An Exploration of Non-Euclidean Gradient Descent: Muon and its Many Variants

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Bird's Eye View

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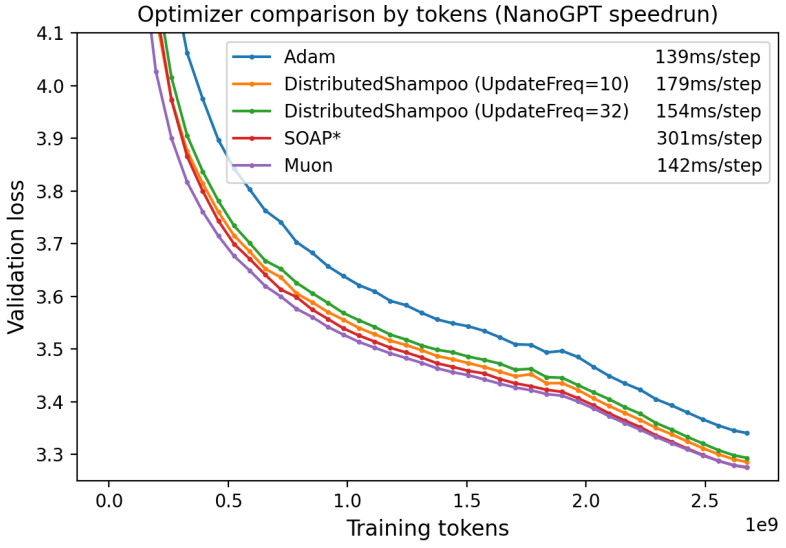
We emerged with:

- A clarified formulation of Muon as a type of steepest descent.
- Polyak-type stepsizes for Muon and its friends.
- A new variant of Muon that is significantly easier to tune.

First Look at Muon

What is Muon?

A neural network optimization algorithm (Jordan et al., 2024), quite efficient for training language models.



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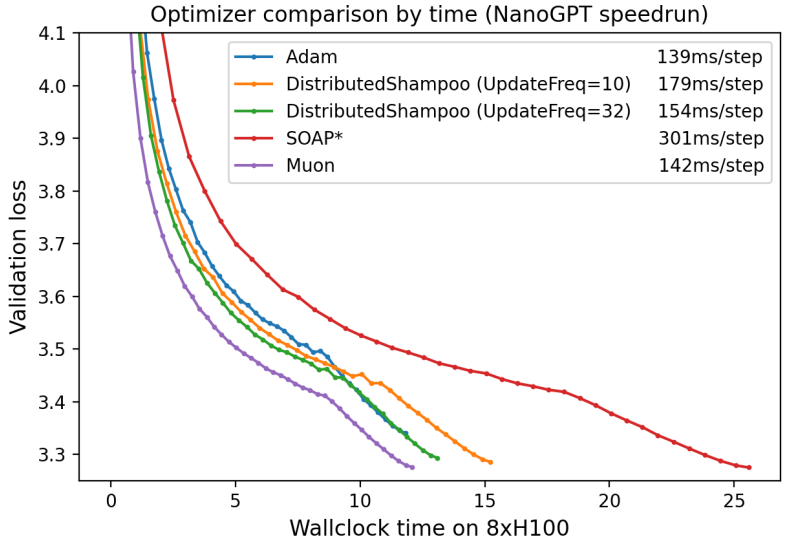

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Muon Definition

Algorithm Muon (for a single weight matrix $\boldsymbol{W}$)

Input: Step size η, momentum β

$\boldsymbol{M}_0 \leftarrow \boldsymbol{0}$
for $t = 0, 1, \ldots, T-1$ **do**
  $\boldsymbol{G}_t \leftarrow$ Stochastic gradient at $\boldsymbol{W}_t$
  $\boldsymbol{M}_t \leftarrow \beta \boldsymbol{M}_{t-1} + (1-\beta)\boldsymbol{G}_t$
  $\boldsymbol{U}, \boldsymbol{\Sigma}, \boldsymbol{V} \leftarrow \mathrm{SVD}(\boldsymbol{M}_t)$
  $\boldsymbol{W}_{t+1} \leftarrow \boldsymbol{W}_t - \eta \boldsymbol{U}\boldsymbol{V}^\top$
end for

(Polar factor is approximated by evaluating a certain polynomial at $\boldsymbol{M}_t$ (Jordan et al., 2024; Amsel et al., 2025))

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Can be interpreted as steepest descent w.r.t. the spectral norm:

$$\begin{aligned}\boldsymbol{W}_{t+1} &= \underset{\|\boldsymbol{W}-\boldsymbol{W}_t\|_2 \le \eta}{\arg\min} \{f(\boldsymbol{W}_t) + \langle \boldsymbol{M}_t, \boldsymbol{W} - \boldsymbol{W}_t \rangle\} \\ &= \boldsymbol{W}_t - \eta \underset{\|\Delta\|_2 \le 1}{\arg\max} \langle \boldsymbol{M}_t, \Delta \rangle.\end{aligned}$$

(using momentum as estimate of full batch gradient $\nabla f(\boldsymbol{W}_t)$)

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Algorithm 1 MuonAdam: where $\boldsymbol{W}^1, \ldots, \boldsymbol{W}^L$ are the weight matrices, and $\boldsymbol{\theta}$ are all other parameters flattened into a vector.

Inputs: $\boldsymbol{W}_0^1, \ldots, \boldsymbol{W}_0^L, \boldsymbol{\theta}_0$, learning rates η_b, η_m, EMA parameters β, β_1, β_2

for $t = 0, 1, \ldots, T-1$ **do**
  $(\boldsymbol{G}_t^1, \ldots, \boldsymbol{G}_t^L, \boldsymbol{g}_t^\theta) \leftarrow \text{backward}(\boldsymbol{W}_t^1, \ldots \boldsymbol{W}_t^L, \boldsymbol{\theta}_t)$
  for $\ell = 1, \ldots, L$ **do**
    $\boldsymbol{M}_t^\ell = \beta \boldsymbol{M}_{t-1}^\ell + (1-\beta)\boldsymbol{G}_t^\ell$
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end for

Things that bother me compared to "steepest descent" perspective:

1. Why normalize?
2. Jump from one matrix to many matrices.
3. Running side-by-side with Adam (with a different learning rate!)

What We'll Do Today

1. Incorporate the three nuisances into steepest descent (and evaluate alternatives).
2. Use this clarified perspective to port Polyak-type stepsizes.
3. Demonstrate easier tuning for language modeling and image classification.

The Many Variants

Normalize or Not?

Remember our steepest descent formulation:

$$\begin{aligned} \boldsymbol{w}_{t+1} &= \underset{\|\boldsymbol{w}-\boldsymbol{w}_t\|\leq\eta}{\arg\min} \{f(\boldsymbol{w}_t) + \langle \boldsymbol{m}_t, \boldsymbol{w}-\boldsymbol{w}_t\rangle\} \\ &= \boldsymbol{w}_t - \eta \underset{\|\Delta\|\leq 1}{\arg\max} \langle \boldsymbol{m}_t, \Delta\rangle. \end{aligned}$$

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$\|\boldsymbol{m}_t\|_* = \max_{\|\Delta\|\leq 1}\langle \boldsymbol{m}_t, \Delta\rangle$ is easy to compute if you know $\arg\max_{\|\Delta\|\leq 1}\langle \boldsymbol{m}_t, \Delta\rangle$.

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Same formulation with full domain $\mathcal{W}$:

$$\boldsymbol{W}_{t+1} = \boldsymbol{W}_t - \eta \arg\max_{\|\boldsymbol{\Delta}\| \leq 1} \langle \boldsymbol{M}_t, \boldsymbol{\Delta} \rangle.$$

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Choice #2: **A norm on the product space $\mathcal{W}$.**

Outer Norm Examples

Some natural choices:

$$\|\boldsymbol{W}\| = \max_{\ell \in [L]} \|\boldsymbol{W}\|_2, \quad \|\boldsymbol{W}\| = \sqrt{\sum_{\ell=1}^{L} \|\boldsymbol{W}\|_2^2}.$$

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Leads to different update scaling across layers:

$\|\cdot\|_\infty$, Constrained:	$\boldsymbol{W}_{t+1}^\ell = \boldsymbol{W}_t^\ell - \eta \mathsf{polar}(\boldsymbol{M}_t^\ell)$
$\|\cdot\|_\infty$, Regularized:	$\boldsymbol{W}_{t+1}^\ell = \boldsymbol{W}_t^\ell - \eta \left(\sum_{j=1}^{L} \|\boldsymbol{M}_t^j\|_{\mathsf{nuc}} \right) \mathsf{polar}(\boldsymbol{M}_t^\ell)$
$\|\cdot\|_2$, Constrained:	$\boldsymbol{W}_{t+1}^\ell = \boldsymbol{W}_t^\ell - \eta \frac{\|\boldsymbol{M}_t^\ell\|_{\mathsf{nuc}}}{\sqrt{\sum_{j=1}^{L} \|\boldsymbol{M}_t^j\|_{\mathsf{nuc}}^2}} \mathsf{polar}(\boldsymbol{M}_t^\ell)$
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Design Choices	Layer ℓ Update	Separable
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If non-separable, need two passes over layers and one of the following:

- Store polar factors across loop steps (extra memory).
- Recompute polar factors during second pass (extra time).

A Simple Trick: Stale Dual Norms

Approximation: store nuclear norms from previous step, use this to compute step size.

$$\boldsymbol{W}_{t+1}^{\ell} = \boldsymbol{W}_{t}^{\ell} - \eta \left(\sum_{j=1}^{L} \|\boldsymbol{M}_{t}^{j}\|_{\mathsf{nuc}} \right) \mathsf{polar}(\boldsymbol{M}_{t}^{\ell})$$

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- Similar time/memory overhead as original Muon.
- Very small change in performance compared to non-stale version.

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MuonAdam (Algorithm 1) is constrained steepest descent with stepsize η_m w.r.t.

$$\|\boldsymbol{W}\|_{\mathsf{muon}} := \max\left(\max_{1\le\ell\le L} \|\boldsymbol{W}^\ell\|_{2\to 2}, \frac{\eta_m}{\eta_b}\|\boldsymbol{\theta}\|_{\mathsf{ada}\infty}\right)$$

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Design Choice #3: **Choose a norm for non-matrix parameters $\boldsymbol{\theta}$.**

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Optimizer design choices:

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Can return to our original motivation: extending GD techniques (particularly Momo (Schaipp et al., 2024)) for Muon and friends.

Model Truncation

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At each step, locally minimize a truncated linear approximation:

$$\begin{aligned}\boldsymbol{w}_{t+1} &= \operatorname*{arg\,min}_{\|\boldsymbol{w}-\boldsymbol{w}_t\|\leq\eta} \{\max\left(f(\boldsymbol{w}_t) + \langle \boldsymbol{m}_t, \boldsymbol{w}-\boldsymbol{w}_t\rangle, f_*\right)\} \\ &= \boldsymbol{w}_t - \min\left(\eta, \frac{f(\boldsymbol{w}_t)-f_*}{\|\boldsymbol{m}_t\|_*}\right) \operatorname*{arg\,max}_{\|\Delta\|\leq 1} \langle \boldsymbol{m}_t, \Delta\rangle\end{aligned}$$

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Can't just drop $f(\boldsymbol{w}_t)$ anymore. But we can estimate it similarly as momentum estimates gradient.

Adaptive Stepsizes with Model Truncation

With momentum, $\boldsymbol{m}_t$ is a convex combination of previous stochastic gradients:

$$\boldsymbol{m}_t = \sum_{i=1}^{t} \rho_{t,i} \boldsymbol{g}_i, \quad \sum_{i=1}^{t} \rho_{t,i} = 1.$$

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$$\boldsymbol{m}_t = \sum_{i=1}^{t} \rho_{t,i} \boldsymbol{g}_i, \quad \sum_{i=1}^{t} \rho_{t,i} = 1.$$

Can similarly estimate $f(\boldsymbol{w}_t)$ as a convex combination of first-order approximations:

$$f(\boldsymbol{w}_t) \approx \tilde{f}_t := \sum_{i=0}^{t} \rho_{t,i} \left(f_i(\boldsymbol{w}_i) + \langle \boldsymbol{g}_i, \boldsymbol{w}_t - \boldsymbol{w}_i \rangle \right).$$

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Resulting update:

$$\tilde{F}_t = \beta \tilde{F}_{t-1} + (1-\beta)(f_t(\boldsymbol{w}_t) - \langle \boldsymbol{g}_t, \boldsymbol{w}_t \rangle), \quad \tilde{f}_t = \tilde{F}_t + \langle \boldsymbol{m}_t, \boldsymbol{w}_t \rangle$$

$$\boldsymbol{w}_{t+1} = \boldsymbol{w}_t - \min\left(\eta, \frac{\tilde{f}_t - f_*}{\|\boldsymbol{m}_t\|_*}\right) \arg\max_{\|\Delta\| \leq 1} \langle \boldsymbol{m}_t, \Delta \rangle.$$

(originally designed for SGD/Adam, but extends naturally to any norm)

Momo for Muon

Now we have Momo for any steepest descent!

$$\text{Constrained :} \quad \boldsymbol{w}_{t+1} = \boldsymbol{w}_t - \min\left(\eta, \frac{\tilde{f}_t - f_*}{\|\boldsymbol{m}_t\|_*}\right) \arg\max_{\|\Delta\| \leq 1} \langle \boldsymbol{m}_t, \Delta \rangle$$

$$\text{Regularized :} \quad \boldsymbol{w}_{t+1} = \boldsymbol{w}_t - \min\left(\eta, \frac{\tilde{f}_t - f_*}{\|\boldsymbol{m}_t\|_*^2}\right) \|\boldsymbol{m}_t\|_* \arg\max_{\|\Delta\| \leq 1} \langle \boldsymbol{m}_t, \Delta \rangle.$$

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Using that MuonAdam is constrained steepest descent w.r.t. $\|\cdot\|_{\text{muon}}$, we finally have MuonAdam-Momo:

$$\begin{aligned}
d_t &= \textstyle\sum_{\ell=1}^{L} \|\boldsymbol{M}_t^\ell\|_{\text{nuc}} + \frac{\eta_b}{\eta_m} \left\| \frac{|\boldsymbol{m}_t|}{\sqrt{\boldsymbol{v}_t}+\epsilon} \odot \boldsymbol{m}_t^\theta \right\|_1 \\
\eta_m' &= \min\left\{\eta_m, \tfrac{\tilde{f}_t - f_*}{d_t}\right\}, \;\; \eta_b' = \min\left\{\eta_b, \tfrac{\eta_b}{\eta_m} \tfrac{\tilde{f}_t - f_*}{d_t}\right\} \\
\boldsymbol{W}_{t+1}^\ell &= \boldsymbol{W}_t^\ell - \eta_m' \,\text{polar}(\boldsymbol{M}_t^\ell) \\
\boldsymbol{\theta}_{t+1} &= \boldsymbol{\theta}_t - \eta_b' \frac{\boldsymbol{m}_t^\theta}{\sqrt{\boldsymbol{v}_t^\theta}+\epsilon}.
\end{aligned}$$

A New Algorithm

A new norm falls from the sky:

$$\|\boldsymbol{W}\|_{\mathsf{MM}} = \sqrt{\left(\max_{\ell \in [L]} \|\boldsymbol{W}^{\ell}\|_2^2\right)^2 + \frac{\eta_m}{\eta_b} \left\| \sqrt{\sqrt{\boldsymbol{v}_t} + \epsilon} \odot \boldsymbol{\theta} \right\|_2^2}.$$

(we originally liked this one for the aesthetics of the resulting steepest descent)

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(we originally liked this one for the aesthetics of the resulting steepest descent)

MuonMax-Momo (regularized steepest descent w.r.t. $\|\cdot\|_{\mathsf{MM}}$, plus Momo):

$$\begin{aligned}
d_t &= \sqrt{\Big(\textstyle\sum_{\ell=1}^L \|\boldsymbol{M}_t^\ell\|_{\mathsf{nuc}}\Big)^2 + \frac{\eta_b}{\eta_m}\Big\|\frac{\boldsymbol{m}_t^\theta}{\sqrt{\sqrt{\boldsymbol{v}_t^\theta}+\epsilon}}\Big\|_2^2}\\
\eta_m' &= \min\left\{\eta_m, \tfrac{\tilde{f}_t - f_*}{d_t^2}\right\},\ \eta_b' = \min\left\{\eta_b, \tfrac{\eta_b}{\eta_m}\tfrac{\tilde{f}_t - f_*}{d_t^2}\right\}\\
\boldsymbol{W}_{t+1}^\ell &= \boldsymbol{W}_t^\ell - \eta_m'\Big(\textstyle\sum_{\ell=1}^L \|\boldsymbol{M}_t^\ell\|_{\mathsf{nuc}}\Big)\mathsf{polar}(\boldsymbol{M}_t^\ell)\\
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\end{aligned}$$

Experiments

Testing Many Variations

Three design choices in our framework lead to many variations, all based on the same principles. Which ones work well for deep learning?

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Training GPT-2 (124M params) for language modeling with FineWeb data (1B tokens):

(SD type, Outer Norm, Backup Norm)	Muon LR	Other LR	Final Loss	Name
(Regularized, $\|\cdot\|_\infty$, $\|\cdot\|_\infty$)	1e-3	1e-5	3.783	-
(Constrained, $\|\cdot\|_\infty$, $\|\cdot\|_\infty$)	1e-2	1e-3	3.599	Scion
(Regularized, $\|\cdot\|_2$, $\|\cdot\|_\infty$)	1e-1	1e-6	4.179	-
(Constrained, $\|\cdot\|_2$, $\|\cdot\|_\infty$)	1e-1	1e-2	3.712	-
(Regularized, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_\infty$)	1e-3	1e-5	3.826	-
(Constrained, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_\infty$)	1e-2	1e-3	3.610	-
(Regularized, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada}\infty}$)	1e-3	1e-5	3.859	-
(Constrained, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada}\infty}$)	1e-2	1e-3	3.604	Muon
(Regularized, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada}\infty}$)	1e-1	1e-4	4.229	-
(Constrained, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada}\infty}$)	1e-1	1e-2	3.748	-
(Regularized, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada}\infty}$)	1e-3	1e-4	3.917	-
(Constrained, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada}\infty}$)	1e-2	1e-2	3.628	-
(Regularized, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada2}}$)	1e-3	1e-4	3.757	-
(Constrained, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada2}}$)	1e-2	1e-3	3.701	-
(Regularized, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada2}}$)	1e-1	1e-3	4.049	PolarGrad
(Constrained, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada2}}$)	1e-1	1e-2	3.664	-
(Regularized, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada2}}$)	1e-3	1e-3	3.791	MuonMax
(Constrained, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada2}}$)	1e-2	1e-2	3.585	-

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Regularized variants lag behind. Aligns with convention: need normalization or clipping.

Testing Many Variations (w/ Momo)

Same setup, using Momo:

(SD type, Outer Norm, Backup Norm)	Muon LR	Other LR	Final Loss	Name
(Regularized, $\|\cdot\|_\infty$, $\|\cdot\|_\infty$)	1e-2	1e-4	3.627	-
(Constrained, $\|\cdot\|_\infty$, $\|\cdot\|_\infty$)	1e-2	1e-3	3.592	Scion-Momo
(Regularized, $\|\cdot\|_2$, $\|\cdot\|_\infty$)	1	1e-5	3.728	-
(Constrained, $\|\cdot\|_2$, $\|\cdot\|_\infty$)	1e-1	1e-2	3.843	-
(Regularized, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_\infty$)	1e-2	1e-4	3.628	-
(Constrained, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_\infty$)	3e-2	3e-3	3.604	-
(Regularized, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada}\infty}$)	3e-2	3e-4	3.578	-
(Constrained, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada}\infty}$)	3e-2	3e-3	3.551	Muon-Momo
(Regularized, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada}\infty}$)	1	1e-3	3.719	-
(Constrained, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada}\infty}$)	1e-1	1e-2	3.737	-
(Regularized, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada}\infty}$)	3e-2	3e-3	3.584	-
(Constrained, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada}\infty}$)	3e-2	3e-2	3.607	-
(Regularized, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada2}}$)	3e-3	3e-4	3.662	-
(Constrained, $\|\cdot\|_\infty$, $\|\cdot\|_{\text{ada2}}$)	1e-2	1e-3	3.701	-
(Regularized, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada2}}$)	3	3e-2	3.613	PolarGrad-Momo
(Constrained, $\|\cdot\|_2$, $\|\cdot\|_{\text{ada2}}$)	3e-1	3e-2	3.602	-
(Regularized, $\|\cdot\|_{\text{hyb}}$, $\|\cdot\|_{\text{ada2}}$)	1e-2	1e-2	3.576	MuonMax-Momo
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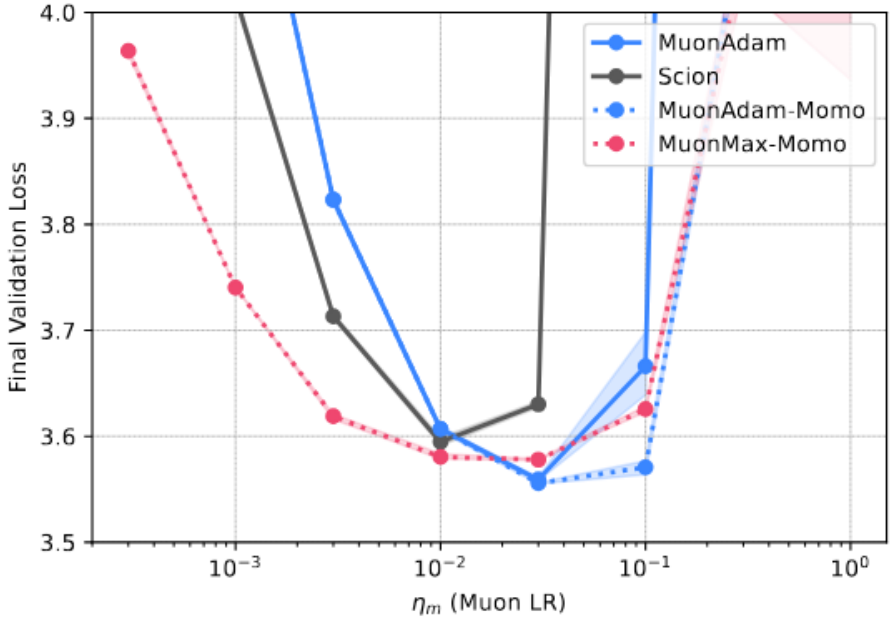
Momo greatly helps regularized variants.

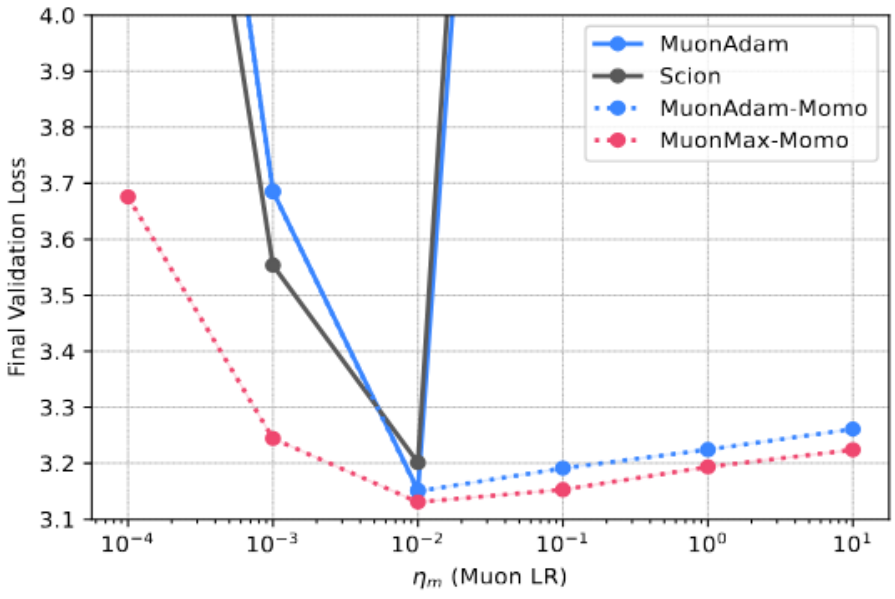
Muon-Momo achieves lowest loss (small margin).

Tuning Sensitivity

FineWeb1B, GPT2-Small



SlimPajama1B, GPT2-Large



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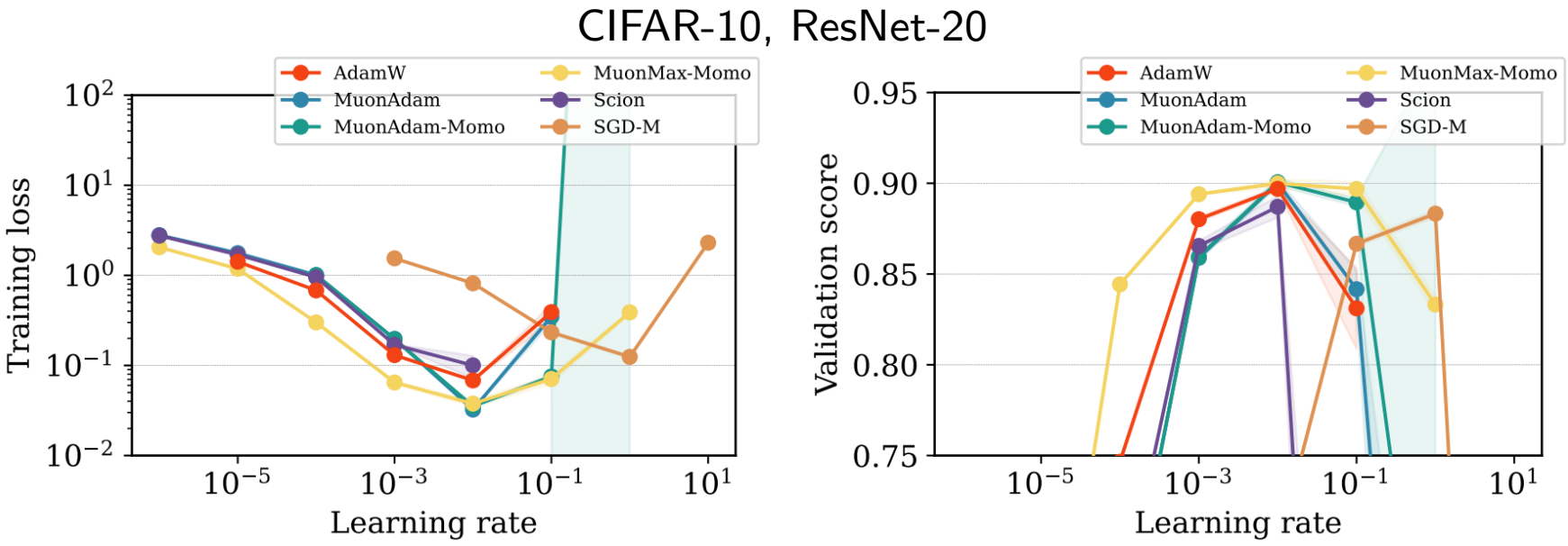

Probing Practical Behavior

Wall-clock time (GPT2-Small, FineWeb1B)

	MuonAdam	MuonMax	MuonAdam-Momo	Scion-Momo	MuonMax-Momo
Original	3.604 (1×)	3.791 (1.09×)	3.551 (1.10×)	3.592 (1.08×)	3.576 (1.11×)
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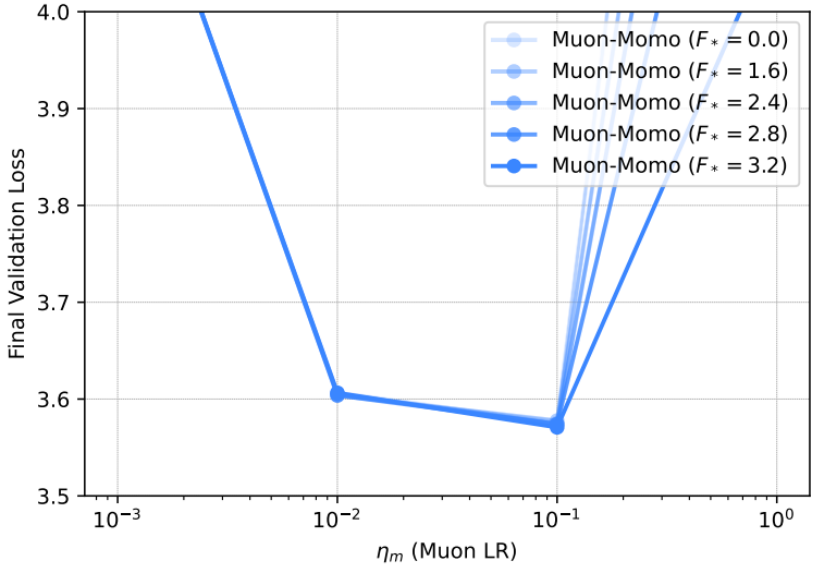
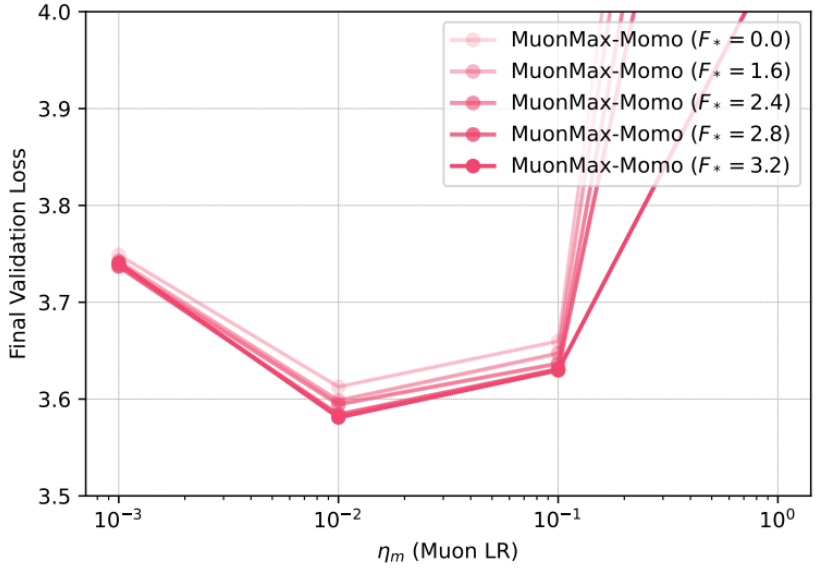
Sensitivity to choice of f_* (GPT2-Small, FineWeb1B):





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We formalized MuonAdam as a steepest descent on the space of all neural network parameters.

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Future directions:

- Why do some of our variations work better than others?
- Why does Muon work better than SGD and Adam???

Thanks!

References

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Jordan, K., Jin, Y., Boza, V., You, J., Cesista, F., Newhouse, L., and Bernstein, J. (2024). Muon: An optimizer for hidden layers in neural networks.

Schaipp, F., Ohana, R., Eickenberg, M., Defazio, A., and Gower, R. M. (2024). Momo: Momentum models for adaptive learning rates. In *International Conference on Machine Learning*, pages 43542–43570. PMLR.