac{-a_x x}{1+c(\cdot)}$
- $\tau = \tau_0(1 + c(\cdot))$
- $c(\cdot)$ function of the state that determines when to stop / start. For $c(\cdot) = 0$ nominal execution, while large values pause execution.

- $c = a_\tau \|y - p_{robot}\|^2$



DMP Properties: Coupling terms

- Adaptation via additional terms

- Velocity / acceleration terms.

$$\begin{aligned}\tau \dot{\mathbf{z}} &= a_z(\beta_z(\mathbf{g} - \mathbf{y}) - \mathbf{z}) + \mathbf{S}_g \mathbf{f}(x) + \mathbf{c}_a \\ \tau \dot{\mathbf{y}} &= \mathbf{z} + \mathbf{c}_v\end{aligned}$$

- Dynamically modify the generated trajectory to generate adapted behaviors.

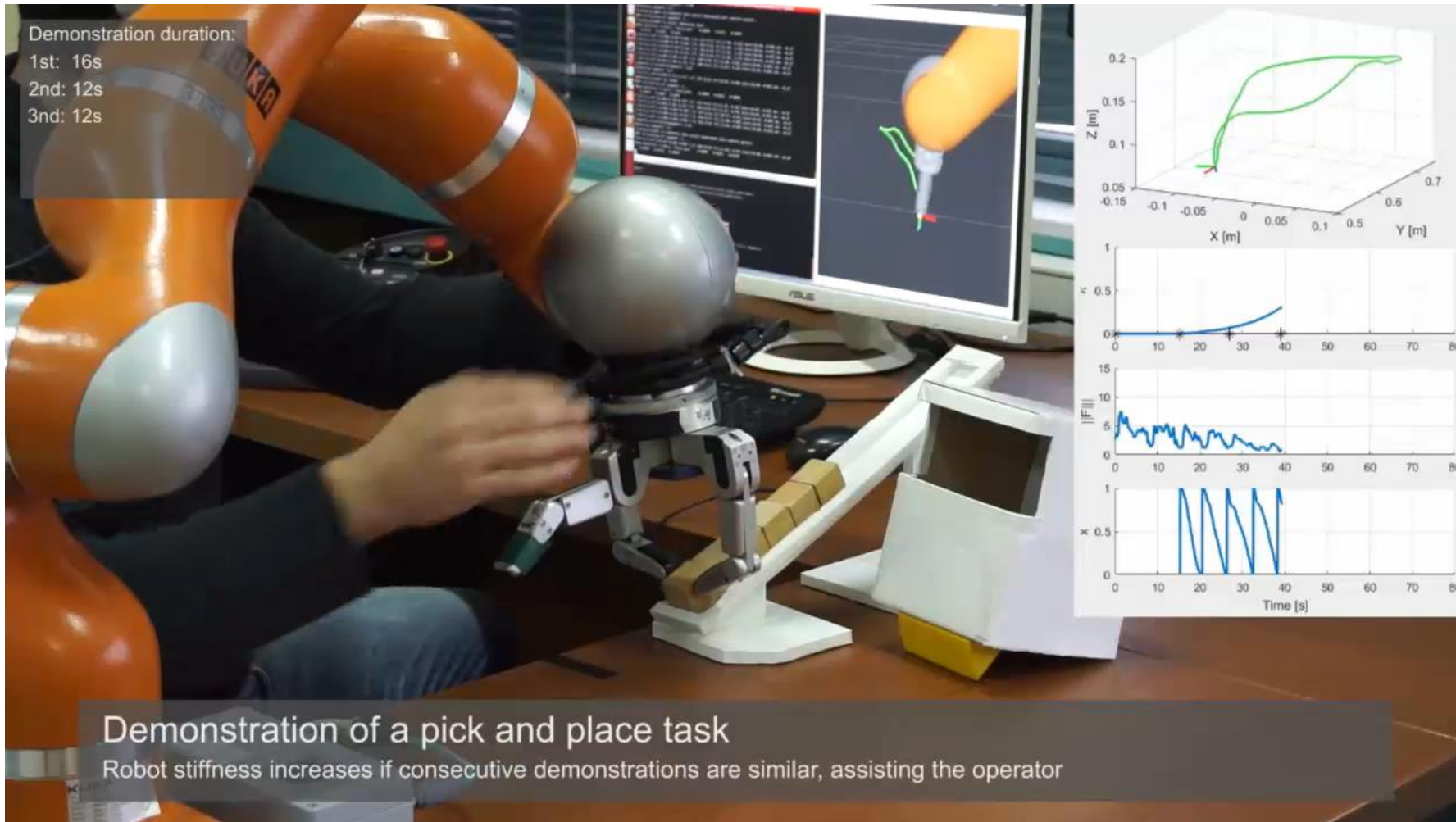
\mathbf{c}_a : acceleration (force) coupling term

\mathbf{c}_v : velocity coupling term

- Applications: obstacle avoidance, force feedback, constraint enforcement etc.

DMP Applications (ARL)

Progressive automation of robotic tasks.



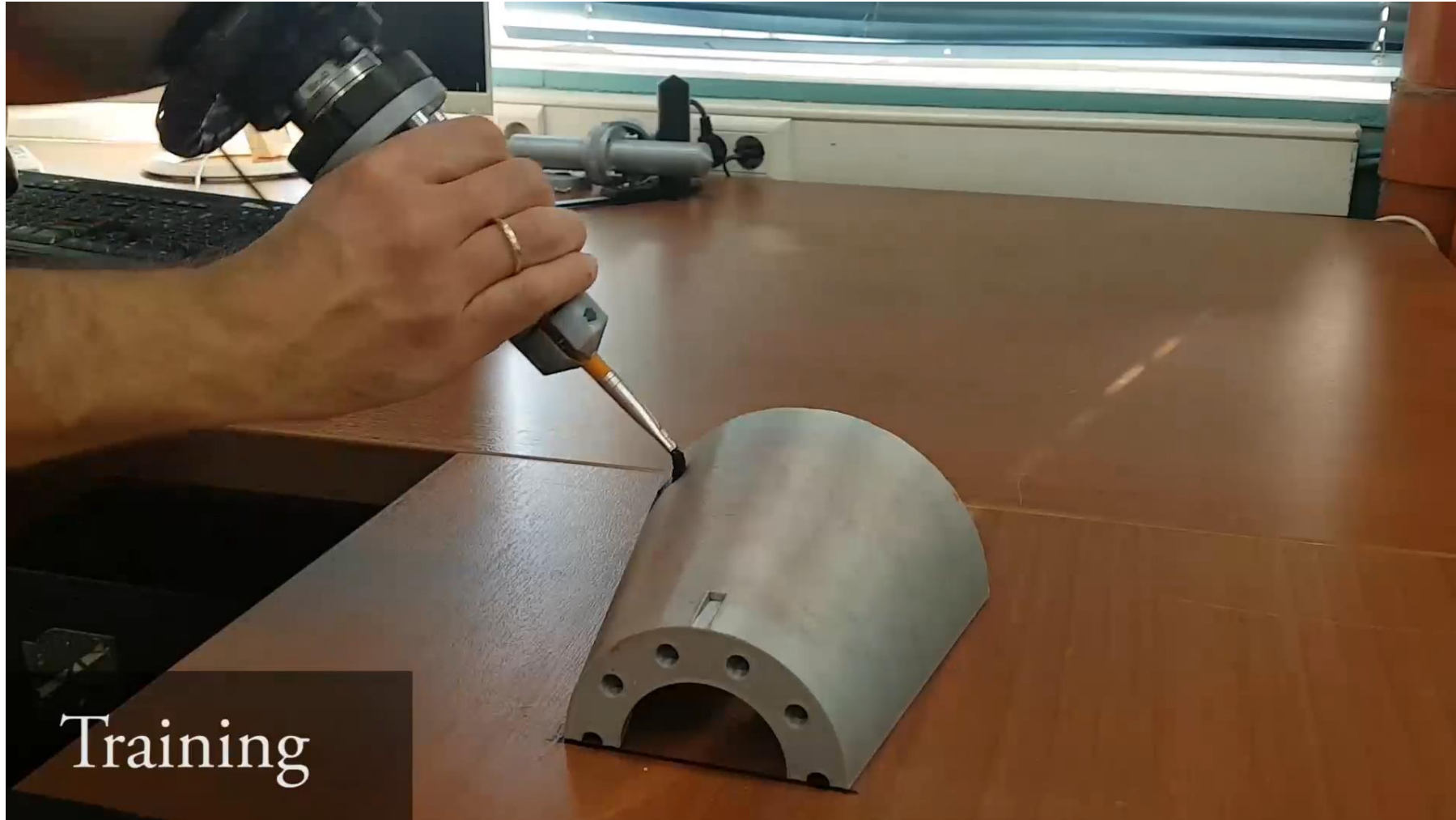
DMP Applications (ARL)

Human-robot collaborative object transfer.



DMP Applications (ARL)

DMP for surface tasks



Contents

- DMP for orientation.
- DMP are built for stationary goals. Extension to moving goals.
- DMP spatial scaling.
- DMP applications.

Orientation DMP formulation

- Goal: Second order dynamics similar to linear dynamics.

$$\tau \dot{\boldsymbol{\eta}} = \alpha_z (\beta_z 2 \log(\mathbf{Q}_g * \bar{\mathbf{Q}}) - \boldsymbol{\eta}) + \mathbf{S}_g \mathbf{f}(\mathbf{x})$$

- Unit quaternions.

$$\tau \dot{\mathbf{Q}} = \frac{1}{2} \begin{bmatrix} 0 \\ \boldsymbol{\eta} \end{bmatrix} * \mathbf{Q}$$

- Orientation error?

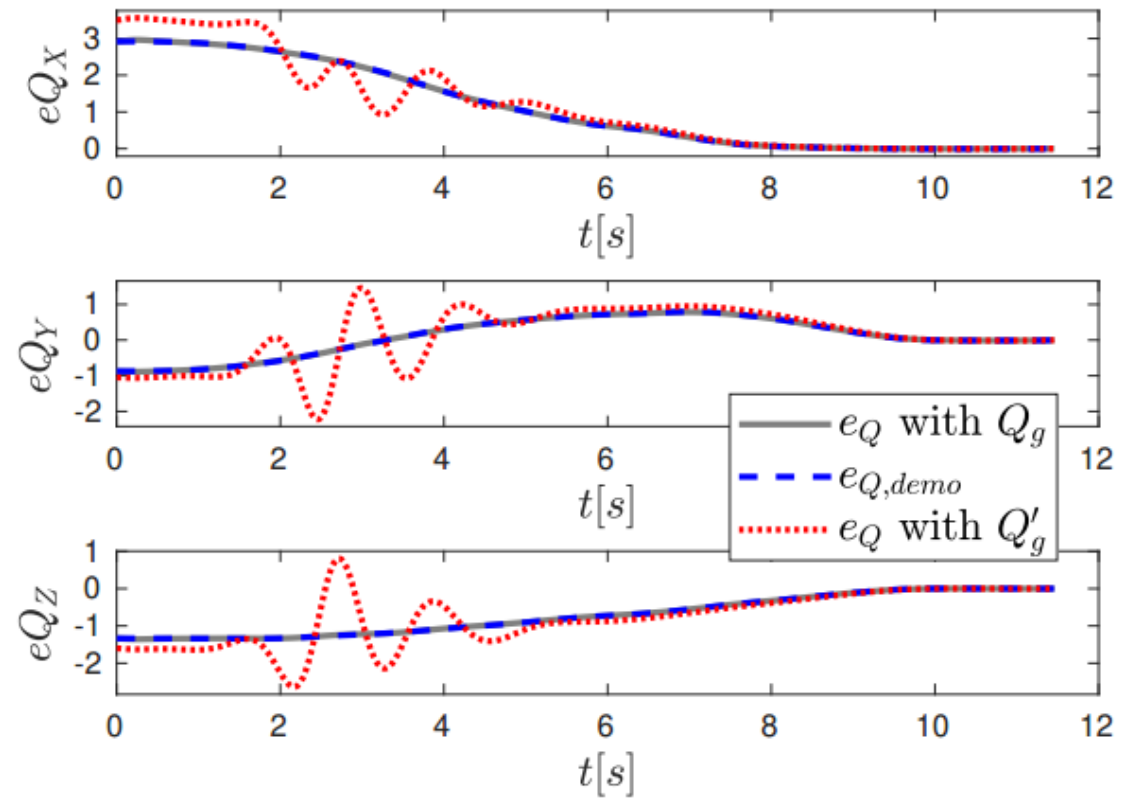
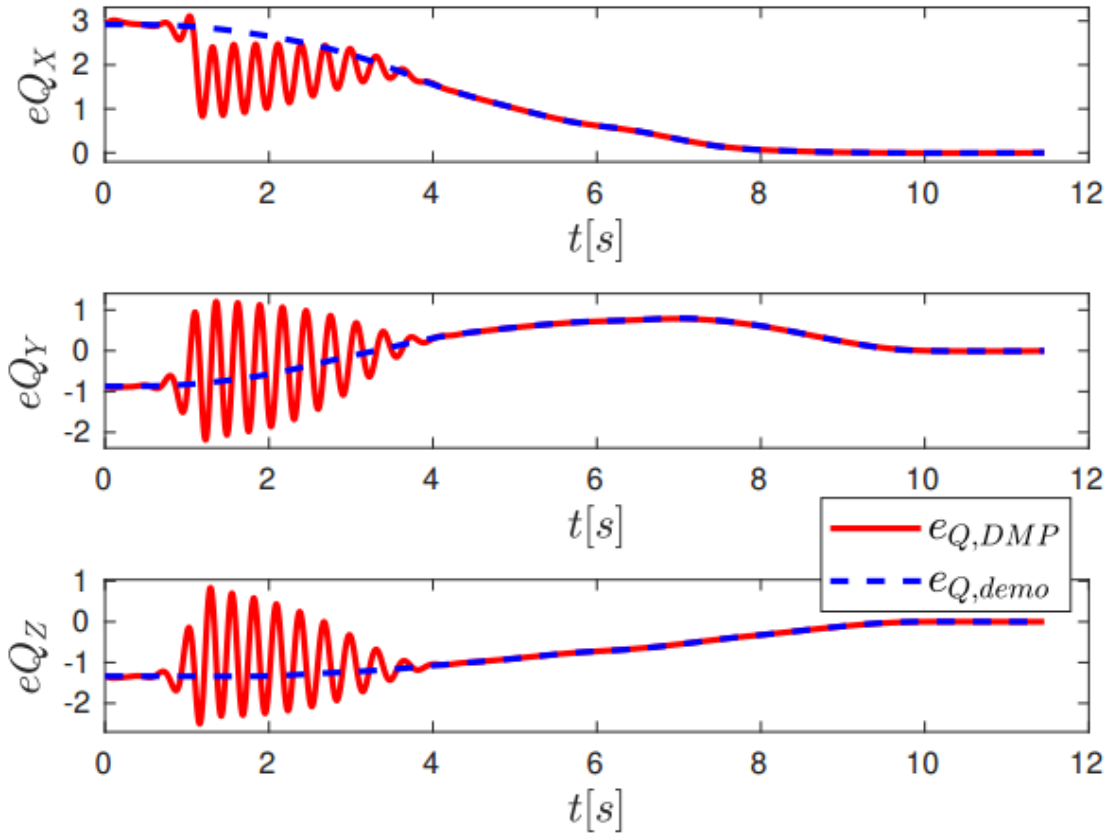
- Logarithmic orientation error (Lie Algebra).

$$\mathbf{S}_g = \text{diag}(\mathbf{Q}_g * \mathbf{Q}_0) \text{diag}(\mathbf{Q}_{gd} * \mathbf{Q}_{0d})^{-1}$$

- Non-linear forcing term / canonical system remain the same.

$$\boldsymbol{\omega} = 2 \text{vec}(\dot{\mathbf{Q}} * \bar{\mathbf{Q}})$$

Oscillatory Behavior



System Analysis (position DMP)

- Assume negligible training error and $\mathbf{g} = \mathbf{g}_d, \mathbf{y}_0 = \mathbf{y}_{0d}$. Then:

$$\mathbf{f}(x) = \mathbf{f}_d(x) = \tau^2 \ddot{\mathbf{y}}_d - a_z(\beta_z(\mathbf{g}_d - \mathbf{y}_d) - \tau \dot{\mathbf{y}}_d)$$

- Closed loop tracking dynamics:

$$\begin{aligned} \tau^2 \ddot{\mathbf{y}} &= a_z(\beta_z(\mathbf{g} - \mathbf{y}) - \tau \dot{\mathbf{y}}) + \mathbf{S}_g \mathbf{f}(x) \Rightarrow \\ \tau^2 (\ddot{\mathbf{y}} - \ddot{\mathbf{y}}_d) + \tau a_z (\dot{\mathbf{y}} - \dot{\mathbf{y}}_d) + a_z \beta_z (\mathbf{y} - \mathbf{y}_d) &= \mathbf{0} \end{aligned}$$

- Linear tracking error dynamics lead to perfect trajectory tracking.

System Analysis (Orientation DMP)

- Assume negligible training error and $\mathbf{Q}_g = \mathbf{Q}_{gd}$, $\mathbf{Q}_0 = \mathbf{Q}_{0d}$. Then:

$$\mathbf{f}(x) = \mathbf{f}_d(x) = \tau^2 \dot{\boldsymbol{\omega}}_d - a_z(\beta_z 2 \log(\mathbf{Q}_{gd} * \bar{\mathbf{Q}}_d) - \tau \boldsymbol{\omega}_d)$$

- Closed loop tracking dynamics:

$$\begin{aligned} \tau^2 \dot{\boldsymbol{\omega}} &= a_z(\beta_z 2 \log(\mathbf{Q}_g * \bar{\mathbf{Q}}) - \tau \boldsymbol{\omega}) + \mathbf{S}_g \mathbf{f}(x) \Rightarrow \\ \tau^2 (\dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_d) + \tau \alpha_z (\boldsymbol{\omega} - \boldsymbol{\omega}_d) + \alpha_z \beta_z (2 \log(\mathbf{Q}_{gd} * \bar{\mathbf{Q}}_d) - 2 \log(\mathbf{Q}_g * \bar{\mathbf{Q}})) &= \mathbf{0} \end{aligned}$$

- Non-linear tracking dynamics cannot guarantee trajectory tracking leading to oscillatory behavior.

Proposed Orientation Formulation

Introduce linear second order dynamics.

$$\tau \dot{\mathbf{z}} = -\alpha_z (\beta_z \mathbf{e}_Q + \mathbf{z}) + \mathbf{S}_g \mathbf{f}(\mathbf{x})$$

$$\tau \dot{\mathbf{e}}_Q = \mathbf{z}$$

Integrate the linear system and get the unit Quaternion from the error formula.

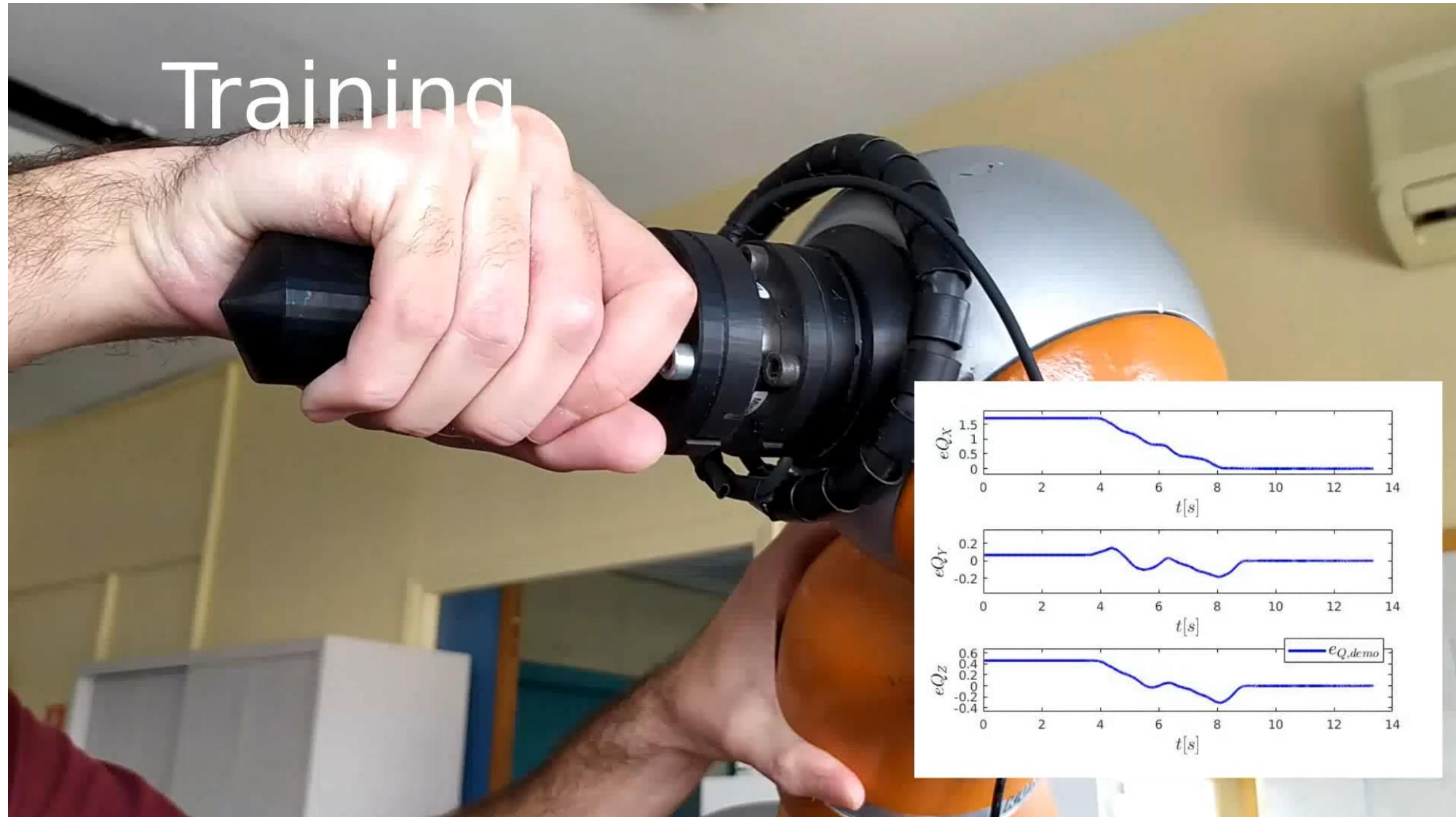
$$\mathbf{S}_g = \text{diag}(\mathbf{e}_{Q0}) \text{diag}(\mathbf{e}_{Q0d})^{-1}$$

$$\mathbf{e}_Q = 2 \log(\mathbf{Q}_g * \bar{\mathbf{Q}})$$

DMP properties are maintained.

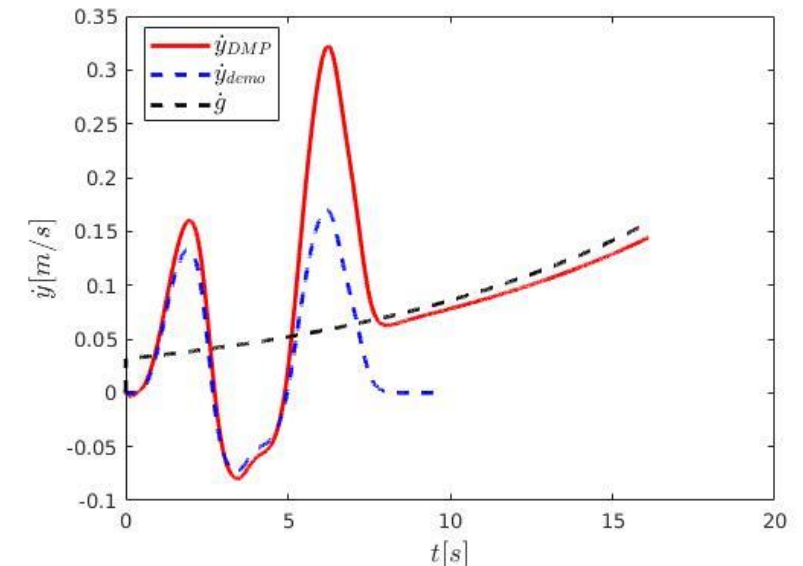
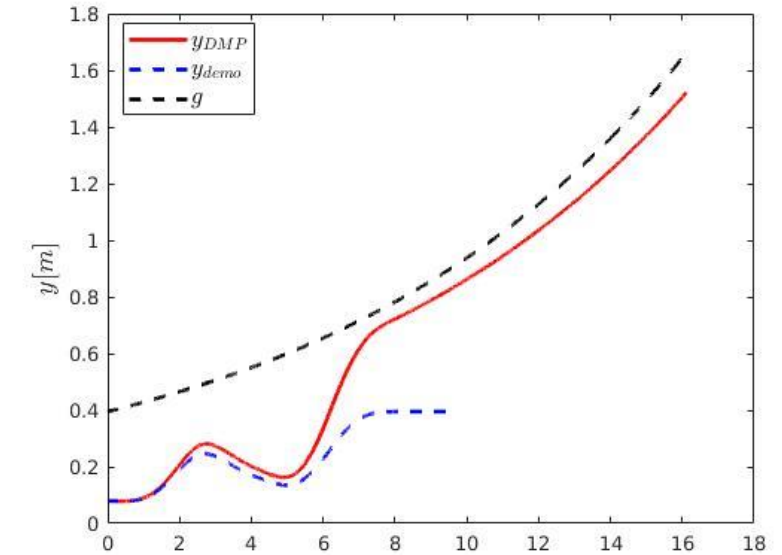
$$\boldsymbol{\omega} = \mathbf{J}_l \mathbf{e}_Q$$

Experimental Results



Real Time Adaptation to Moving Goals

- Original DMP:
 - the linear attractor system does not yield zero tracking error
 - A goal moving away from the initial position would induce high velocities due to the spatial scaling of the DMP
- Previous works:
 - predictors or estimators for the goal's position at the time of interception.
 - estimation errors which may worsen the system's performance.
 - they do not consider the possible large velocity scaling rendering the system unsafe for human environments.



DMP for Moving Goals

- Rewrite the linear system with respect to the error.
- Train the DMP with a stationary goal.
- Adaptation of the temporal scaling parameter.
- Exponentially stable through contraction analysis.

$$\begin{aligned}\tau \dot{\mathbf{z}} &= -\alpha_z(\beta_z \mathbf{e} + \mathbf{z}) + \text{diag}(\mathbf{g} - \mathbf{y}_0) \mathbf{F}(\mathbf{x}) \\ \tau \dot{\mathbf{e}} &= \mathbf{z}\end{aligned}$$

$$\tau \dot{x} = -\alpha_x x$$

$$\dot{\tau} = -\alpha_\tau(\tau - \tau_g) + \dot{\tau}_g$$

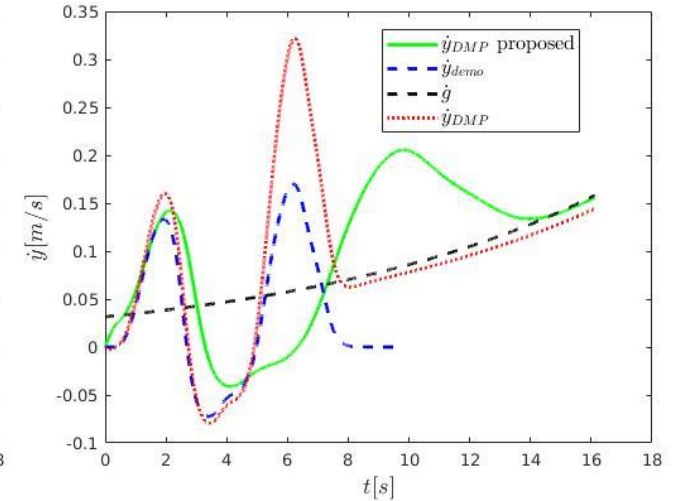
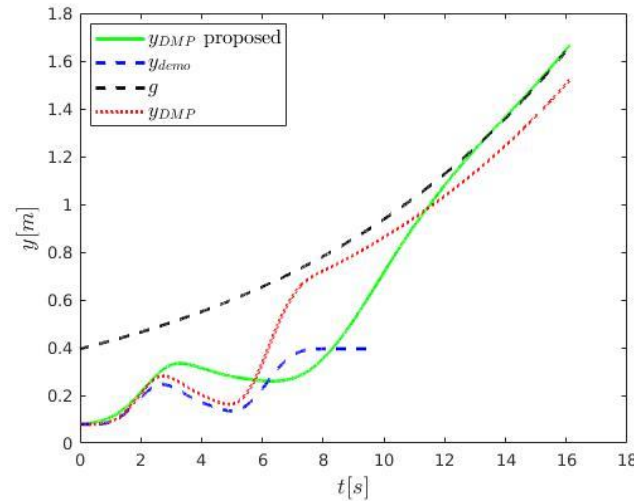
$$\tau_g = \frac{\|\mathbf{g} - \mathbf{y}_0\|}{\|\mathbf{g}_d - \mathbf{y}_{0d}\|} \tau_d$$

Temporal Scaling Adaptation

$$|g - y_0| \uparrow \Rightarrow$$

$$\tau \uparrow \Rightarrow$$

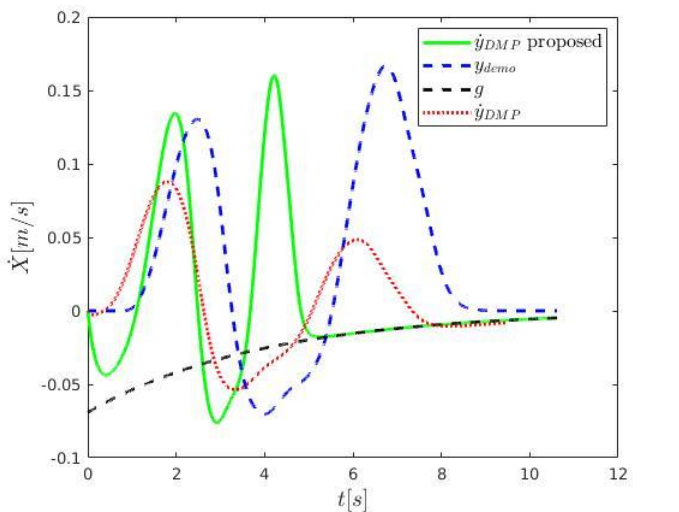
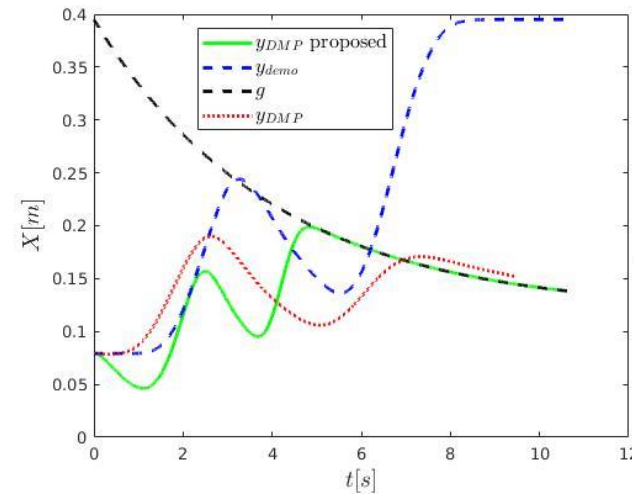
$$|\dot{y}| \downarrow$$



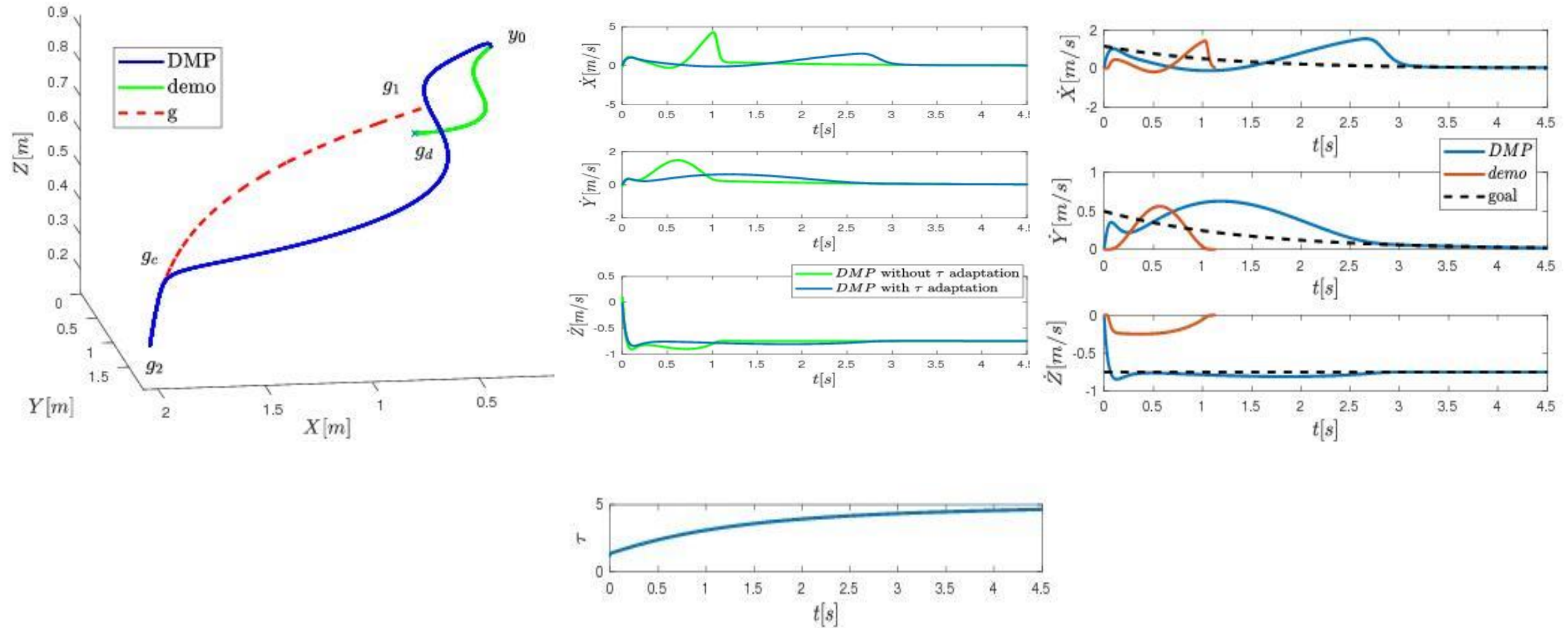
$$|g - y_0| \downarrow \Rightarrow$$

$$\tau \downarrow \Rightarrow$$

$$|\dot{y}| \uparrow$$



Simulation Results



Experimental Results

Dynamic Movement Primitives for moving goals with temporal scaling adaptation

Leonidas Koutras and Zoe Doulgeri

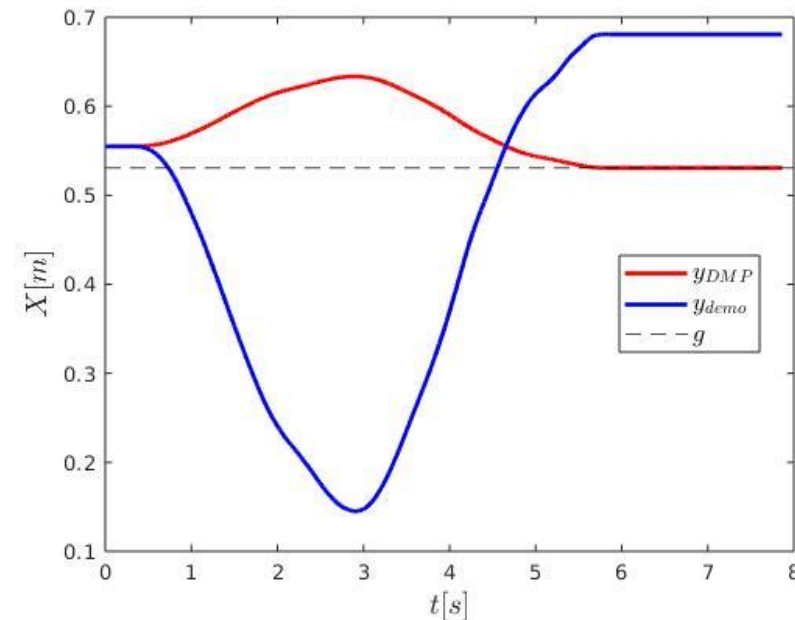
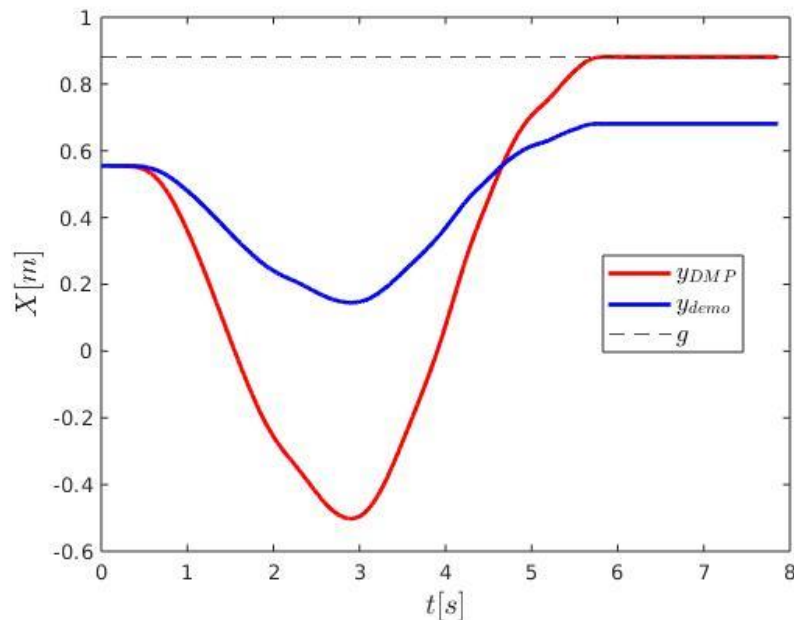


Aristotle University of Thessaloniki,
Department of Electrical and
Computer Engineering, Greece.

Original DMP Spatial Scaling

- Assume negligible training error. Closed loop tracking dynamics:

$$\tau^2(\ddot{\mathbf{y}} - \mathbf{S}_g\ddot{\mathbf{y}}_d) + \tau\alpha_z(\dot{\mathbf{y}} - \mathbf{S}_g\dot{\mathbf{y}}_d) + \alpha_z\beta_z(\mathbf{y} - \mathbf{S}_g\mathbf{y}_d) = \alpha_z\beta_z(\mathbf{y}_0 - \mathbf{S}_g\mathbf{y}_{0d})$$
- Diagonal \mathbf{S}_g : trajectory flipping, large scaling when a coordinate of \mathbf{g} and \mathbf{y}_0 is similar, trajectory flipping when the sign of $\mathbf{g} - \mathbf{y}_0$ changes.



Biologically Inspired DMP

$$\begin{aligned}\tau \dot{\mathbf{z}} &= a_z(\beta_z(\mathbf{g} - \mathbf{y}) - \mathbf{z}) + \alpha_z \beta_z(\mathbf{s}_g + \mathbf{f}(x)) \\ \tau \dot{\mathbf{y}} &= \mathbf{z}\end{aligned}$$

- Proposed to correct the scaling issues of the original DMP.

$$\mathbf{s}_g = -(\mathbf{g} - \mathbf{y}_0 - \mathbf{g}_d + \mathbf{y}_{0d})x$$

- Additive scaling rather than multiplicative.

$$\mathbf{f}(x) = \frac{\sum_i \mathbf{w}_i \Psi_i(x)}{\sum_i \Psi_i(x)}$$

$$\Psi_i(x) = \exp(-h_i(x - c_i)^2)$$

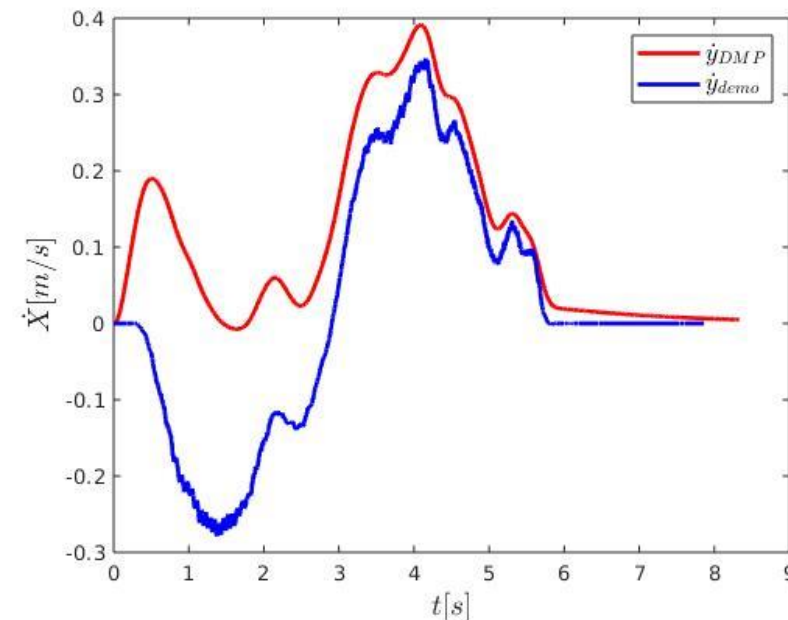
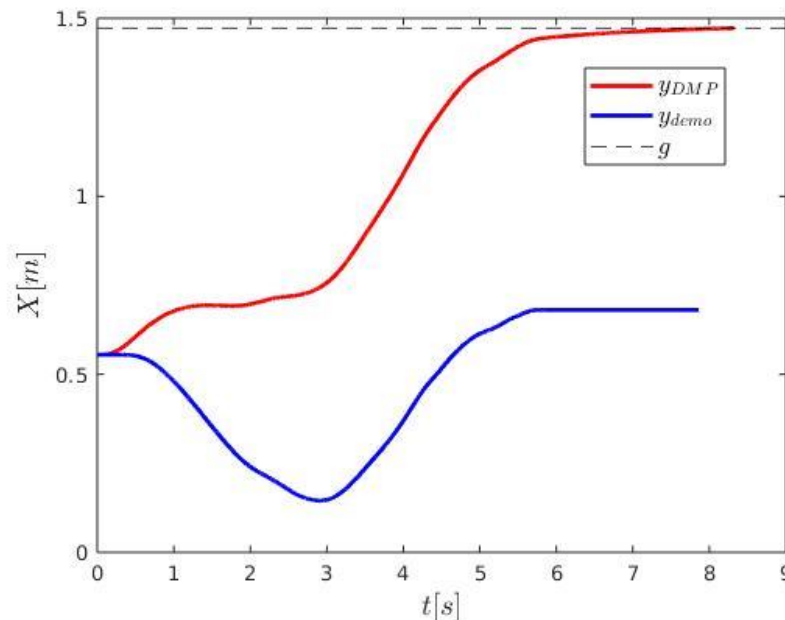
$$\tau \dot{x} = -a_x x$$

Biologically Inspired DMP Spatial Scaling

- Assume negligible training error. Closed loop tracking dynamics:

$$\tau^2(\ddot{\mathbf{y}} - \ddot{\mathbf{y}}_d) + \tau\alpha_z(\dot{\mathbf{y}} - \dot{\mathbf{y}}_d) + \alpha_z\beta_z(\mathbf{y} - \mathbf{y}_d) = \alpha_z\beta_z(\mathbf{g} - \mathbf{g}_d) - \alpha_z\beta_z\mathbf{s}_g$$

- Additive \mathbf{s}_g : works well for small variations, dominates large ones.



Proposed Formulation for Spatial Scaling

- Instead of scaling each coordinate independently, we scale based on the magnitude of the difference between initial and goal position.

$$\begin{aligned}\tau \dot{\mathbf{z}} &= a_z(\beta_z(\mathbf{g} - \mathbf{y}) - \mathbf{z}) + \mathbf{S}_g \mathbf{f}(x) \\ \tau \dot{\mathbf{y}} &= \mathbf{z}\end{aligned}$$

$$\mathbf{S}_g = s_g \mathbf{R}_g$$

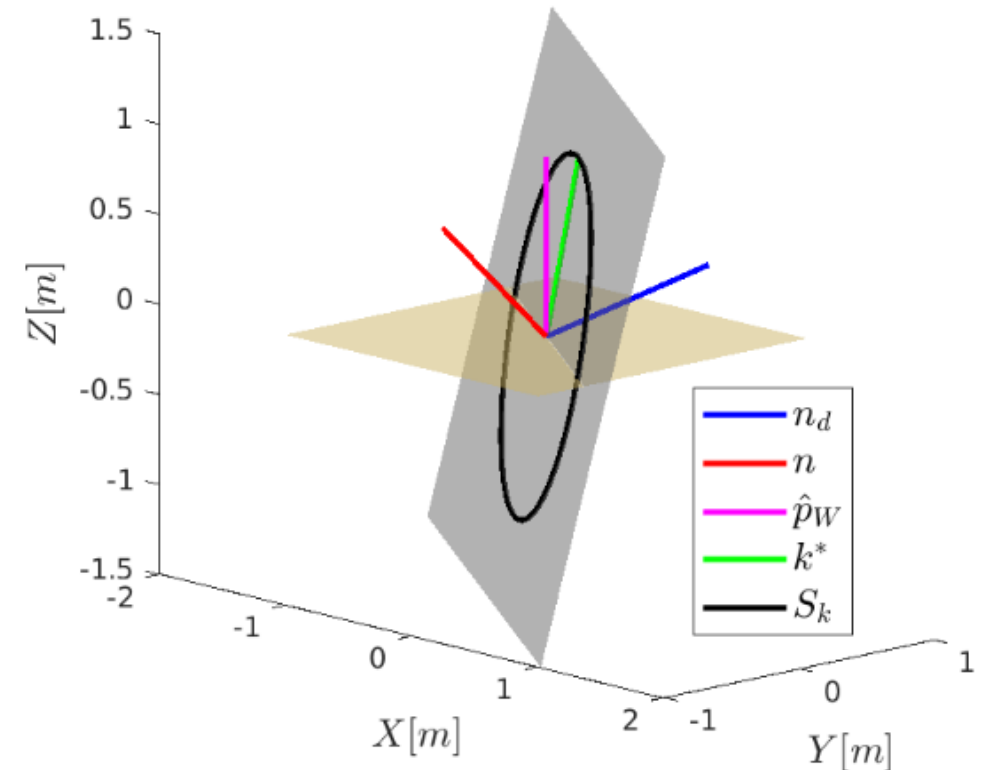
- The trajectory is also rotated according to rotation matrix \mathbf{R}_g related to rotation axis \mathbf{k} and angle θ .

$$s_g = \frac{\|\mathbf{g} - \mathbf{y}_0\|}{\|\mathbf{g}_d - \mathbf{y}_{0d}\|}$$

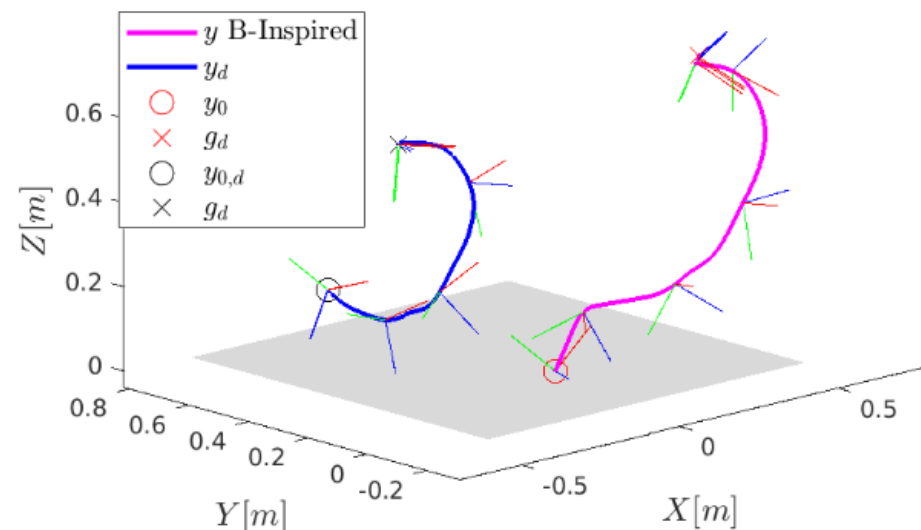
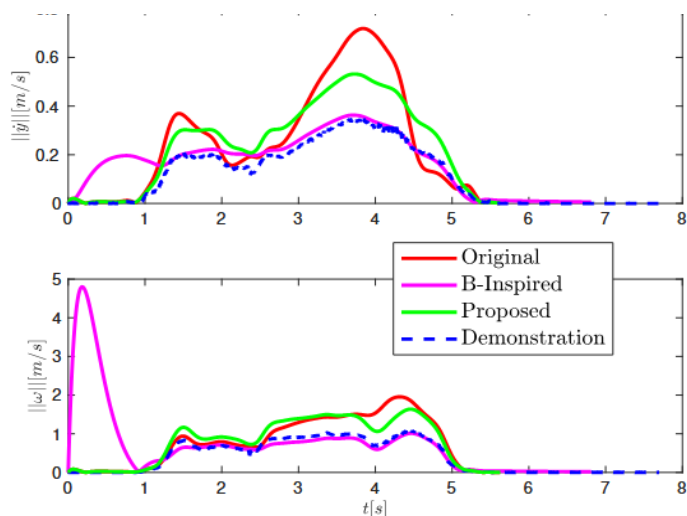
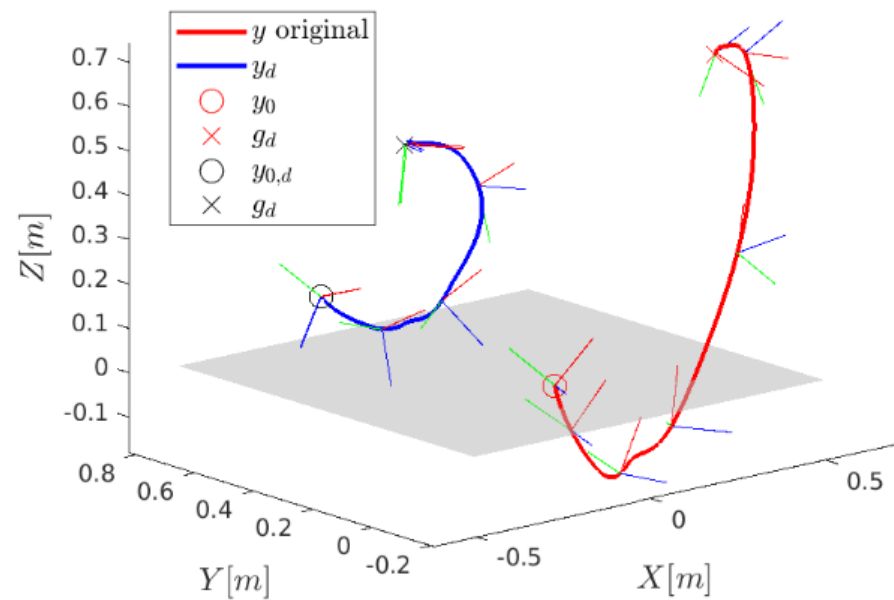
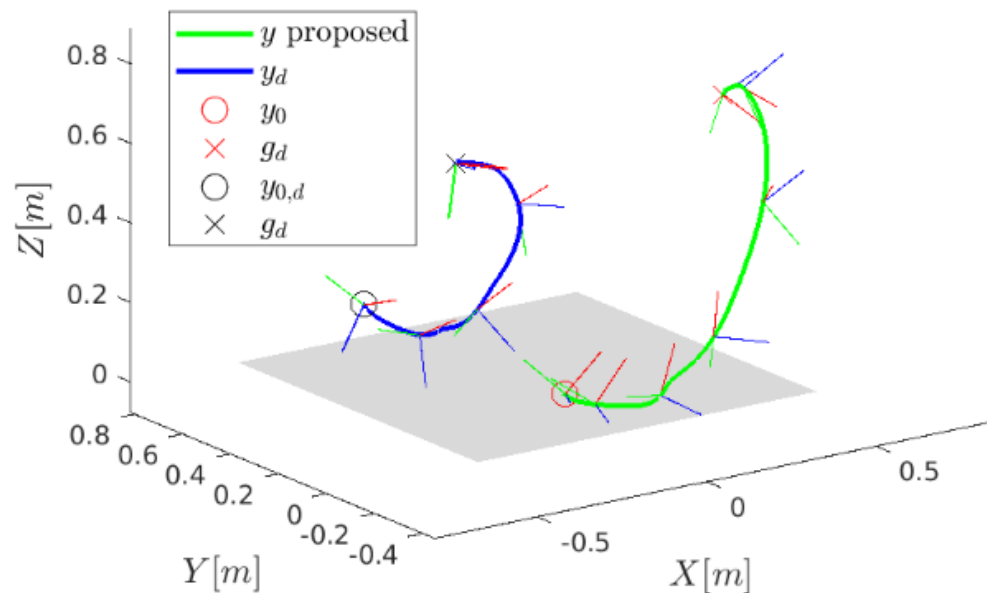
$$\mathbf{R}_g = \mathbf{I}_3 + \mathbf{S}(\mathbf{k}) \sin(\theta) + \mathbf{S}^2(\mathbf{k})(1 - \cos(\theta))$$

Rotation Axis Selection

- Let $\mathbf{n} = \frac{\mathbf{g} - \mathbf{y}_0}{\|\mathbf{g} - \mathbf{y}_0\|}$, $\mathbf{n}_d = \frac{\mathbf{g}_d - \mathbf{y}_{0d}}{\|\mathbf{g}_d - \mathbf{y}_{0d}\|}$, $\hat{\mathbf{n}} = \frac{\mathbf{n} - \mathbf{n}_d}{\|\mathbf{n} - \mathbf{n}_d\|}$
- Requirement: $\mathbf{n} = \mathbf{R}_g \mathbf{n}_d$
- $\mathcal{S}_k = \{\mathbf{k} \in \mathbb{R}^3 \mid \|\mathbf{k}\| = 1, \mathbf{k}^T \hat{\mathbf{n}} = 0\}$
- $\theta = \cos^{-1} \left(\frac{\mathbf{n}_d^T (\mathbf{I} - \mathbf{k} \mathbf{k}^T) \mathbf{n}}{\|(I - \mathbf{k} \mathbf{k}^T) \mathbf{n}_d\| \|(I - \mathbf{k} \mathbf{k}^T) \mathbf{n}\|} \right)$
- Free motion: $\mathbf{k} = \mathbf{n}_d \times \mathbf{n}$
- Tasks over a surface: $\mathbf{k} = \frac{(\mathbf{I} - \hat{\mathbf{n}} \hat{\mathbf{n}}^T) \mathbf{n}_s}{\|(\mathbf{I} - \hat{\mathbf{n}} \hat{\mathbf{n}}^T) \mathbf{n}_s\|}$



Simulations



Experiments

A novel DMP formulation for global and frame independent spatial scaling in the task space

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Compliant Impedance Control

- Robot system under Impedance control:

$$\mathbf{M}_d \begin{bmatrix} \ddot{\mathbf{e}}_P \\ \ddot{\mathbf{e}}_o \end{bmatrix} + \mathbf{D}_d \begin{bmatrix} \dot{\mathbf{e}}_P \\ \dot{\mathbf{e}}_o \end{bmatrix} + \mathbf{K}_d \begin{bmatrix} \mathbf{e}_p \\ \mathbf{e}_o \end{bmatrix} = \mathbf{f}_e + \mathbf{f}_{ext}$$

- $\mathbf{e}_p = \mathbf{p}_{robot} - \mathbf{y}, \mathbf{e}_o = 2 \log(\mathbf{Q}_{robot} * \bar{\mathbf{Q}})$
- Low stiffness implies compliant behavior when in contact with the environment exerting forces \mathbf{f}_{ext} .
- However, it also means low accuracy due to unmodelled dynamics and uncertainties \mathbf{f}_e .

Compliant and Accurate Motion

- DMP with a coupling term \mathbf{e}_p :

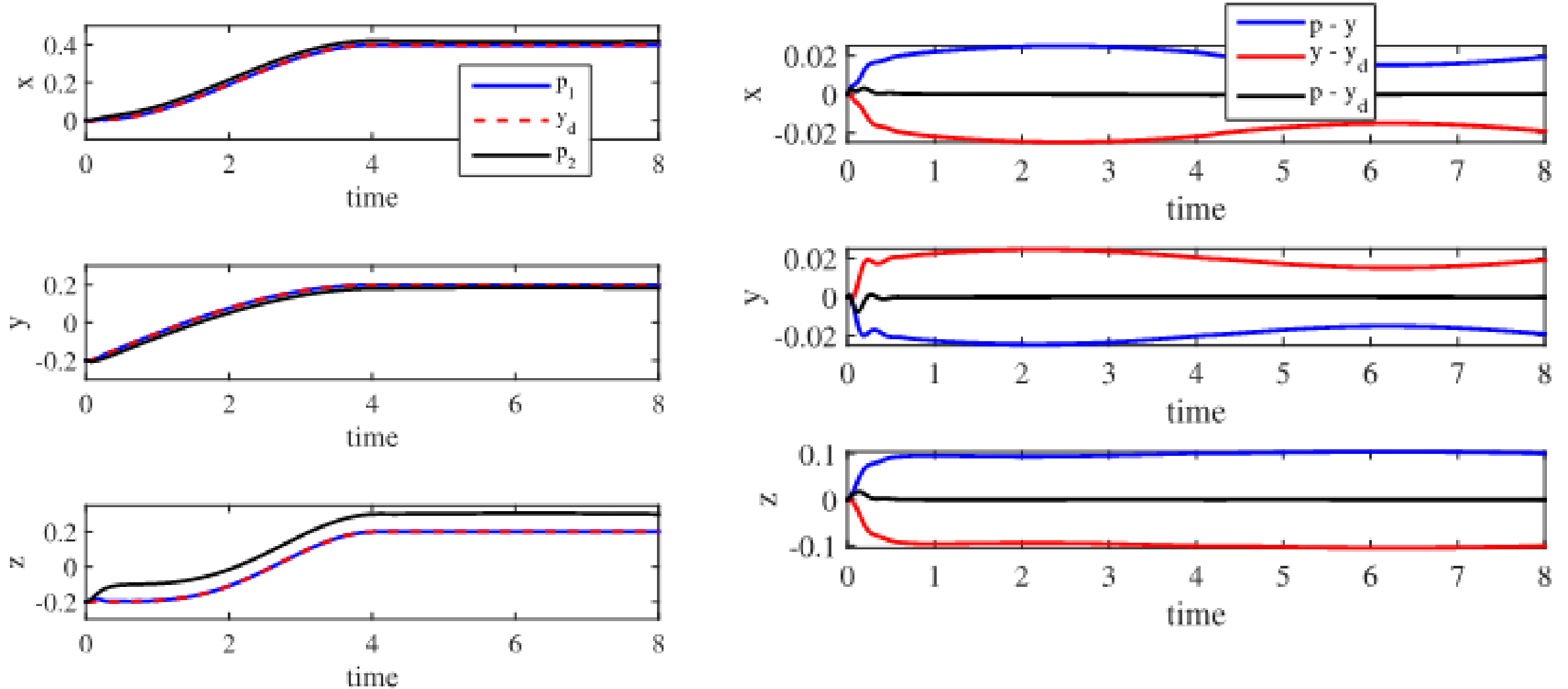
$$\begin{aligned}\tau \dot{\mathbf{z}} &= \alpha_z (\beta_z (\mathbf{g} - \mathbf{y}) - \mathbf{z}) + \mathbf{S}_g \mathbf{f}(\mathbf{x}) - \beta_z \alpha_z \mathbf{e}_p \\ \tau \dot{\mathbf{y}} &= \mathbf{z}\end{aligned}$$

- The coupling term generates a virtual reference trajectory that compensates for the errors due to the modelling uncertainties.

$$\tau = \tau_0 (1 + \exp(\alpha_{sig} \|\mathbf{e}_p\| - c_{sig}))$$

- Temporal scaling adaptation ensures that the system evolution stops when large errors are detected, which are attributed to collisions.

Simulation Results



Experiments

A control scheme with a novel DMP-robot coupling achieving compliance and tracking accuracy under unknown task dynamics and model uncertainties

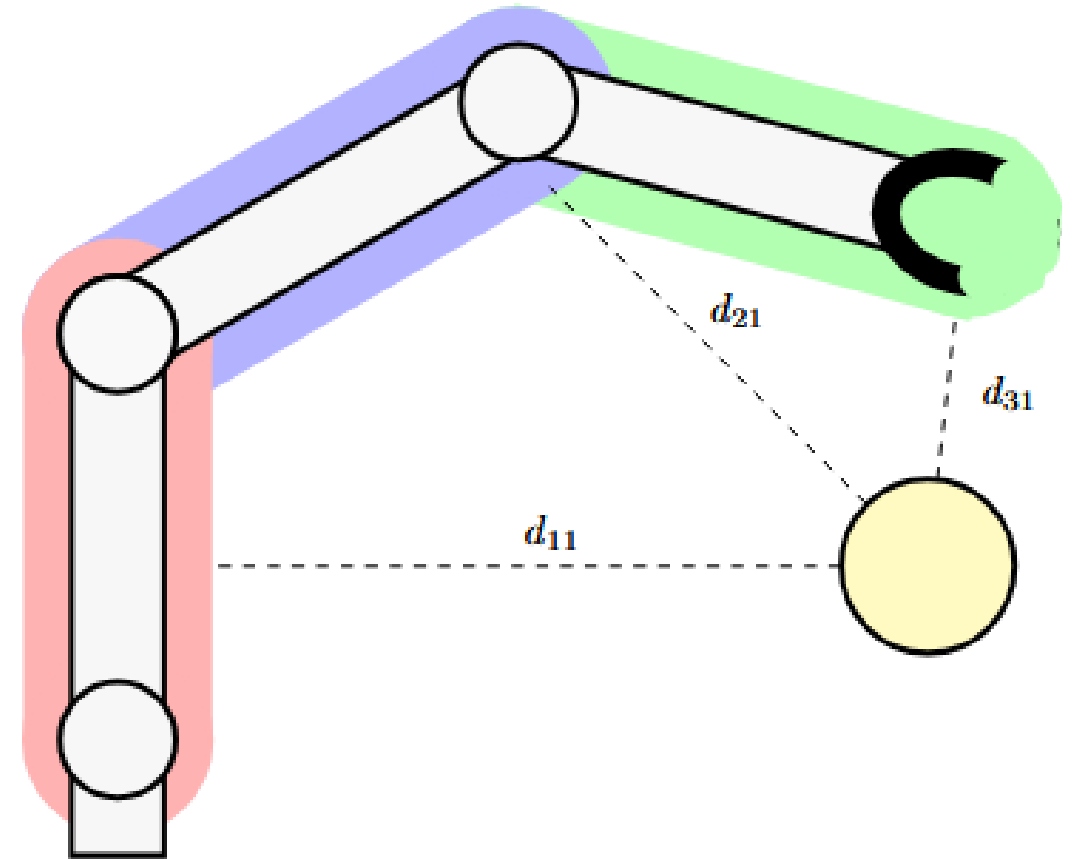
Konstantinos Vlachos and Zoe Doulgeri



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Full Body Obstacle Avoidance

- Body of the manipulator is covered by M capsules. N moving obstacles are in its workspace. Let d_{ij} be their distances.
- $m(t) = h(h_{w1}, h_{w2}, \dots, h_{wN})$
- $$h_{wi}(d_{i1}, d_{i2}, \dots, d_{in}) = \frac{\sum_{j=1}^n w_j}{\sum_{j=1}^n \frac{w_j}{d_{ij}}}$$
- $$\dot{q}_{obs} = -\kappa \frac{\ln(1+\xi)}{1+\xi} \frac{1}{\left\| \frac{\partial m(t)}{\partial q} \right\|^2} \left(\frac{\partial m(t)}{\partial q} \right)^T$$
- $$\xi = \frac{m(t) - \rho}{\rho}$$



Compliant Motion and Obstacle avoidance

$$\begin{aligned}\tau \dot{\mathbf{z}}_p &= \alpha_z (\beta_z (\mathbf{g} - \mathbf{y}) - \mathbf{z}_p) + \mathbf{S}_g \mathbf{f}_p(x) - \beta_z \alpha_z \mathbf{e}_p \\ \tau \dot{\mathbf{y}} &= \mathbf{z}_p + \mathbf{u}_{obs,p}\end{aligned}$$

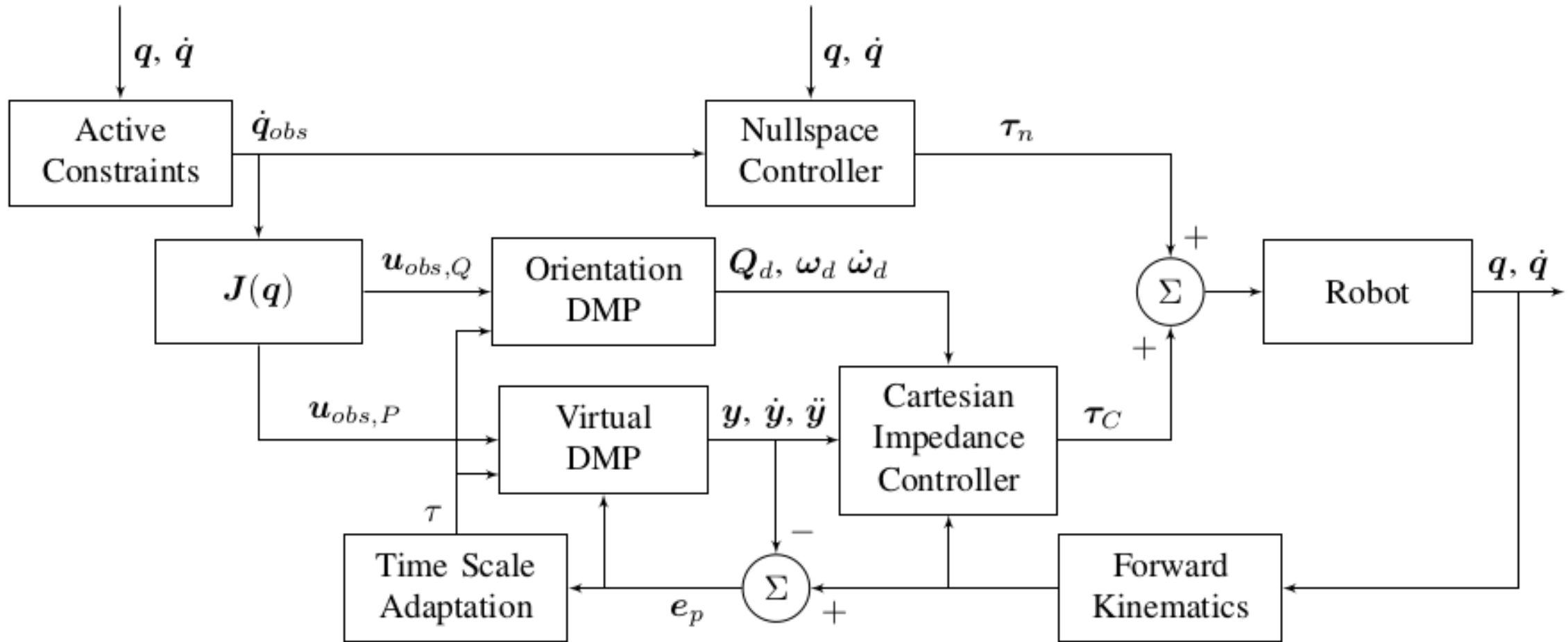
$$\begin{aligned}\tau \dot{\mathbf{z}}_Q &= -\alpha_z (\beta_z \mathbf{e}_Q + \mathbf{z}_Q) + \mathbf{S}(\mathbf{e}_{Q,0}) \mathbf{F}_Q(x) \\ \tau \dot{\mathbf{e}}_Q &= \mathbf{z}_Q - \mathbf{u}_{obs,Q}\end{aligned}$$

$$\tau = \tau_0 (1 + \exp(\alpha_{sig} \|\mathbf{e}_p\| - c_{sig}))$$

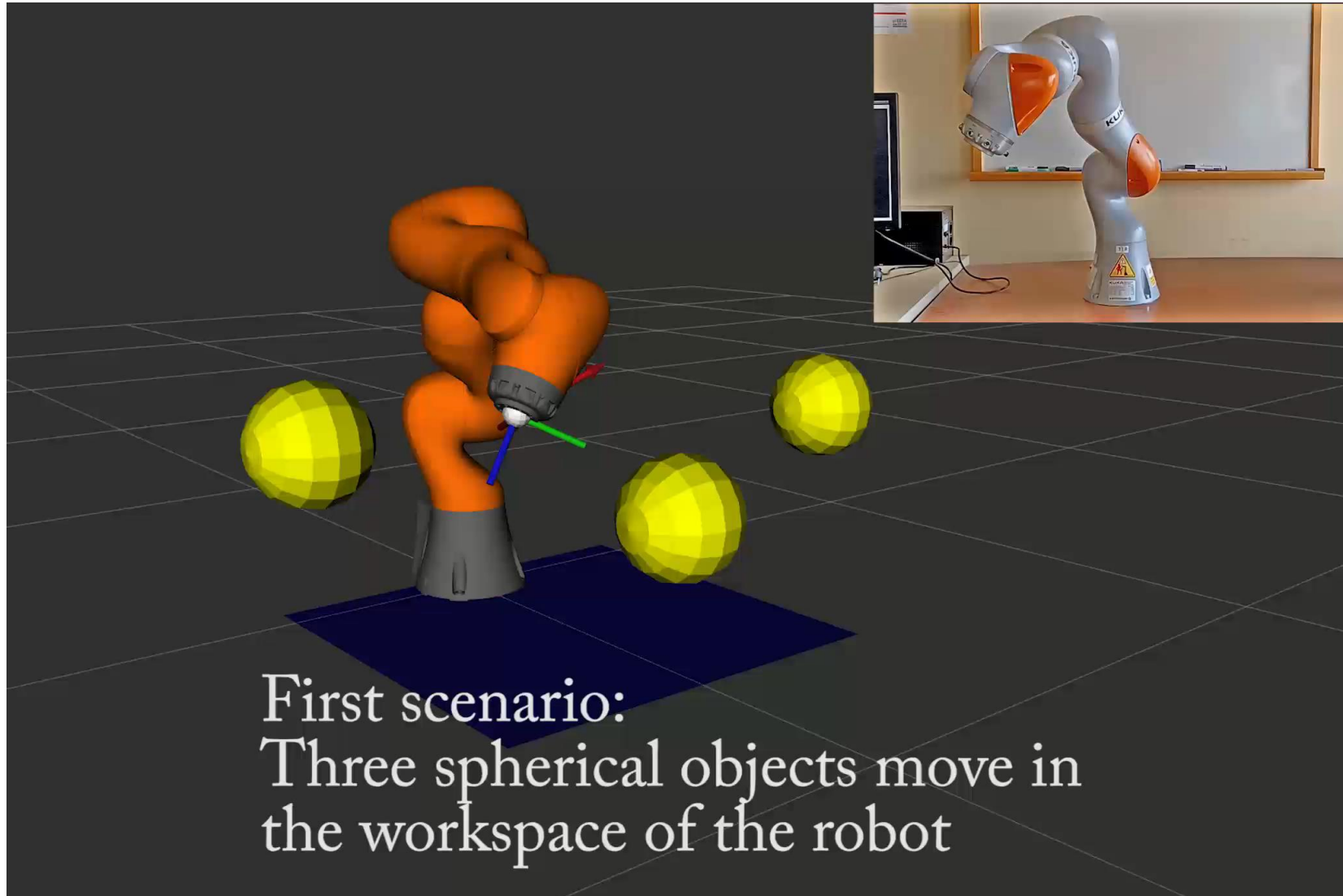
$$\mathbf{u}_{obs} \begin{bmatrix} \mathbf{u}_{obs,p} \\ \mathbf{u}_{obs,Q} \end{bmatrix} = \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}_{obs}$$

$$\boldsymbol{\tau}_n = -\mathbf{N}^T \mathbf{D}_N \mathbf{N} (\dot{\mathbf{q}} - \dot{\mathbf{q}}_{obs})$$

Proposed Architecture

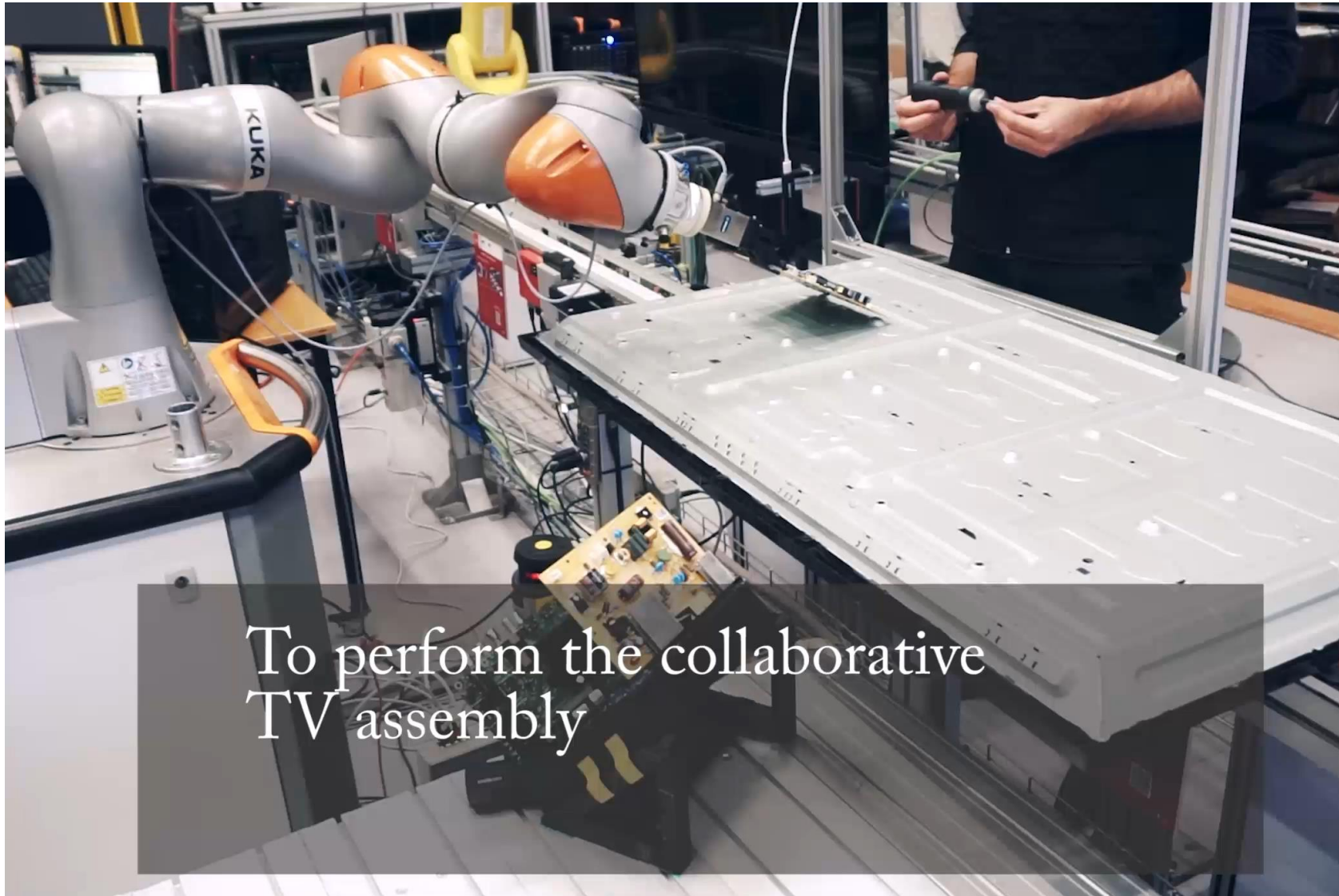


Experiments: Simulated Obstacles



First scenario:
Three spherical objects move in
the workspace of the robot

Experiments : Industrial Assembly Scenario



Conclusions

- Problems with the existing DMP formulation were showcased and a proposed formulation was presented.
- A DMP extension to moving goals was proposed.
- Problems with the spatial scaling of existing DMP were demonstrated and a new method was proposed.
- A way to include full body obstacle avoidance to compliant robotic systems was proposed.

Thank you for your
attention

Any Questions?